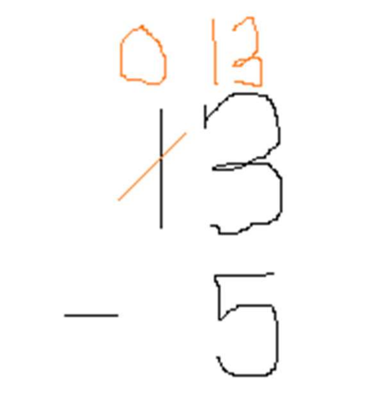
An Inquiry concerning Mathematics

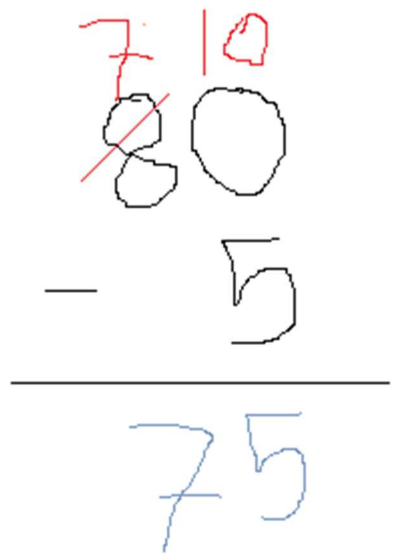
Von Neumann once told his student, “Young man, in mathematics you do not understand things. You just get used to them.” Mathematics is abstract, more so than the reality. Do we make sense of this abstractness through perseverance or study of its application or a combination of both? This begs the question: “Is mathematics learned or adapted?” Is mathematics really understood? One would not normally think such stuffs. Most hate mathematics and most of the rest get by with mathematics through robust calculation and parroting process to solution. Only few love mathematics and only few among these ponder upon such profound questions.

I love mathematics but I would not consider myself among those few of the few until recently. A 5-year-old cousin of mine woke me up from my dogmatic slumbers! I was asked to help her with an arithmetic calculation. The problem seemed simple to me: “13-5 =?” I proceeded to teach her how to arrive at the answer but I was not getting through her. The problem for her was that because 5 can’t be subtracted directly from 3, a carry-over from 1 would change 3 to 13 (and 1 to 0) and then we are back to the original problem (13-5=?). With this method, which she was taught, a step towards solution retraces the problem ad infinitum.

Here is a depiction of her method:



This problem does not recur in other cases such as: 80-5=?, depicted as:



Such subtractions are trivial for adults like me. But are they really? After all, I was once her age and since I have no memory of then, I wonder how I learnt. With my cousin, I see two problems hidden beneath the problem-in-itself: 1) the bluntness and therefore lack of creativity in her method, and 2) an epistemological issue.

Addressing the first problem as such, my cousin is at a great disadvantage. The method she is taught only brings mechanical robustness in her. You see, a child is at the highest level of curiosity in a man's life. I am always amazed at children's ability to learn their native language. With no a priori knowledge of the language but only broken fragments from adults, children start making sense of what they are hearing. However, without visual aid, it is difficult: it has been shown that vision critically affects language development in children. Children pick up the sign by making association of the signifier and the signified. When they are being, for example, shown an apple, the sound "apple" which is a signifier is being associated with its signified, the apple that is being shown at. The same is true for the counting numbers children learn. By this way, a different but easy method would be to teach children such subtractions through visual aid: using 13 apples, taking 5 away and then asking children to count the remaining ones. I tried using this method but since I had no apples at that moment, I had to start with "Suppose you have 13 apples" and my cousin did not really like it that way. She just wanted the solution!

The visual aid method I was talking about is a natural method but it has its limitations: making sense of beyond counting numbers ($\sqrt{3}$ apples sound odd!) but mainly an epistemological one, which brings us back at the second problem I mentioned earlier. How do we know what we know? Why do we know the way we know? When we say these are 2 apples, what we are really doing is assigning a number (2 in this case) to certain quantity of apples. Assigning that same quantity the number 3 or any other number would not change the state of the apple or our ability for calculation. After all, I could say those are 10 apples, which is true in base 2 system. We would just get used to this. Therefore, mathematics is to reality as language is to communication. Consequently, mathematics is a language of reality.

The epistemological question seems cryptic and just an unnecessary musing but it is not so. Why does it matter that we know how we know? Which is more important: the solution, the process, or their dissection? After we have moved past the bluntness of a method and start using vigor of creativity in problem solving, we could stop at that point and just be with it. However, creativity, by definition, cannot stop at that point, for then progress would be impossible. Throughout the history of humankind, an ardor for inquisitiveness has been the cause of progress. So then, removing past stagnancy, we can dissect the problem solving techniques or the bases of our most trivial postulates. This is when we begin to question whether the bases are solid and by addressing this epistemological uncertainty, we become more confident on wonders of mathematics.

Mathematics, an abstract language, does help us in empirical sciences such as physics or engineering. How can it be that an abstract definition, something that is posited without any regard to reality, helps us in making sense of that very reality? How can it be that, for example, Riemann geometry finds application in Einstein's General Relativity years later? Mathematics does calculation in "n" dimensional spaces but space-time, the dimension of our reality, is 4 dimensional. Let me say this more precisely, the dimension of our reality at the moment works out to be 4 dimensional but as we move forward, we might discover there is more to reality and then only mathematics can save us from the despair of lack of this knowledge. We once thought that the space we live in was Euclidean!

It seems so fascinating and spooky to me that something created to understand reality ensembles reality in itself. It's as if reality itself is a part of that which was once a part of reality. An overwhelming oddity! Such is the power of mathematics. How is the finitude of our being able to tackle the infiniteness of the reality? All credit to mathematics.