Journey of an amateur

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> *"The progress of our knowledge of numbers is advanced not only by what we already know about them, but also by realizing what we yet do not know about them."*
>
> -Sierpinski

A conjecture in Mathematics is a guess, one may find a brilliant pattern which could endure the test of time and efforts by the greatest mathematicians. A well grounded conjecture is supported by vast numerical evidences and plenty of standard heuristics. Modern day mathematicians have very rigorous methodology to propose conjectures, while amateurs don't. Mathematicians work day and night to resolve any famous conjecture in their domain, or to find any exotic pattern and later propose it in the form of conjecture. Curiosity of amateurs may be transient in nature, but they too share similar interests as that of mathematicians mentioned above.

> *"Mathematics, and especially Number Theory, and not least - the prime numbers, are indeed exceptionally full of patterns, but our very human capacity to spot patterns is even more exceptional!"*
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> - David Wells

Let's have a look at the Goldbach's Conjecture which was posed in 1742 and states that every even integer greater than 2 can be expressed as the sum of two primes. It has very compelling numerical evidences having been tested up to 4×10^{18}. Although many significant progresses have been made in order to prove this conjecture, it remains unsolved till date. In Mathematics one can find some pre-existing results on their own because the destination is pre-determined and so is the path to reach it, while the mode of transportation varies as per individual's choice. Once, some little corner of the mathematical world caged in dimensions of four walls witnessed a similar case when an amateur accidentally re-discovered the Goldbach's Conjecture, which was followed by some obvious disappointments. Realization was that his discovery didn't serve as the source of origin of the conjecture.

Line of distinction is drawn when someone explores the unexplored paths, which consequentially leads them to unknown results. Few months later, that same amateur came up with a generalized version of Goldbach's Conjecture as follows - *For every $2n = a + b$ and for $n > 1$, where $a, b, n \in N$, let d be the factors of $2n$ then there always exists minimum one pair (a, b) for every d such that both $|d - a|$ and $|d - b|$ are primes simultaneously with $gcd(a, b) = 1$ where the values of $|d - a|$ and $|d - b|$ ranges over all the primes* and replacing d by 2n results in the Goldbach's Conjecture. Such amateurs are privileged to bear the public humiliation on the ground of educational qualifications, age, grade, achievements and so on...Jens Fehlau (the founder of Flammable Maths) was more or equally surprised as me, we didn't know that such generalizations even existed.

The Collatz Conjecture, posed in 1937, is a conjecture in mathematics that concerns a sequence defined as follows: start with any positive integer n. Then each term is obtained from the previous term as follows: if the previous term is even, the next term is one half of the previous term. If the previous term is odd, the next term is 3 times the previous term plus 1. The conjecture is that no matter what value of n, the sequence will always reach 1. According to mathematicians, they don't have enough tools to handle this problem, clueless from where to attack and how to attack it. But there was another such amateur who thought about - whether there exists a similar system in which no matter what the value of n is, it never loops? After testing lots of expressions in place of 3n+1 , he ended up at 9n+1.By replacing 3n+1 by 9n+1 in the Collatz Conjecture setting, we can witness the striking contrast between them, no matter what the value of n is, the elements in the sequence will be distinct till infinity.

Euler discovered in 1772 that the quadratic expression $x^2 + x + 41$ is prime for x = 0 to 39, now famously known as Euler's quadratic expression. The following expression is my personal favorite because it never fails to amaze anyone who has witnessed it:
Consider the expression $a+b^2+1$, where 40 = a+b.Including the case a=b and a = 0 , b = 40, we have the following sequence of numbers which the above expression generated - (41, 43, 47, 53, 61, 71, 83, 97, 113, 131, 151, 173, 197, 223, 251, 281, 313, 347, 383, 421, 461, 503, 547, 593, 641, 691, 743, 797, 853, 911, 971, 1033, 1097, 1163, 1231, 1301, 1373, 1447, 1523, 1601). Surprisingly, they all are prime numbers which after plugging x = 0 to 39 (inclusive) into the Euler's quadratic expression yields.So far there's nothing which

could be appealing to any mathematician... but is of great interest to amateurs and those who share a non mathematical background.

In Mathematics the more obvious we get, the tougher it gets to prove something. Above mentioned things are not of much obviousness, but anyone could have arrived at them, if not today then sometimes later. Our main purpose should be to analyze the real depth of mystery they hold, despite of being so called obvious like others, they remained untouched. Mathematics is not just about concrete logics. It's about ideas and how we use them to connect more deeper areas of mathematics. Though nobody bats an eye upon the works by such amateurs, their works are relatively safe on platforms like AoPS community, unlike other places where they suffer bans ranging from 3 days to forever. I've been competing in Mathematical Olympiads since a long time. I was unaware of such a beautiful side of Mathematics that amateurs are possibly happy for. Like consider these two distinct conjectures featuring two distinct polynomials,

Conjecture 1 : For every $2n = a + b, n \in N$, there always exists minimum one pair (a, b) such that $2(a + n) + ab$ is prime.
EXCEPTION: 2n = 42

Conjecture 2 : For every $2n = a + b, n \in N$, there always exists minimum one pair (a, b) with gcd(a,b) = 1 such that $n^2 + n + (ab + 2)$ is prime.
EXCEPTION: 2n = 42

Note that they both share a common exception. It has been tested[1] upto 1.2 millions and no other exceptions are found, isn't it more than a mystery?

It's rare that such extraordinarily deep insight meets all the present day mathematical tools. Mathematicians and people like them enjoy the responsibility of using it within much of limited frames, while amateurs don't. I don't experience that burden anymore. I enjoy the bliss of prime numbers and significant works by "the amateurs".

[1]Tables and numerical evidences have been extracted out for the concern of word limits only. The reader may refer to the original articles made on Art of Problem Solving community for detailed analysis.