Geometry and symmetry

As you and I already know that mathematics is the amazing tool to understand any natural phenomena to abstract content. Mathematics is the language of the universe. When I read first time the physics book in high school a quote was written there and it was “Physics is the king of the science while mathematics is the queen”.

Now when it comes to mathematics for me, I like all the parts of mathematics. I like number theory, algebra, geometry, topology, calculus, etc. Now I’m interested in number theory more but I have chosen here geometry and symmetry because I think geometry is the only one branch of mathematics in which you can literally understand distinct types of problems of different branches of mathematics by making geometric interpretation of the problems. It is so easy to understand any complex topic if we interpret that topic geometrically and the reason once you interpret that complex topic in the language of geometry by drawing or making geometric sketch of it. Let me give you an example

Now differentiation of function $f(x)$ at $x=a$ is denoted by $f'(a)$.

$f'(a)$ represents the instantaneous rate of change of $f(x)$ at $x=a$.

Now at the first look it looks like abstract for all those who are not familiar with it but if you interpret it geometrically then it becomes easy to grasp. Let’s interpret it geometrically and you will see how it becomes completely understandable,

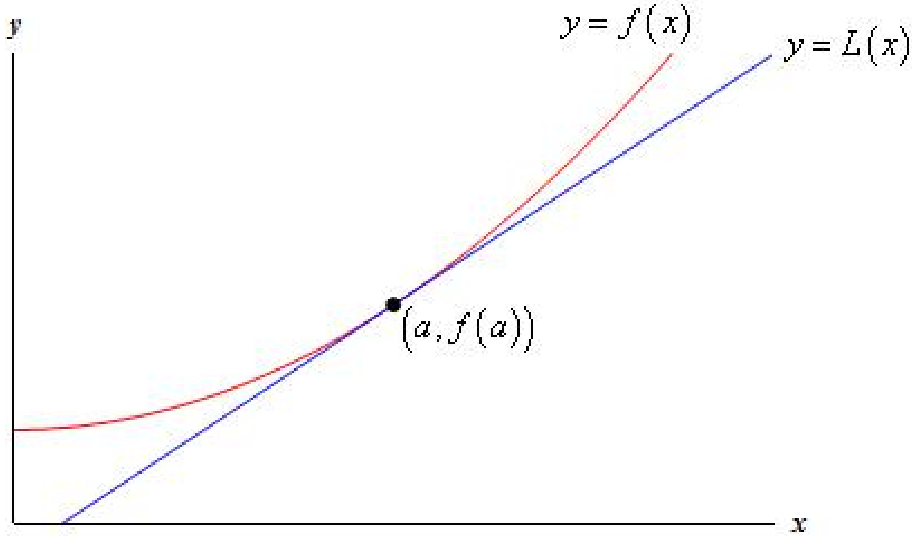

Fig-1

As you look at the fig we have got a curve y=f(x) and a tangent to the curve is drawn at the point (a ,f(a)) now one may raise a question what is the value of slope of the tangent to the curve y=f(x) at x=a? and the answer is f′(a). In other word differentiation of y=f(x) at x=a is nothing but representing value of slope of the tangent to the given curve at some point where we are interested to find derivative if the given function representing the given curve.

Now let us make an analogy similar to the above look at the algebraic equation $x^2+y^2=a^2$ now this equation is abstract notion of something which is very much familiar to us. It represents a circle whose center is at origin having radius a now when someone looks the equation for the first time it will be meaning less unless you take different appropriate value of x and obtain values of y and the draw graph of it. When you make a graph using that equation you will get a circle whose center is at origin and having radius a.

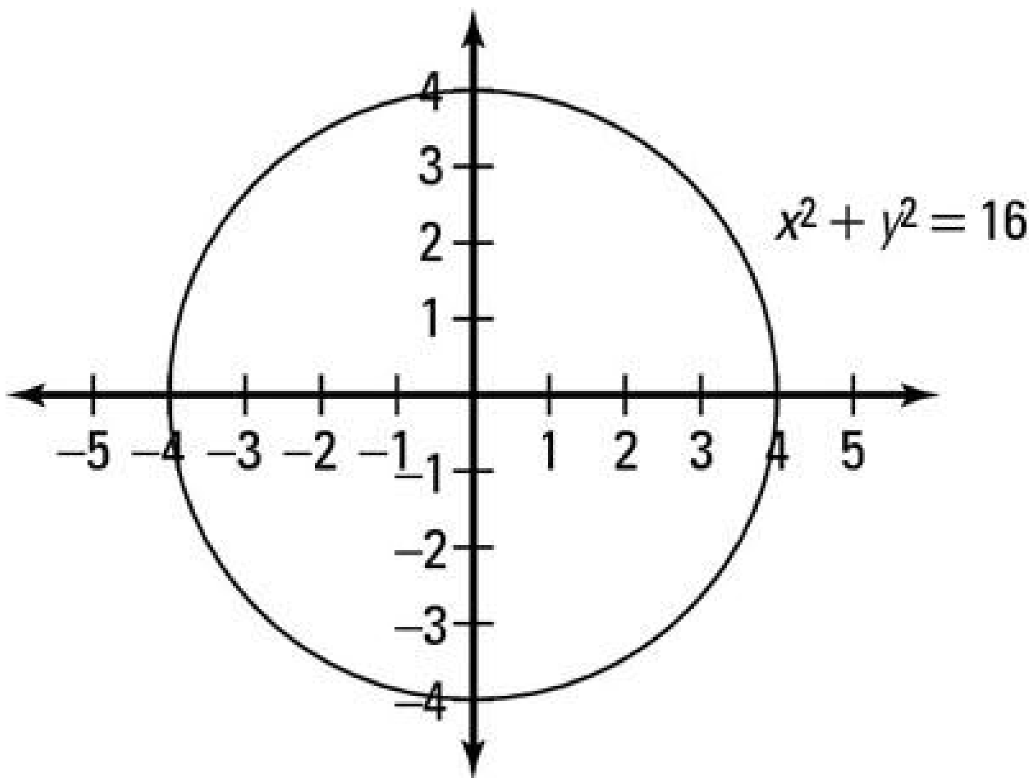

Fig-2 graph of algebraic eq $x^2+y^2=4^2$

In the fig-2 we have represented the graph of algebraic equation and as you can see that once you interpret that equation geometrically you will get geometric fig which nothing but a circle having radius 4 and center at the origin. Now it is easy to grasp because we can visualize that equation because we interpreted it geometrically.

When I studied triple scalar product for the first time it was hard to grasp that topic it was easy to remember but I had no idea how could I visualize it but when I see that it is nothing but representing the volume of a parallelepiped it becomes not only to easy to understand but also easy to remember as well

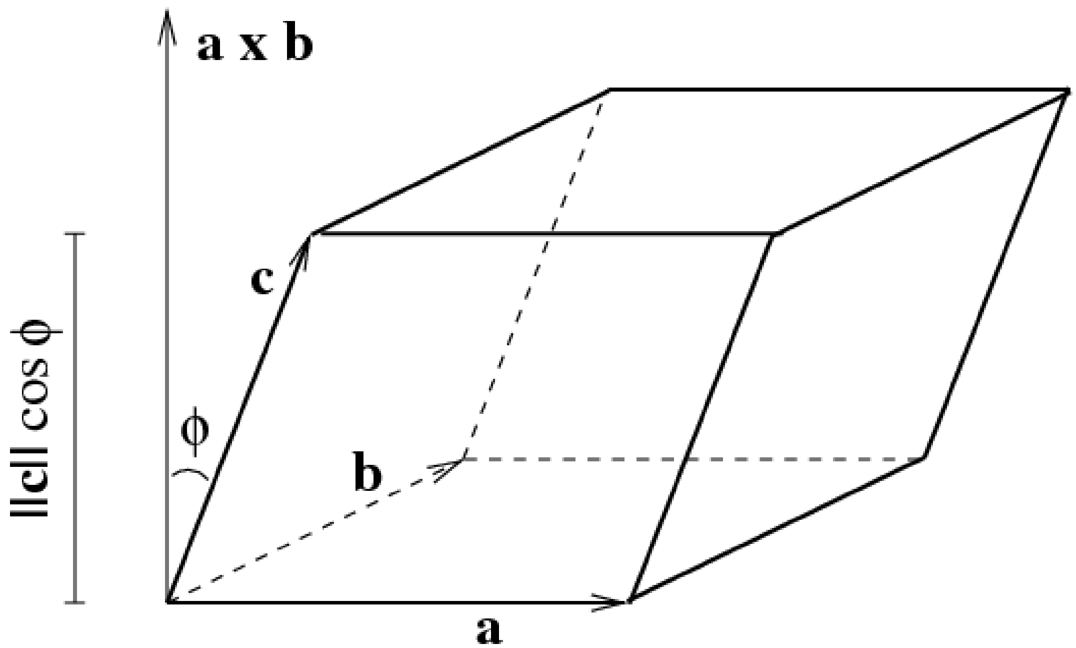

Fig-3

In fig-3 we have a parallelepiped whose adjacent sides are vectors a, b and c. Now triple scalar product of these vectors is (a×b)·c and that's nothing but volume of the above parallelepiped.

I have also included symmetry in the title because symmetry is the essential thing to understand and derive conclusions easily from any kind of complex geometrical structure. There are two types of symmetries one is discrete and other is continuous let us first talk about continuous symmetry.

First of all I think it is better to do define what do we mean by symmetry in real world symmetry means something which looks perfectly shaped or very much look like when you look at it from different angles or whatever. In physics symmetry appears in the equations you change something and final result remains same then we say that eq is symmetrical under that operation or change and in physics for every fundamental symmetry we have conservation law kind of amazing isn't it.

When we rotate a circle about its center with any angle and we will still get the same beautiful structure that means circle is symmetrical under rotation about its center and that's an example of continuous symmetry. Now in discrete symmetry we have mirror symmetry for example out face is symmetrical if you look right and left part from the nose or you see butterfly which are examples of mirror or more general discrete symmetry. There are other symmetries as well translation symmetry let's consider one more example of discrete rotational symmetry.

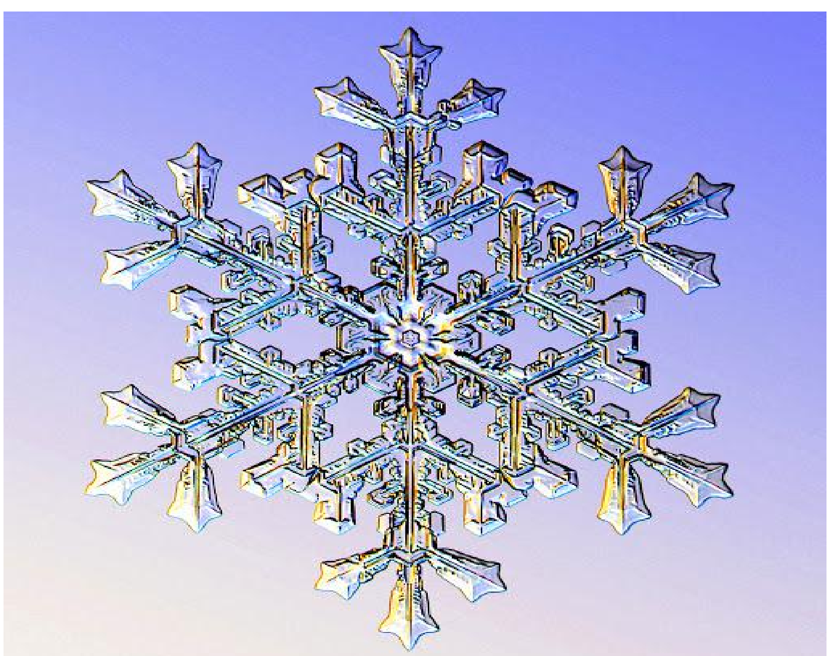

Fig-4

In fig-4 we have shown a snowflake which is symmetrical under 60degree rotation and also if you look at it by considering some axis passing through middle you will see left and right and similarly bottom and up parts are very much look like all three symmetries are discrete out of them two are mirror and one is rotational symmetry.

From the above discussion we can understand how symmetries are important because once you interpret any complex topic geometrically you need help of symmetry to make complete analysis and once you find pattern it becomes completely totally understandable.

I like geometry because I also like topology as well now one may ask this doesn't make any sense since there is no connection between them because in geometry we study about metric spaces while in topology we study about topological spaces geometry has local structure while topology has global structure but after all they are both the fields of the study of physical objects with some exceptions that we do trivial non trivial deformation in it actually there is conjecture which connects these both fields. It's Poincare conjecture and it's about the shape of the universe so seems like there is geometry and since space time is dynamic object so we should have to consider topology as well

Now it would be pretty amazing if we can solve a well- known problem and it's is there any hidden pattern in prime numbers if there is then is it possible to find out that using geometry?

Well I think if we could interpret prime numbers geometrically or study geometric figure which has some deep relation to prime numbers then pattern or in other by studying all the symmetries of that fig might lead us to the hidden pattern of the prime numbers I think that's what we should work on because prime numbers are at the heard of the number theory and most of the conjectures in the number theory are all about prime numbers. It's kind of philosophy but neat idea to start solving some big mysteries of the prime numbers using geometry and symmetry.