Fibonacci and the golden ratio

Sunflowers. The universally recognised yellow burst of happiness. Home to the golden ratio. A perfect mathematical number. Too perfect.

In the 12th century, a boy named Leonardo Pisano was born to an Italian couple. He soon developed an interest in mathematics, and wrote several books, contributing greatly to number theory. In the year 1202, he wrote his book, Liber Abaci, which includes the famous Fibonacci sequence.

The Fibonacci sequence is a sequence in which each term is a sum of the two previous terms. The basic way to write it, is $F_i = F_{i-1} + F_{i-2}$

In 1843, a French mathematician named Jacques Philippe Marie Binet published Binet's formula, but was already known in the 18th century by Daniel Bernoulli, Leonhard Euler and Abraham de Moivre, which allows us to calculate Fibonacci terms quickly.

$$F_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right)$$

However, Leonardo was not the first one to come up with this sequence.

In fact, the Fibonacci sequence actually originated in ancient India, thousands of years before Fibonacci published it. The correct name would be the Pingala-Hemachandra series, giving credit to Acharya Pingala and Hemachandra.

The Fibonacci sequence, starts at zero, and continues on, 1,1,2,3,5,8,13,21,34,55,89,144 and so on, and so forth. It's a simple pattern, but it appears to be a kind of built-in numbering system to the cosmos.

Many more patterns appear within the Fibonacci set as well. Squaring two Fibonacci terms in order, and adding them together, gives us a new Fibonacci number. For instance, squaring the numbers 2 and 3, and adding those squares together, will give us 13, which is a Fibonacci number. Squaring 5 and 8 and adding those together, gives us 89.

The ratio between any Fibonacci number, and the number preceding it, tends to the golden ratio which is a unique irrational number.

$$\varphi = 1.618034...$$

The Golden Ratio is most commonly represented as the Golden Rectangle, a rectangle with side-length ratio of 1.618:1

Golden Rectangles also have the property that if you cut off a square, you'll be left with another Golden Rectangle.

But why is this number so important?

Take a hive of bees, for example. If we divide the number of female bees, by the number of male bees, the answer will always equal 1.618. Sunflowers have a 1.618 ratio between the diameters of each rotation.

If beauty had a number, this would be it.

One of the special properties of the Golden Ratio is that it can be defined in terms of itself. The Golden Ratio can be written as an infinite continued fraction. This also makes it the most irrational number possible.

$$\varphi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{\varphi}}}}$$

The golden ratio even show up in the physics of black holes! It is precisely the point where a black hole's modified heat, changes from positive to negative, and it is part of the equation for the lower bound on black hole entropy.

The Fibonacci sequence shows up in a surprising number of places too. The number of petals almost all flowers have, is a Fibonacci number. The way tree branches split, it's main trunk will grow until it produces a branch, which creates two growth points, and one of the new stems branches into two, while the other one lies temporarily inactive. Seashells grow in the Fibonacci spiral shape. Pineapples. Spiral galaxies also follow this pattern. Hurricanes. Spiderwebs. The human face. Many animals grow in the golden ratio proportion, including dolphins, ants, starfish, sea urchins, and honeybees. Even DNA molecules measurements are Fibonacci numbers!
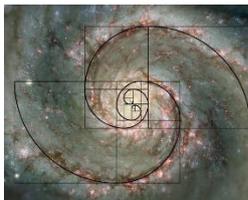
Composers and instrument makers have been using the Fibonacci Sequence and the Golden Ratio for hundreds of years to compose and create music. Mozart, for instance, based some of his classical music on Phi.

Leonardo Da Vinci is another famous artist who included this sequence in his creations. The well known Mona Lisa used the golden spiral as a grid to make it more pleasing to the human eye.

What exactly is the golden spiral?

This is a special spiral, a self-similar curve which keeps its shape at all scales, should you imagine it spiraling out forever. It is called equiangular, since a radial line from the center always makes the same angle to the curve. We derive it by first drawing the Fibonacci numbers as squares, placing them next to each other and spiralling around to ensure a golden rectangle at all times.

The Fibonacci sequence even shows up in the famous Mandelbrot set, which is a complex function, being iterated infinitely to end up as a fractal. When we look at the hyperbolic components of the fractal, starting with the largest component, we see that the number of tendrils increase in the

Fibonacci pattern. The reason for this, is because there's a way to turn this set into a disk. Now, we draw a line from the centre point, half way around the circle for example, this will lead to the second largest component. Drawing a line a third way around, will map to the third largest component. Looking for the next largest number, between the two components, will in fact be the mediant of the two previous fractions, which is a similar rule we use to find the Fibonacci sequence.
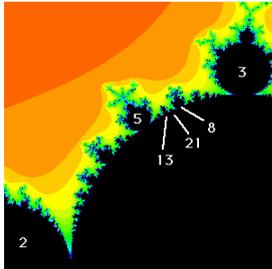
There's also someone else special which includes this pattern, called the magical Fibonacci fraction, which is equal to 1/999,999,999,999,999,999,999,998,999,999,999,999,999,999,999,999

This is magical since when we solve this as a number, we see the Fibonacci sequence coming up in every 24 spaces. This works because $1/(1 - x - x^2)$ is what is known as the generating function for the Fibonacci sequence. And the reason it's this polynomial in the denominator, is the recursion formula that the Fibonacci sequence obeys,

$F_i = F_{i-1} + F_{i-2}$, when it's written as, $F_i - F_{i-1} - F_{i-2} = 0$

If we write the operator that diminishes the index by one, as Δ, this can be written as $(1 - \Delta - \Delta^2)F_i = 0$

We can even use the Fibonacci set in the stock market, called Fibonacci retracement. A tool that technical analysts use to guide their outlook about buying and selling behavior in markets, corresponding to Phi percentages.

Pascal's triangle also includes this sequence. When we add up the numbers on the diagonals, they are the Fibonacci numbers in order.
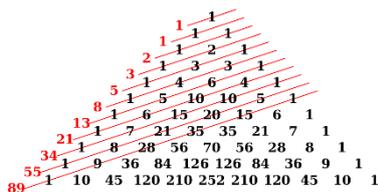
Fibonacci day is celebrated on 23 November every year. It's this day where the first four Fibonacci numbers can be represented as a date in mm/dd format (11/23).

We see that there's definitely something beautiful and quite mystical with this pattern.

I can only hope that this beautiful sequence and ratio, will be included in the theory of everything.