Wild Possibilities at Mysore Zoo

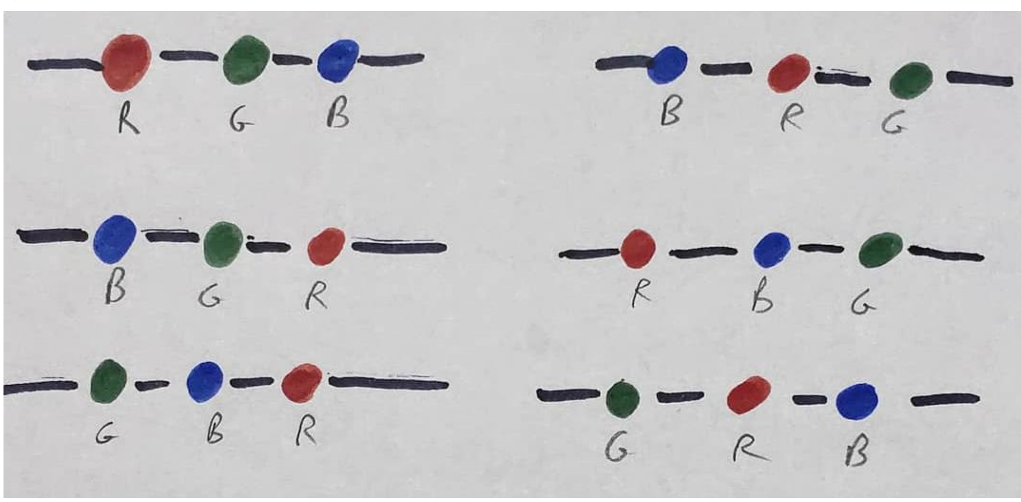
Essay By: Aadil Ahmed, Age: 16 years; Ahmedabad, India.

Hi Friends, by simply clicking on my essay you have successfully run a part of the choose function or as the folks at school like to call it, the ‘combinatorial function’!

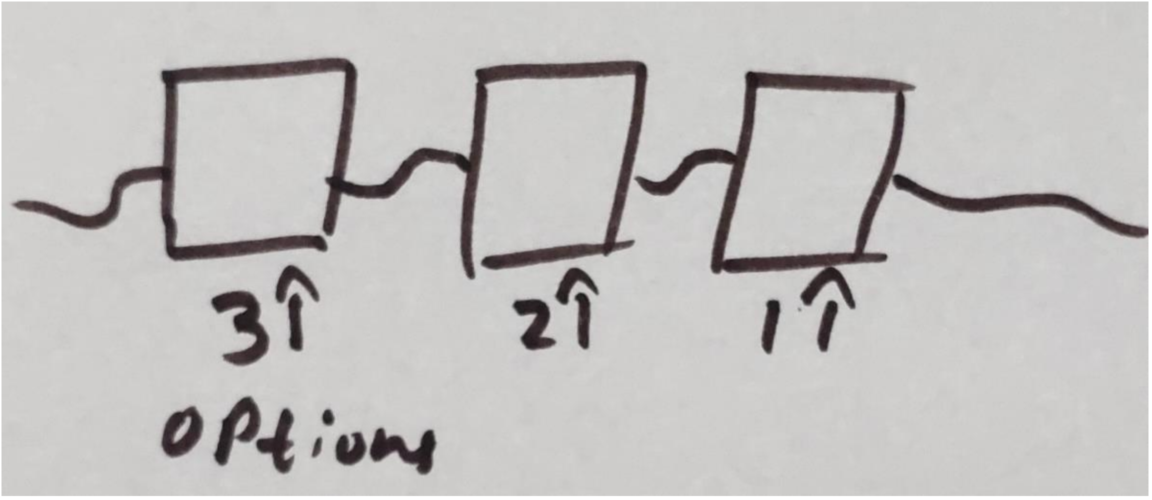
So what is combinatorics about?
As you might have guessed, it's all about selecting options; just the way you selected my essay of the many others. Formally put, it is the measure of the number of arrangements and assortments for a given set of things.

The word ‘permutations’ refers to the number of ways you could order or arrange things where the position of arrangement matters
And the word ‘combinations’ refers to how many ways you could assort something.

Let's say that you have 3 beads of colour red, green and blue; now try placing them on a string such that you get a new pattern every time...how many ways can you do that?
With a bit of finger twistin’ you would come to know that there are 6 patterns you could make using these beads, namely
RGB
BGR
GBR
BRG
RBG
GRB

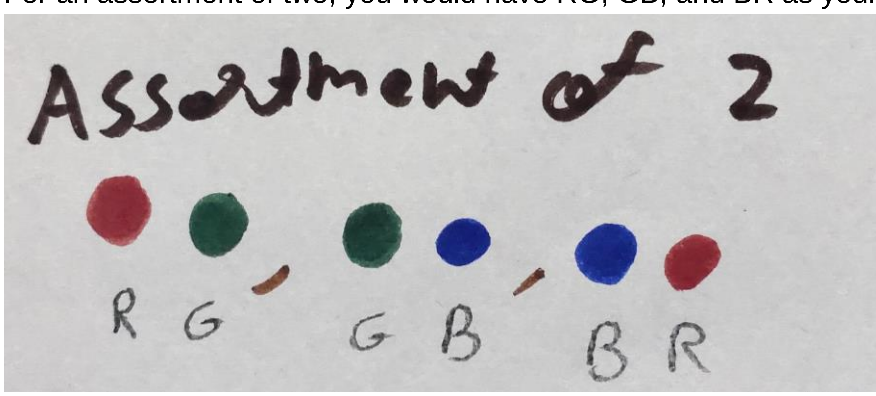


The above were the number of permutations of the three beads
Now to approach this mathematically and to turn this nice activity in a maths ‘problem’ we can consider imaginary boxes drawn upon our string and really think about



After drawing our string with its imaginary boxes on the windowpane (like mathematicians do) we can see that in the first box, we can insert red, green or blue beads; that is we have three options here.
Once we put in our bead of choice in the first box, we can move on to the second box where we would now have 2 options for we already put in one bead in the previous box, there would be only one bead once we reach the last box so, one option for the last box
The options in the first, second and third box contribute to the total number of patterns i.e.
$3 \times 2 \times 1$=6.

Now to figure out the number of ways of combinations, we remove the string and think about how many ways you could assort them in hand (position does not matter).
So if we wanted an assortment of three, there'd be only one way to do that for having taken all of them together.
For an assortment of two, you would have RG, GB, and BR as your assortments.



In this essay we will be dealing with mainly combinations and try to gain some insights about the way it works and the mysteries it beholds.

Puzzle #1

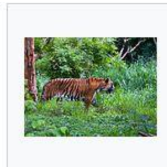
The Zoo problem

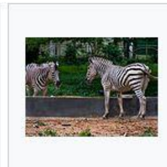
One of India’s most popular zoos, the Mysore zoo houses 168 different species of animals which include Indian elephants,Leopards,Giraffes,Gaurs and Lions.

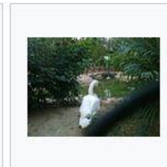

Indian elephant


Leopard


Giraffes feeding

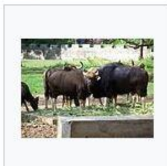

Tiger in lush vegetation


Zebras


Swan


American white pelican

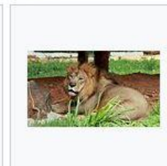

White rhinoceros

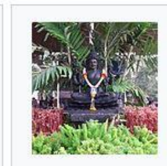

Gaur


Elephant

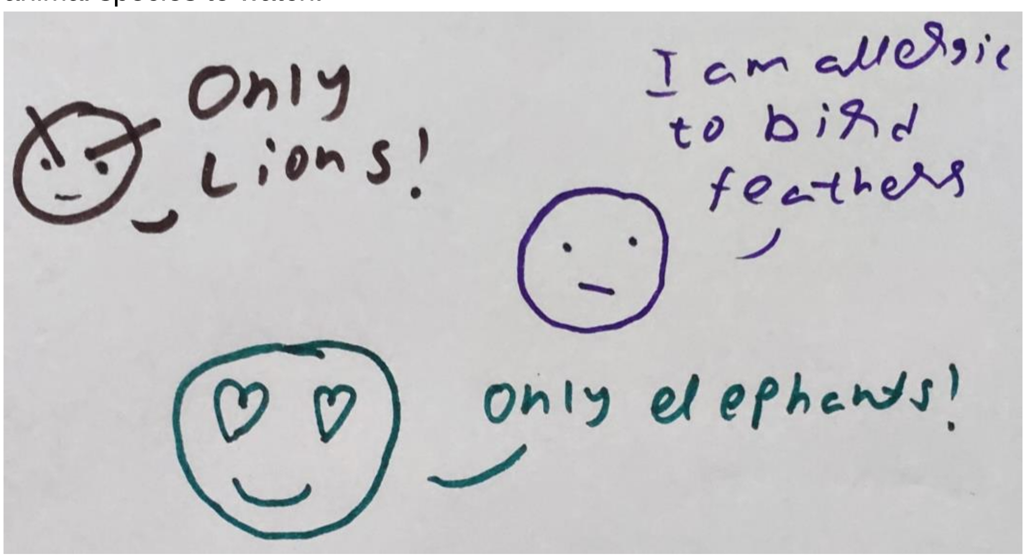

Rhino


White tiger


Lion

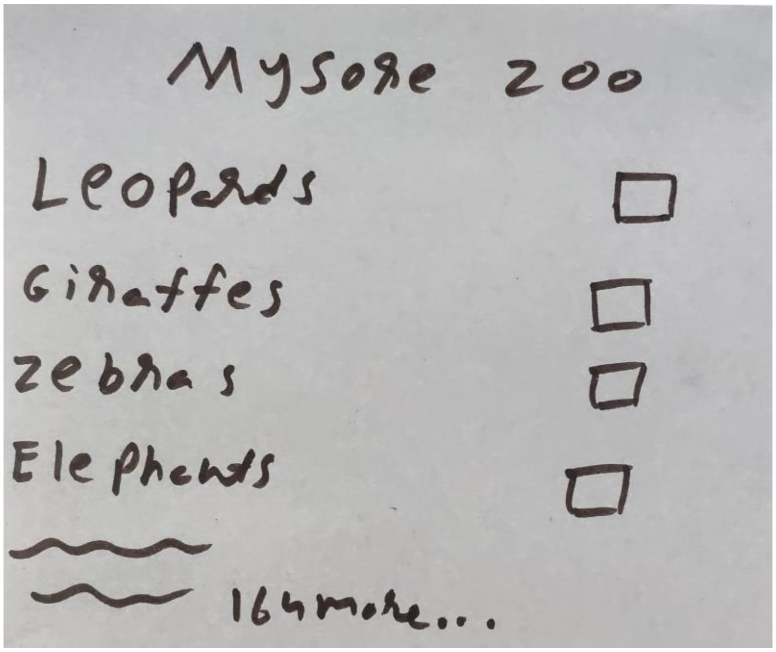

Statue located at Mysore Zoo

Say you and your friends plan on visiting this zoo and have an argument about what variety of animal species to watch.



So the question is, how many ways can you and your friends spectate the different types of species (exhibits) and how much time would it take to explore all the different combinations for visiting exhibits?

We have two choices when it comes to visiting an enclosure: to visit the enclosure or to not. Let's make a checklist which has a box against each of the animals' names, one can either tick the box or cross it; so essentially what we’re doing is calculating the number of ways one can fill this checklist.



So the number of ways one can fill this checklist must be a factor of two, there are 2^n(ranging from seeing no animals to seeing all of them) ways to fill that checklist where n is the number of exhibits;so for 168 exhibits it comes out to be 2^{168}i.e multiplying 2 by itself 168 times.
The above is just the number of subsets of our animal exhibit set.

2^{168}When computed comes out to be 374 quindecillion or about 3.71×10^{50} in scientific notation, which is an enormous number and can be compared with even the number of nucleons that make up our massive earth (4×10^{51} nucleons).

Now to figure out how much time it would take to explore all of these combinations:
Suppose if visiting each exhibit takes 10 minutes, how long would it take you to try all the 2^n or in this case all 2^{168}combinations?

Observing the patterns we get from combining exhibits, we see that each exhibit is visited in half of them (just in different combinations).
So you will visit each exhibit 2^{n-1}times (i.e.$2^n \div 2$).Since you do this for n exhibits, you will visit a total of $n \times 2^{n-1}$Multiplying the term by time will fetch you the total time needed.

So for n=168 and time to observe each exhibit=10 minutes;

Time taken to explore all combinations=$168 \times 2^{168-1} \times 10$ minutes=5.979×10^47 years ≈

4.3×10^37 × age of the universe (≈ 14 Gyr)

Which does not include the hours of waiting in line and bathroom breaks...yikes!
Pretty mind boggling; if you ask me.

Puzzle #2

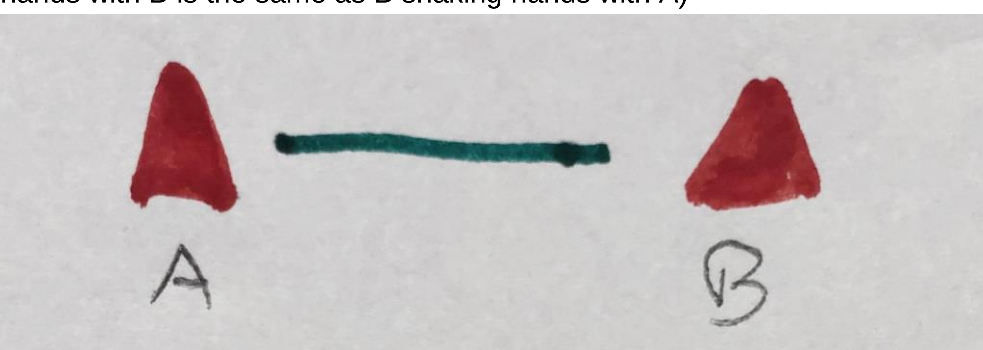
For the virus season is on, zoologist Dr.Sundari is busy studying the interactions between animals. However, she got distracted when a monkey a stole her delicious vada during her tea break and lost her count; for you were wearing a numberphile shirt, she asks you put a lower bound on her count [there are a total of 1320 animals in the zoo].

Mysore Zoo (Sri Chamarajendra Zoological Garden)

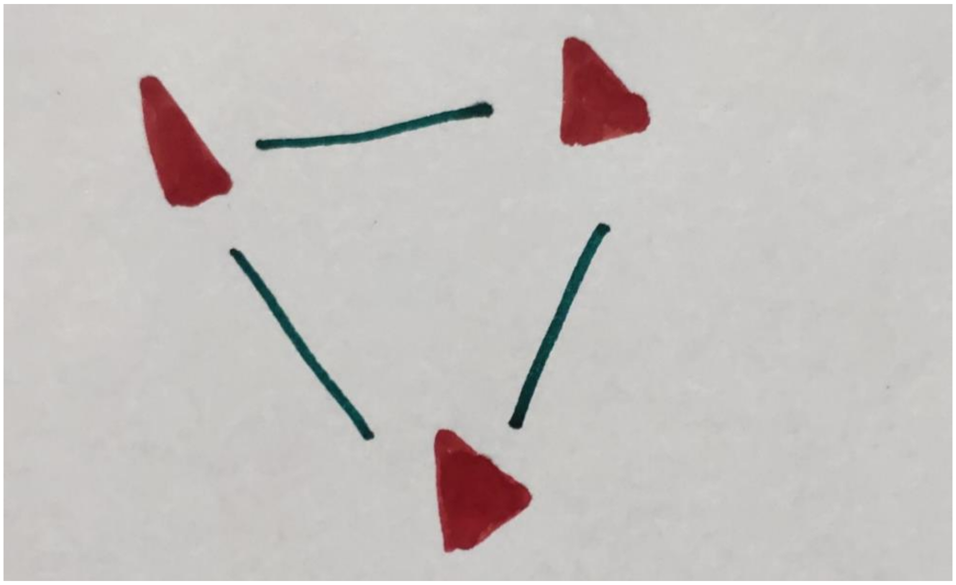
Date opened	1892[1]
Location	Mysore, India
Coordinates	12.3008°N 76.6677°E
Land area	157 acres (64 ha)[2] + 113 acre
No. of animals	1320
Memberships	CZA[3] / WAZA/ ZAK
Website	www.mysorezoo.info

To solve this, one can imagine the interactions to be handshakes (or tail-shakes we talk about reptiles, or trunk-shakes if we talk about elephants, ah! Don't get me started) and try count the number of handshakes.

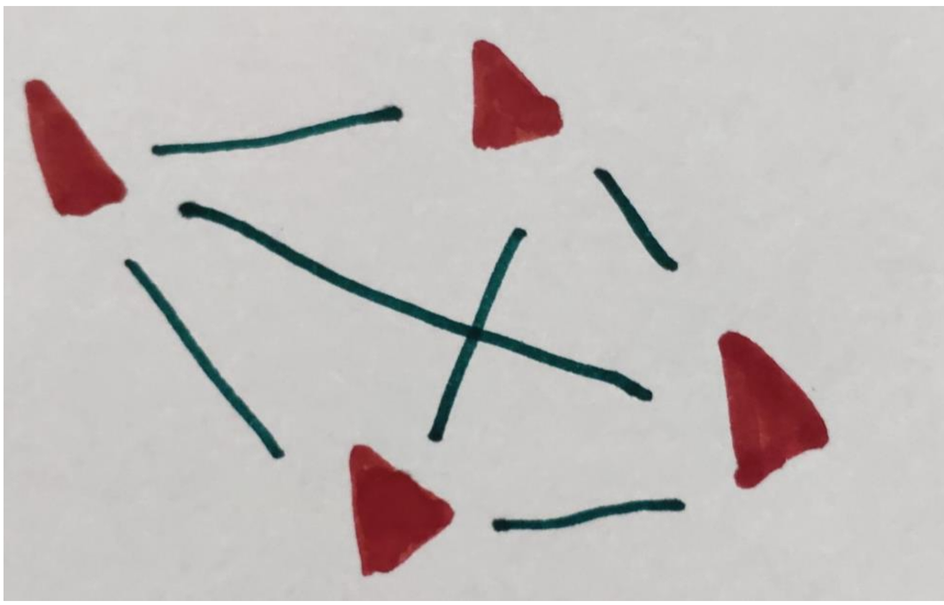
If there are 2 animals there would be only one handshake (handshakes are mutual, A shaking hands with B is the same as B shaking hands with A)



For 3 animals there would be 3 handshakes



For 4 animals there would be 6 handshakes

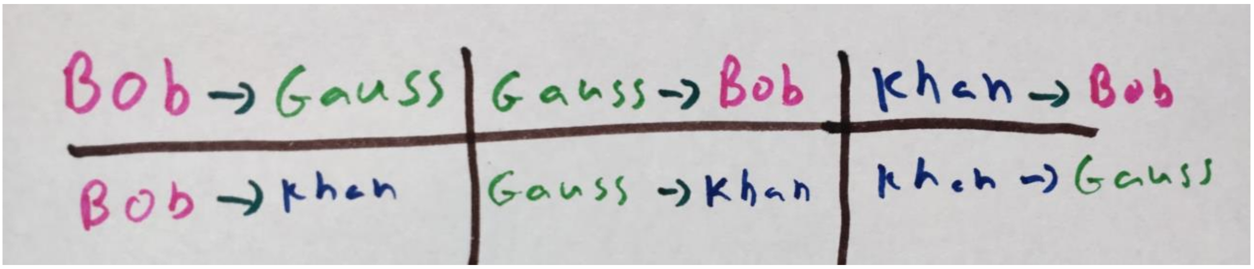


As you might have observed, the handshakes are denoted by an edge and animals by triangles (here, a vertex); this kind of a presentation is called a graph which consists of vertices and edges.

As we move on, counting edges becomes a bit difficult, so to suffice we can find a formula that will help us find the number of handshakes for any number of animals

Let's call in our friends Bob the Sheep, Gauss the Gaur and Khan the Lion for a little experiment.

Each of the 3 animals shake hands with the 2 others. That’s $3\times 2=6$ handshakes in total



So for n animals, the number of handshakes appears to be $n \times (n-1)$

That however is incorrect because we observe that Bob shaking hands with Gauss is the same as Gauss shaking hands with Bob.

This tells us that we have accounted for each handshake twice! If we divide the $n \times (n-1)$by 2, we will have the total number of handshakes made by n animals!

So, for 1320 animals, the total number of interactions made will be

$1320 \times (1320-1) \div 2$=8,70,540 handshakes

Which is close to the world record to the world record made by a handshake relay of over 1800 people in UAE by the Abu Dhabi Police on 29 January 2020(also the day Tom made the essay competition announcement, yay!)

Combinatorics is sure fun!

But on a serious note, combinatorics proves to have many applications in real life ranging from cryptology to biology.

I’m so glad that you've reached the end of my essay, a big thank you for that; I hope you enjoyed reading this just as much I did enjoy writing it,

Cheerio.