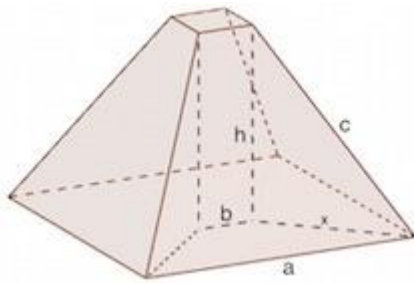


# COMPLEX NUMBERS

“The shortest path between two points in the real domain passes through the complex domain.”

-Jacques Hadamard

In the late 16<sup>th</sup> century Gerolamo Cardano was figuring out a solution for third degree equations, when he came across a tricky problem. What Cardano faced would later be recognized as complex numbers. Later an Italian mathematician Rafael Bombelli took up the topic. He concluded addition, subtraction, multiplication and root formation. But perhaps the first recorded observation was in the late first century AD by Heron of Alexandria. He came up with a formula to calculate the frustum of a pyramid, which is as follows,



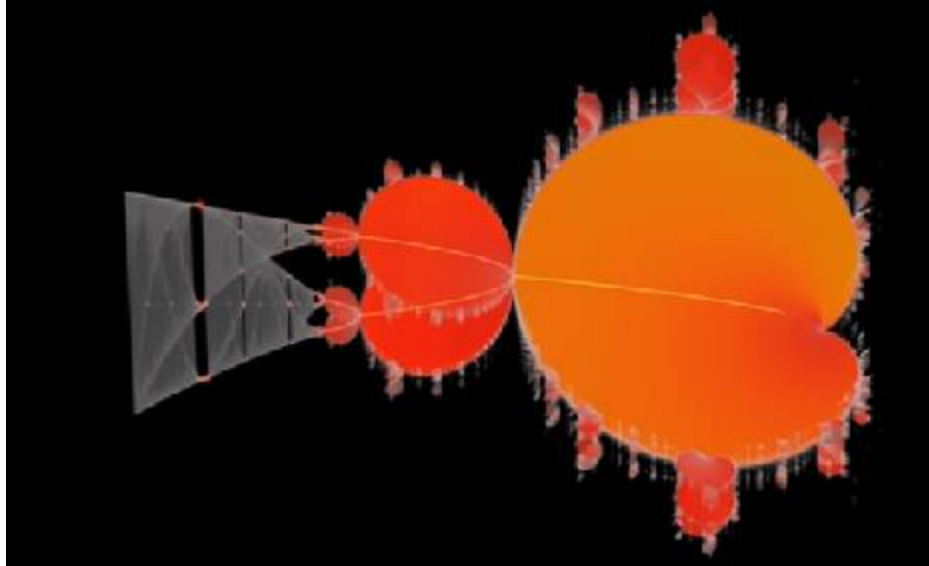
$$h = \sqrt{c^2 - \frac{(a-b)^2}{2}}$$

now plugging in  $a = 28$ ,  $b = 4$  &  $c = 15$  gives the end result of  $\sqrt{-63}$  of which he took the magnitude, closely missing the chance to study the unique properties of these numbers. But by far the most famous application of complex numbers was done by Euler in his equation,

$$e^{i\pi} + 1 = 0$$

which is regarded by some mathematicians as one the most beautiful equations of all times.

It is utilized in various fields ranging from circuit analysis to fluid flow simulations to quantum mechanics, but let us leave the hardcore math for another composition. Arguably the most famous fractal of all times, the Mandelbrot Set, exploits the properties of complex numbers. Now, as you can see in the image under, on the set tends to hold a weird mesh like



structure between its needles and a fine thread passes through its inner bodies. This particular 3D diagram joins the chaos theory to the complex plane. To build the set we use the Mandelbrot equation

$$Z_{n+1} = Z_n^2 + C$$

The points on the graph which doesn't approach infinity, rather starts oscillating around the origin, is a part of this set. The line through the big yellow bubble is the line where predictions can be made, or the region of order. But on the needle that mesh structure is the where everything goes chaotic. A very practical application of this is found in medical research. Scientists have determine the exact time the heart beat goes to chaos and apply a shock accordingly, returning it to normal.

But besides very ancient mathematician and complicated fractals, complex numbers offer brilliant observations to keen observers. It in itself a brilliant hobbies of mine. Let us start with the simplest of the observations in complex numbers. In the Argand plane the negative complex axis is the positive axis mapped through the function  $f(x) = x^3$ . In fact, all the other axis are just the complex axis mapped through the function  $f(x) = x^2$  (negative real axis) and  $f(x) = x^4$  (positive real axis). Thus the entire Argand plane can be wrapped in on the positive complex axis. Thus the unique beauty of the complex numbers has and will always attract mathematicians.

"I tell you with complex numbers we can do anything." – John Derbyshire