Bubbles, Exotic Fruit and Voronoi Diagrams

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Imagine you're in charge of organising which children go to which school in Melbourne. How would you go about doing this? You rightly assume that the students should go to their nearest school, and one way of doing this would be to use Voronoi Diagrams.

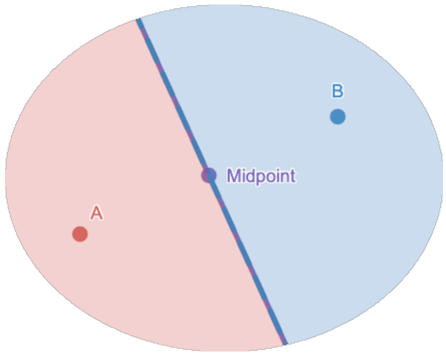


Figure 1: Area divided by the closest school to each point. We find the midpoint of AB and then take the negative reciprocal of its gradient to form the equation of the line.

Well, let's start with a simpler scenario. If there were only two schools A and B, what would we do then? This event is a lot easier to solve. We can simply find the perpendicular bisector of the two schools and split the area down the middle. Everything on the lefthand side is closer to school A, and everything on the righthand side is closer to school B. Anything on the line itself is equally close to either one of the two schools. Job well done.

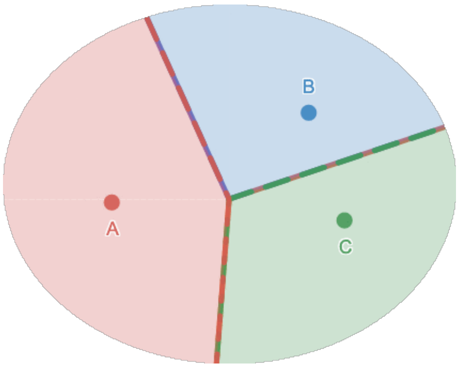


Figure 2: Area divided by the closest school to each point, but with three schools.

What about three schools? Well, we can simply repeat what we did in the first scenario. By finding the perpendicular bisector between each of the three schools (A and B, A and C, B and C), we divide the area into three sectors. Following the same principles as before, everything in the red sector is closer to school A, the blue sector to school B and the green sector to school C.

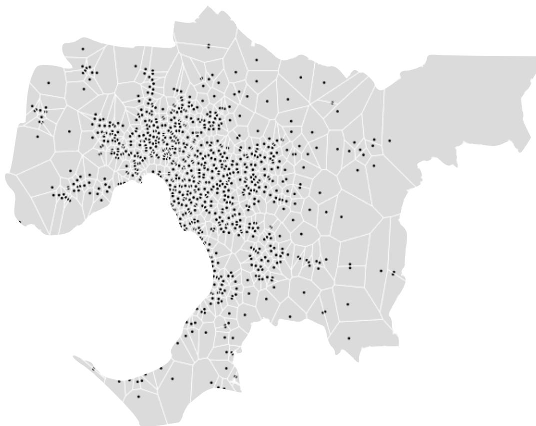


Figure 3: Voronoi Diagram of Primary Schools in Greater Melbourne, 2018. *Copyright (c) 2016 Aleksey Bilogur,* MIT License*, via geoplot.*

If we keep repeating this for all the different schools, we might get something similar to *Figure 3*. These illustrations are known as Voronoi diagrams, and they're very useful in

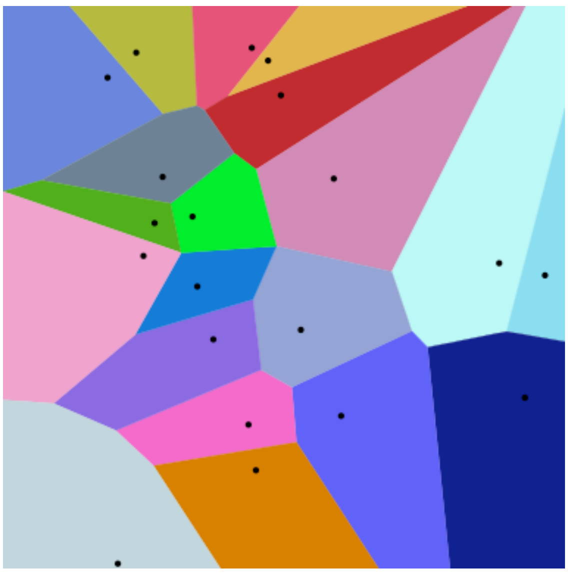


Figure 4: Area divided into 20 sections by closest point. *Balu Ertl,* CC BY-SA 4.0*, via Wikimedia Commons.*

Georgy Voronoy

Figure 5: A portrait of Georgy Voronoy. *Unknown author, Public domain, via Wikimedia Commons.*

Voronoi diagrams are named after the mathematician Georgy Voronoy (1868-1908), who is well known for his contributions to number theory in particular. Some consider him as one of the founders of the Geometry of Numbers.

Although the use of Voronoi diagrams can be traced back much earlier in history, Georgy Voronoy studied them and helped to define them rigorously around 1907-08.

dividing areas by how close they are to certain points. We can then determine which school each child should attend.

For more general scenarios, these diagrams can result in some really beautiful patterns such as in *Figure 4,* where in any given section, the black dot is the closest school.

Some of you might have noticed a small problem by now. We're always assuming you're travelling as the crow flies. In reality, some routes might be more complicated than this. One way around this issue is to use taxicab geometry. Here, instead of going straight to the destination, we take the total horizontal distance along the x axis and add it to the total vertical distance along the y axis. This gives the effect of moving along a grid, which is how boroughs like Manhattan are laid out. See *Figure 6* for an example.

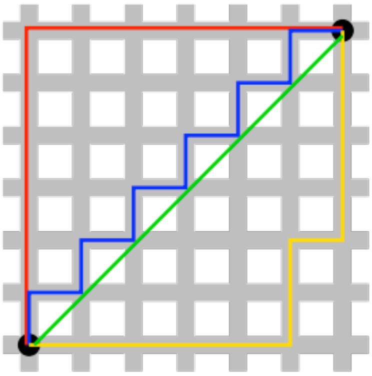
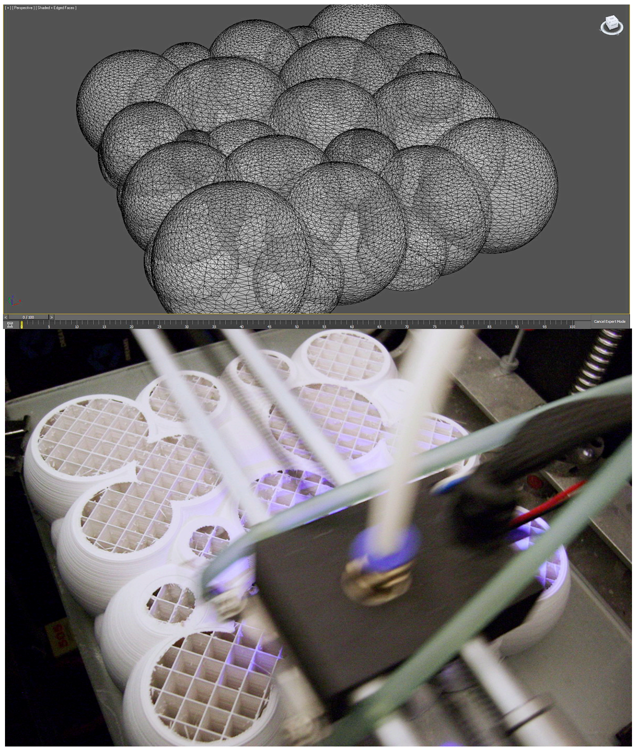


Figure 6: Taxicab versus *"crow flies"* distance. The red, blue, and yellow lines all have the same length (12) whereas the green line has length $\sqrt{72} \approx 8.5$ *User:Psychonaut, Public domain, via Wikimedia Commons.*

Why don't we now look at a more…exotic scenario. One of the most beautiful and intriguing uses of Voronoi diagrams is in the world of baking! Originally an architect and 3D visualiser, Dinara Kasko chose to go into the world of patisserie. Using various geometric concepts she had already acquired, such as Voronoi diagrams, helped her to produce desserts that are both stunning and delicious, including one of her most famous cakes, *"The Bubbles with Exotic Fruit"*.

Figure 7: The production process of the silicone mould. *https://www.dinarakasko.com/the-bubbles-with-exotic-fruit/*



As displayed in the top image of *Figure* 7, Voronoi diagrams are used to divide the plane into bubbles, which can then be used to determine the structure of the mould. It is then 3D printed and cast, and the resulting template can then be used to make the cake.

"I've used geometric constructing principles such as triangulation, the Voronoi diagram, and biomimicry…Tasty and beautiful, that's great."

- Dinara Kasko

Figure 8: The final result of Dinara Kasko's cake, "The Bubbles with exotic fruit". *https://www.dinarakasko.com/the-bubbles-with-exotic-fruit/*

Figure 9: The mould being using to form the cake. *https://www.dinarakasko.com/the-bubbles-with-exotic-fruit/*

As an A-Level Computer Scientist, I've naturally found myself drawn to Voronoi diagrams because of their numerous applications in the subject. Take for instance the exciting new world of autonomous robotics. How would you program a robot to travel without crashing into objects? How would you allow them to determine an optimal route?

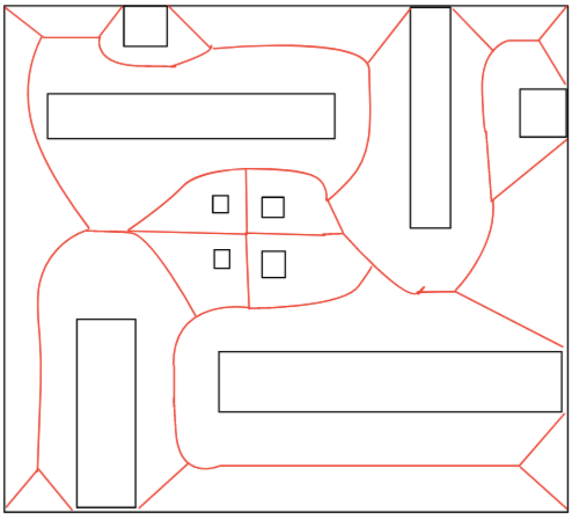


Figure 10: An example of a generalised Voronoi diagram. *https://www.usna.edu/Users/cs/crabbe/SI475/current/simple-model/path_planning/voronoi.pdf*

Using similar principles as before, we can form our Voronoi diagram as shown in *Figure 10* where the black objects are to be avoided and the red lines act as boundaries between the regions. These lines are our potential routes for the robots to take. This illustration is known as a Generalised Voronoi Diagram since we dividing the area around big objects rather than specific points.

This works well, but as you can probably imagine, it can be very complicated to produce this path. In our first few examples, all our edges were straight lines that were nice and easy to calculate. Here, we have curves which makes things a bit more challenging.

One way around this is to instead approximate the path rather than finding it exactly. In *Figure 11*, the white lines and dots represent the obstacles, and the coloured regions show which obstacle is the closest. As an example, any point within the pink region would be closest to the white curve obstacle. The process of producing this diagram can be done very quickly on some modern computers.

Figure 11: An approximation of a general Voronoi diagram. *https://www.usna.edu/Users/cs/crabbe/SI475/current/simple-model/path_planning/voronoi.pdf*

In general however, there can be issues with using Voronoi diagrams in this problem. The concept of keeping as far away from obstacles as possible can lead to less efficient routes, and it might be hard to apply this method to previously unmapped regions. In these examples, we're fortunate enough to already know the layout.

However, since Voronoi diagrams provide a path far away from the obstacles, they also supply one of the safest routes. It is normally very straightforward to implement this method, and it results in a smooth path for the robot to follow.

We've looked at real world applications of Voronoi diagrams in government planning, baking, robotics, and this only scratches the tip of the iceberg. With the rapidly increasing use of technology, Voronoi diagrams will only become more important through its relevance in machine learning, wireless networking and other areas. Each of these everyday uses, and it all comes back to GCSE perpendicular bisectors.

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