

## Mathematics vs Reality: the pursuit of perfect representation

Applied mathematics has always been the most interesting part of maths to me. Taking the ideas of mathematics, which are built on foundations of such simplicity, and using it to explain the complicated experiences of reality is to me basically magic. In essence, applied mathematics is about finding a mathematical model that explains observed events. Once these models are established there is only one thing that humans want to do. Take the model and extrapolate it into the future so that predictions can be made. A model that is unique and entirely certain would be as accurate in both directions of time. The holy grail is a mathematical “theory of everything” that explains every single aspect of reality and to achieve this with complete accuracy. In this essay I will explore if this dream is attainable and examine the problems that face this goal practically.

To begin with careful consideration needs to be given to the idea of accuracy. Pure mathematics and observed reality have differences. The platonic idea of the world of forms is a very useful analogy. Theoretical mathematics is something that is separate from “our” world. This separation is noticeable when trying to give a definitive value to any irrational numbers. Ironically, pi (maths’ beloved constant) can be defined perfectly with language descriptions but if you try to give it representation numerical it will never be definitive, just a poor projection of the truth. Problems arise because it is impossible to do calculations with pure concepts. Therefore, an approximated translation must take place when maths is applied to the real world. This approximation can be achieved to an absurd degree of accuracy (in some cases) but it remains impossible to achieve perfection. For this a halting solution must be applied. This involves calculating (translating) to a sufficient degree of accuracy then just accepting the result at that point. Essentially saying “erm... that’s close enough”.

So is that the answer then. Reality cannot be conveyed by maths; they are distinct from each other and they just won’t integrate properly. Well don’t fear I have an optimistic side as well. The material reality that we observe is entirely finite. There are incomprehensibly small particles all over an incomprehensibly large universe. However, there is an entirely finite amount of them. And therefore, a finite number of possible events. Numbers however have no such restriction. The bounds of mathematics never end and conceptually there are infinite uses and developments to be made. The example of platonic incompatibility previously used showed that some mathematical concepts cannot be represented by humanities numerical notations. This importantly doesn’t mean that that the inverse, that our reality is impossible to represent mathematically, is true. Despite the immense complexity and size of our reality, it isn’t even a drop in the ocean of a true infinity. This should mean that maths must have the ability to encompass any parts of our reality. But is this actually true.

Before exploring problems applied mathematical models face I thought it would be best to start with the success models have had. The first place to start when looking at humanities achievements in mathematical representations of reality must be mechanics. Newton’s laws of motion have explained movement across the board for absolutely everything a human brain can perceive (I’ll get onto the other cases later). These laws are simple enough that they are taught to 14-year olds across the world, yet they have applications in every problem that involves movement. So is mechanics humanities strong-hold on reality. Simple, elegant and a perfect model for movement in our world. Well yes and no. Anything solid we’ve got you covered. But if you’re dealing with a fluid you’ve got a nasty shock ahead of you. And unfortunately for daydreaming mathematicians a fair amount of reality is made of fluids. Fluid mechanics is a completely different game to its fixed state cousin. The problem with fluids lies in their internal forces - really a solid is just a fluid that has no internal forces. Fluids don’t break Newton’s laws. They just have additional laws that must be applied. Luckily

for humanity some plucky European mathematicians figured out the equations for how fluids behave. Only problem is no one can make these equations work consistently (give it a go, if you can you'll find yourself a hell of a lot richer). Fluid mechanics can (rather lazily) be explained as just like regular mechanics; except rather than a few objects you have millions, they interacting among themselves with certain forces, they also interact under as a collective under different forces, plus it'd be nice to work this out with only a handful of variables if you could. Just typing that out is cause for a substantial headache! The easier (and rather more succinct) explanation for these headaches is one word. CHAOS.

Bit dramatic I know but I couldn't help myself. Chaos theory is a field where a battle over accuracy is being contested. "Chaos" is observed when tiny changes in conditions become multiplied to give massively different consequences (the butterfly effect is the infamously quoted example). The problem this creates is that the before mentioned inaccuracies that were "close enough" become amplified from having a small impact to being significant and noticeable. The chaotic feature of fluid dynamics is turbulence. Turbulence occurs when excessive internal energy is present and causes the fluid to behave chaotically. So is this the point we lose hope in the search for the holy grail. Well overcoming chaos isn't as desperately difficult as the name may suggest. Chaotic motion obeys exactly the same laws as simple motion, it is just that even small internal variables and force must be accounted for precisely and individually (whereas simple calculations work just as well with resultant forces). This challenge can actually be solved with enough calculation power. It isn't particularly elegant but with computer programs every input can be accounted for by brute processing power. (in a process of starting with larger averages and then reducing boundaries). Even though it is challenging, fluid mechanics and chaotic phenomena do not break the bounds of mathematical models. On top of this, developments and improvements in the field of chaos theory are significant and reasonably regular. With this the accuracy of modelling is being continually improved.

So then, mechanics is humanities success story. While there is still room for improvement, these improvements are both possible and probable. Well not quite. About a century ago a new theory emerged that quite literally shook the foundations of the world as we know it. Quantum mechanics. It deals with the actions on the very smallest scale and completely breaks the established laws of physics everyone was so confident were universal. Quantum mechanics has provided an impasse to any fantasy that mathematics can explain and predict reality with complete confidence. The nature of randomness and superpositions has meant it is physically impossible to know both velocity and position of particles. There must always be uncertainty. This isn't the case of inaccuracy discussed before, where it's like chasing an ever-shifting finish line. In quantum mechanics there is no finish line. Built into the nature of these particles is an uncertainty that cannot be overcome. These ideas are basically incompatible with our natural intuition. So why is it accepted and used. Well the fact is the maths works, it is derived from well-defined origins and the explanation fits the practical evidence perfectly. So, what does this mean for the dream of a perfect model. It's hard to say. On the one hand it breaks all possibility of certainty and confidence in the future which was a foundational aspect of a perfect model. But on the other hand, it is a theory that is thoroughly mathematically defined. In a way you could think of quantum mechanics as the bleeding of the world of maths into our reality. Uncertainty and probabilities in any other field are conceptual ideas used to quantify data and statistics. Yet in quantum mechanics uncertainty is physically manifested and probability waves are thought by some to have physical existence in our reality. These are really the only examples of our reality seeming to emerge out of mathematics rather than the other way around. However, I think caution needs to be taken when trying to analyse the implications of quantum theory. It is still a young field that has a lot of questions to answer and future developments may completely change everything we think we know. With that considered to my understanding

quantum mechanics is probably an insurmountable blockade to the goal of perfect modelling. These probabilities could be input into a model, but the lack of certainty means these models just present possibilities rather than being a representation of our reality.

Beyond the more mathematically dense issues that challenge the possibility of applied maths; it is definitely worthwhile considering whether there are elements of our reality that exist outside the sphere of mathematics. It is necessary that the world be entirely determined, for a singular universal mathematical theory to be possible. This means saying goodbye to the notion of free will. Even with improving scientific evidence that hormones, neurons and behavioural psychology are responsible for decision making. There is something deeply unsettling about the idea that there is no unique soul in humanity. Surely there's something extra special about us. In its nature the argument for free will is empirically indefensible; instead it relies entirely on humans perceiving their own actions as being made by our own will power. Personally, I cannot see the space free will fills anymore. Our explanations of actions and psychology are continually improving, and everything is following the rules of physics and maths. I think the reason for this illusion of free will is based upon emergent complexity. Features of our reality are built upon layers and layers of complexity. From sub-atoms, to structured atoms, then chemicals, then genetic sequence, then cells, then brains, then societal systems built by many individuals. These emergent traits build and build until the correlations between the contingent parts are difficult to see. However, the language and laws that connect everything here is mathematics. So, when you dig deep into these problems and reduce their nature back to basics, it isn't a soul that emerges. It is maths.

Overall, I don't think mathematical models can ever be perfected. There will always be some level of disconnect and inaccuracy. For me the issues in quantum mechanics just can't be resolved (though I would love to be proven wrong). However, I think the lack of a universal and complete theory is preferable. There will always be more to discover in our reality and new challenges that we could never predict. For me this is the only version of reality that would be at all interesting. With ever a new avenue to investigate and theories to improve. Mathematical imperfection is cause for excitement not frustration.