Fractal geometry

Fractals are basically never-ending patterns which are self-similar in all scales - it means that we can take a piece of a fractal and it looks the same as the whole shape. Their name comes from the fact that they don’t have whole number dimensions, instead they have fractional dimensions They are created by repeating the same process over and over again.

1. Fractals in nature

Fractals are very frequent in nature and can be found between the familiar dimensions. Some phenomena that possess fractal features are coastlines, blood vessels, mountain ranges and snowflakes. They all seem quite random at first, but, as all fractals, have an underlying pattern which determines what they look like.

2. Chaos Theory

Fractals are strictly connected with the theory of Chaos. It studies the unpredicted events and teaches how to cope with them. It deals with non-linear things which are normally impossible to predict or control, such as turbulence or the stock market. It states that within the most unpredictable complex systems, there are some certain features, which let us deal with such systems, such as self-similarity, self-organization, feedback loops, interconnectedness. Fractals can be used to show the images of dynamic systems (chaotic systems, pictures of Chaos) when they are driven by recursion (applied within their own definition).

3. Techniques of generation of fractals

One of probably the most used methods for generating fractals is Iterated Function Systems (IFS) which uses fixed geometric replacement rules. It's used for example while generating the Koch Curve or Sierpinski Triangle which are both discussed in more detail below. Another method is strange attractors which use the solutions of a system of initial value differential or difference equations which exhibit chaos - this is very complicated and used in multifractal images or the logistic map. Next method are L-systems which use string rewriting and are helpful when it comes to coastline calculations. Some of the other types are random fractals and escape-time fractals.

4. Calculating fractal dimensions - Koch Curve

Koch Curve (Koch Snowflake) can be constructed starting with a straight line of length equal to 1 unit, called initiator. Then the line is divided into three equal segments, each of which has a length of $\frac{1}{3}$. The middle segment is then removed and replaced by two lines of length equal to $\frac{1}{3}$ pointing upwards (which would create an equilateral triangle with the just removed line as a base). Finally, there are 4 lines each of length equal to $\frac{1}{3}$. This construction is called the generator because the same rule is used then to generate new forms. Each of the 4 lines are then replaced with the generator, which results in 16 lines. The process can be repeated infinitely. We can derive a formula for this which is $N=r^D$, where N is the number of segments, r the reciprocal of the value by which we reduce the figure, D the dimension which we are looking for. In the case of Koch Curve, N=4 and r=3. When we take the logarithms of both sides, we obtain $\log(4)=\log(3^D)$, from which after calculations, we can see that the fractal dimension for Koch Curve, D=1.26.

5. **Calculating the fractal dimension - Sierpinski Triangle**

The Sierpinski Triangle can be constructed starting with an equilateral triangle. Then we have to find the mid-points of all the three sides and remove the inner triangle resulting from joining these three points. Then the process has to be repeated removing the inner triangle in other triangles and so on. From this we know that for Sierpinski Triangle, N=3 and r=2, so $\log(3)=\log(2^D)$ and from that we obtain that D=1.58.

6. **Coastline paradox**

Coastlines show fractal properties because the closer we look at them, there are more details visible. How long a coast is depends on how closely you look at it or how long measuring tool you use. It is impossible to measure the exact value of the coastline - if you measure it using a ruler on a map, you'll get a certain value for the perimeter, if you use another ruler, the value will be (slightly) different and if you walk around the coast, the value will be different as well. Although the last one will be the most accurate, it will still be far from the real value. In order to obtain a value closest to the real value, you would have to take into account the length around every boulder, rock and grain of sand, which is also a fractal on the microscopic level. This means that the perimeter of a coastline increases indefinitely - it approaches infinity.

As it can be seen, fractals are found everywhere around us. They have amazing properties which differ from what we usually learn about a lot and therefore they are a very interesting topic.