Teddy Rocks maths competition - Flo Brooke

1, 2, Tree

An exploration into the relationship between mathematics, beauty and nature.

"*Wherever there is number, there is beauty.*" – Proclus.

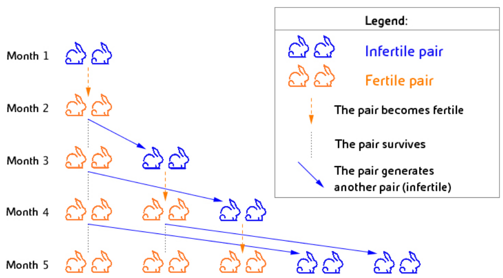
Nature sometimes comes across as complete chaos, however it only requires a cursory examination to realise that this is not the case. Maths is inherent in the natural world. Symmetry, fractals, sequences and constants like pi or the golden ratio pop up everywhere you look. In this way it is clear to see that maths, beauty and nature go hand in hand. So how are they linked?

The Fibonacci Sequence and the Golden Ratio

The Fibonacci sequence is probably one of the most well known sequences in the history of mathematics. Most people, mathematician or not, will have heard of it, whether from a Dan Brown novel (which was where I first encountered it!), or a maths class at school, but few people realise how omni-present this sequence is in our world.

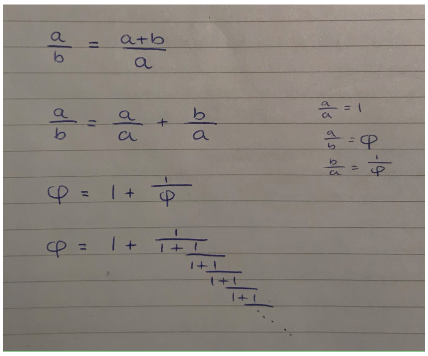
Before we get into that, what actually is it? The Fibonacci sequence forms by adding the previous two terms of the sequence together. So starting at 1, it goes: 1, 1, 2, 3, 5, 8, 13, 21, 34 etc. As an iterative formula, the sequence can be written as $F_{n+2} = F_{n+1} + F_n$.

The Fibonacci sequence was named after Leonardo of Pisa, nicknamed Fibonacci, after he introduced the sequence to the Western world in the early 1200s, however the origins are rooted in ancient Sanskrit texts from decades before Leonardo, in around 450-200BC. Leonardo explained the sequence in his book, the Liber Abaci, by writing about the rate of a group of Rabbits reproducing. He introduced a problem where it begins with two rabbits who reproduce to birth two new rabbits, and each new pair of rabbits reproduce one month after they are born, resulting in a Fibonacci sequence mapping the growing population of a slightly incestuous family of rabbits.

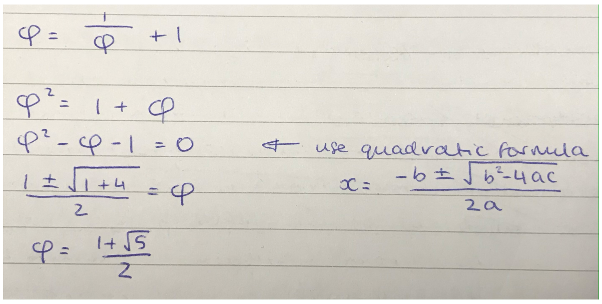


You may be wondering what the significance of Fibonacci is. What makes this seemingly mundane sequence of numbers so interesting is how prevalent it seems to be in nature.
It plays a significant role in Phyllotaxis, a field of study that examines plants like flowers' and trees' petal or branch distribution. If you count the number of petals on flowers, or the number of spirals on a pine cone, the result will often be a Fibonacci number. For example, lilies and irises have three petals, parnassias and rose hips have five, cosmea flowers have eight, etc.

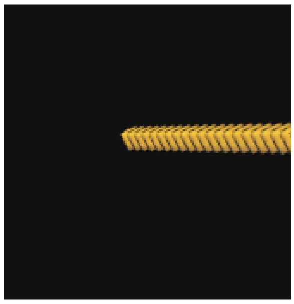
There is a very well-known mathematical constant linked to beauty and nature that can be derived from the Fibonacci sequence: the Golden Ratio. By dividing two consecutive numbers in the sequence, the resulting value tends towards the number 1.61803398875[...], or its inverse 0.6180339[..], the higher up the sequence you go. 13 divided by 8 is 1.625, 34 divided by 21 is 1.619, 144 divided by 89 is 1.618, and so on. The most famous name for this number is the golden ratio, however the word ratio is a bit deceiving as it is an irrational number (ie cannot be written as a simple ratio). Other names for it are the divine proportion, the divine mean, or Phi (φ).



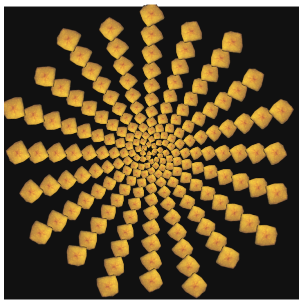
You can also find the golden ratio by splitting a line so that the long part divided by the short part is the same as the total length divided by the long part: $\frac{a}{b} = \frac{a+b}{a}$. This proportion created is the golden ratio.

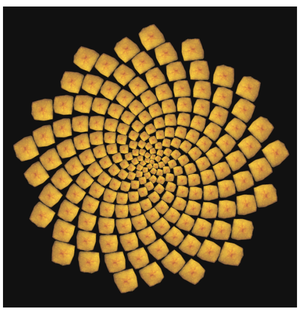


There are ways of manipulating this equation to express the golden ratio differently. Above I have shown how it can be seen as some sort of infinite fraction as it can be written as 1 + 1/itself. Solving this equation as a quadratic shows that the number can be expressed as $\frac{1+\sqrt{5}}{2}$ (see workings, right).



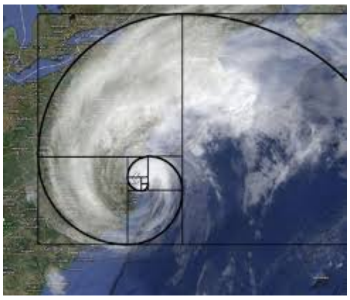
The golden ratio is seen in nature in the spirals of petal growth and pine cone wood scales. The number of turns around the centre made before laying another petal, seed etc is usually close to the golden ratio. To explain this, let's use seeds in the centre of a sunflower. Starting at the centre, if you made a turn of 1 before placing each new seed, the result would just be a single line of seeds (see right). If you made a half turn there would be two spokes protruding out. You can use this useful website to try out different numbers and see the pattern of seeds produced: https://www.mathsisfun.com/numbers/nature-golden-ratio-fibonacci.html. What happens for lots of numbers is that the seeds spiral for a bit before forming a number of distinct spokes (see right for a turn number of 1.45).

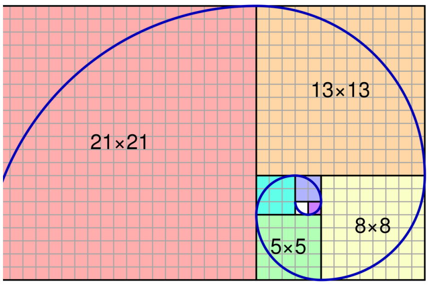




From nature's perspective, this isn't very useful as it's not a very economical use of space and if a plant's leaves grew like this, only the top leaves would get light and be able to perform their function (photosynthesis). As it turns out, the number of turns which results in the most evenly distributed seeds is, you guessed it, the golden ratio. Although spiralling spokes are still visible, the pattern that results

(left) is both beautiful and hypnotic to look at, and displays the most efficiently distributed seed pattern.
Furthermore, if you count the number of spiralled spokes protruding from the centre in the far right image below, there are 13 clockwise turning spiralled spokes, and 21 anticlockwise turning spiralled spokes. Both 13 and 21 are numbers of the Fibonacci sequence.
The Fibonacci sequence and the golden ratio can also be displayed as a spiral or a rectangle (sometimes called the golden ratio). The rectangle is constructed by placing squares with Fibonacci-number side lengths together. By drawing a smooth curve from corner to corner of the squares, you can create a Fibonacci spiral.
As we said, the ratio between consecutive Fibonacci numbers tends towards the golden ratio so you can also use this constant as a growth factor to create this Fibonacci/golden spiral. This spiral is seen in sea shells, snail shells, succulents and other plants, hurricanes and more.

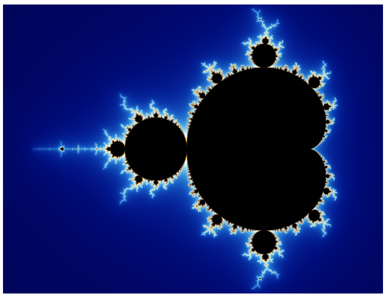




Some people believe that this is a sign of some higher power or programming in our world, however this is extremely unlikely. A more probable reason for this is that the golden ratio is just the most economical and efficient way for plants to arrange their seeds/petals/branches and so this number has appeared through the process of evolution. Furthermore, often people will make major approximations and claim that Fibonacci or the golden ratio is present somewhere it simply is not.
Despite this fact, it still is interesting to consider the links between this mathematics and why we find it so aesthetically pleasing and beautiful. Architects and artists including, notably, Leonardo Da Vinci, have been using the golden ratio to appeal to the human eye for centuries. Artists of the renaissance used the Golden ratio for the frame proportions of their paintings as well as finding the most aesthetically pleasing face construction. Some claim that it's merely a coincidence that the Mona Lisa contains proportions linking to the golden ratio, and that the Parthenon in Athens is circumscribed by golden rectangles, but whether or not it's intentional is irrelevant when it comes to beauty.
A professor of mechanical engineering, Adrian Bejan, claims to know why we are so attracted to and pleased by this ratio. He argues that, biologically, the ratio happens to be the best proportions for the brain to understand and understand quickly. As a result, when we see architecture, art, nature etc. that are modelled using the golden ratio, our brains can quickly perceive and digest the information, which results in us feeling pleasure that contributes to our sense of its beauty. The beauty we see is a result of us being able to view it without difficulty.

Fractals

Fractals are another part of geometry that are ubiquitous in nature. They are geometric patterns that have repeated patterns within themselves. For example, you could have a circle with four smaller circles around it, each of which have further circles around them, and those further circles also have circles around them etc. If you zoomed into one part of the fractal, the pattern would just repeat more and more minutely. In nature there are many examples of fractals including ferns, tree branches, romanesco broccoli, lightning, clouds, snowflakes and rivers.



Studies have shown that patients in hospitals with a view looking on nature will recover more quickly than those without, and some will argue that this is because of fractals. Due to the fact that the natural world is where we belong, and where we have come from, seeing and being in nature releases serotonin which makes us more happy and relaxed. As a result of our experiences to do with nature and specifically fractals within nature, our brains have learnt to process them with ease (just the same as the golden ratio) so that when we perceive them we feel happy and relaxed, because we are in our comfort zone, and thus consider them to be beautiful.

So next time someone says that maths is beautiful, don't be so sceptical!

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