Set Theory - My Experience Of Passionate Mathematics.

Set Theory is one of the basic math topics you'll attempt if you are a math major in Nigeria. I fell in love with that topic on the very first day my lectures began in the Federal University of Technology, Akure, Nigeria. I was sitting on the second row of the first column from the left of the 750capacity Lecture theatre when we had our set theory class. Dr Dawodu had gone bald, an advantage to me because his style of teaching still shines through my mind as does his ever radiant forehead. He had a great sense of humor and everything he said and did that day in class made me realize that I was in for a nice treat as a math major.

Dr Dawodu noticed everyone was somehow unsettled, he then tactfully bought our attention by screaming out a rather controversial definition for mathematics. He said out loud: "***Mathematics is the art of reasoning round and round using letters, numbers, and figures as entities.***" Mixed reactions he got from the audience, but most of us soon realized that this is just one of his many tactics he uses to make students fall in love with his lectures and give him their undivided attention. Well, it worked!

Prior to that class, I had made my own research and I discovered that when mathematicians talk about set theory, the name George Cantor must be mentioned. It was with Cantor's work that set theory came to be put on a proper mathematical basis. Cantor published a six part treatise on set theory from the years 1879 to 1884. This work appears in ***Mathematische Annalen*** and it was a brave move by the editor to publish the work despite a growing opposition to Cantor's ideas. The leading figure in the opposition was Kronecker who was an extremely influential figure in the world of mathematics. Kronecker's criticism was built on the fact that he believed only in constructive mathematics. He only accepted mathematical objects that could be constructed finitely from the intuitively given set of natural numbers. For example when Lindemann proved that π is transcendental in 1882 Kronecker said: ***"Of what use is your beautiful investigation of π. Why study such problems when irrational numbers do not exist."*** At this point, I would urge you to not be quick to judge Kronecker because people like him pushed people like Cantor and Lindemann to make sure their work was free from condemnations even down to our time.

As Dr Dawodu continued his magic, I realized in no time that the notion of set is so simple that it is usually introduced informally, and regarded as self-evident. He took our minds back to secondary high school indicating that an object x is a member of a set A, one writes $x \in A$, while $x \notin A$ indicates that x is not a member of A. A set may be defined by a membership rule (formula) or by listing its members within braces. For example, the set given by the rule “prime numbers less than 10” can also be given by {2, 3, 5, 7}. In principle, any finite set can be defined by an explicit list of its members, but specifying infinite sets requires a rule or pattern to indicate membership; for example, in the set {0, 1, 2, 3, 4, 5, 6, 7, …} indicates that the list of natural numbers ℕ goes on forever. The empty (or void, or null) set, symbolized by {} or Ø, contains no elements at all. Nonetheless, it has the status of being a set. A set A is called a subset of a set B (symbolized by A ⊆ B) if all the members of A are also members of B. For example, any set is a subset of itself, and our good friend Ø(the empty set) is a subset of any set. If both A ⊆ B and B ⊆ A, then A and B have exactly the same members. Part of the set concept is that in this case A = B; that is, A and B are the same set. He moved on to operations on set where he indicated that the symbol ∪ is employed to denote the union of two sets. Thus, the set A ∪ B—read “A union B” or “the union of A and B”—is defined as the set that consists of all elements belonging to either set A or set B (or both). He touched various operations and definitions in set theory including: Intersection of a set, compliment of a set, symmetric properties of a set, relations on a set, Cartesian products, Ordered n-tuples and many more. We even mastered in no time the properties of Set Theory including; Identity Law, Idempotent Law, Domination Law, Distributive Law, DeMorgan's Law,...

To be sincere, many of what he said that day was familiar to us all I guessed. My mind sparked up though, when I heard the word 'Partitions'. I immediately remembered my second favorite math movie 'The Man Who Knew Infinity'. Ramanujan(an unprofessional mathematician) claimed to have cracked partitions in his letter to G.H Hardy at Cambridge. Deep joy seeped into my quiet curious mind and I began to think in my head that now I would be doing the maths of the big bosses. I was disappointed though when I realized the 'the big boss maths' is a little bit far away, because all he did was just to summarize that the number of ways that an integer n can be expressed as the sum of k smaller integers is a partition. For example, the number of ways of representing the number 7 as the sum of 3 smaller whole numbers (n = 7, k = 3) is 4 (5 + 1 + 1, 4 + 2 + 1, 3 + 3 + 1, and 3 + 2 + 2).

After so much done in class that day and after much personal research I realized that in set theory, as is usual in mathematics, sets are given axiomatically, so their existence and basic properties are postulated by the appropriate formal axioms. The axioms of set theory imply the existence of a set-theoretic universe so rich that all mathematical objects can be construed as sets. Also, the formal language of pure set theory allows one to formalize all mathematical notions and arguments. Thus, set theory has become the standard foundation for mathematics, as every mathematical object can be viewed as a set, and every theorem of mathematics can be logically deduced using set theory. I really loved what I learnt.

I am now in my second year in the university and now doing more interesting topics like Combinatorics, Linear algebra, and so on ... And I know Set Theory would soon seize to be my favorite. As for now though, the experience I had that day still remains ever green.