Euler and the 7 bridges of Königsberg

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1 Introduction:

Leonhard Euler, one of if not the greatest mathematician. I can regurgitate his extraordinary life, his extraordinary efforts in making mathematics of his time, well, more mathematical I suppose. He is reckoned to be one of the most prolific mathematicians of all time, producing over 500 books papers during his lifetime. He is also known to have produce what is ought to be the most beautiful equation we have ever seen shown by Figure 1.1. I will be spliting this in to two parts, first, what was the infamous Königsberg problem, second, it's applications and the further development of graph theory.

1.1 Eulers Formula:

$$e^{i\pi} + 1 = 0$$

2 The 7 bridges of Königsberg:

Königsberg, was a Prussian port city, abandoned in world war 2, now known as Kaliningrad. Home to the notable Philosopher Immanuel Kant where his tombstone is located. A pass time for the locals in the 18th century (apparently) was to spend their Sunday afternoons walk around their local town. The people of the town then (again, apparently) decided as a past time to see if they could get around the city only crossing each of the 7 bridges once and thanks to Bernoulli's, he was based in St.Petersburg not far from Königsberg. Now, usually in Mathematics it's usually the case that a Mathematician who is solving a ridculously hard problem which, on the surface, has no applications/ uses. Well, in this case, The tables have turned as Euler was Intrigued in something that had no direct correlation with Mathematics as he put it:

2.1 Infamous quote from the letter he sent to Giovanni Marinoni, an Italian mathematician and engineer that same year:

"This question is so banal, but seemed to me worthy of attention in that neither geometry, nor algebra, nor even the art of counting was sufficient to solve it."

3 Leonards solution:

Euler realised that he could treat the separate land masses as points and the bridges as lines connecting them points. Euler, in summary, proved that it was impossible due to the number of points(birdges) connecting the points(land masses) were odd, the journey of only crossing once only works if the the number of lines connecting the points were even. This is essentially Euler's rule and the bridges of Königsberg failed to satisfy this. So Euler Mathematically proved that it was impossible to cross all 7 bridges at once (at least for Humans!!!! that is). This paper was not just important not just for solving the Königsberg problem, but also solved the much more general case of any network of points/ vertices connected by lines/ arches. This would then later help the development of graph theory and branches out to the field of Topology.

4 "Geometria situs":

Euler thought that this problem had some correlation with a problem Gottfried Wilhelm Leibniz and himself once discussed, what Gottfried Wilhelm Leibniz referred to as "geometria situs" meaning geometry of position. We now refer to this as graph theory. graph theory is one of the most influential and important branches of mathematics and just like anything important in mathematics, came from a problem which had no direct implication to have an effect and use to the world. Graph theory, upon face value, is the study of graphs, seems simple right? Graph theory has profound applications from Computer networking so that your Internet doesn't take an hour to load your 1 hour cat videos compilation, Science, The molecular structure and chemical structure of a substance, the DNA structure of an organism. This is all due to the founding father of graph theory, Euler. From the man with the arguably one of the best beards in Scientific history Kirchhoff, to maps that we use, many have developed and found uses of graph theory. Lets talk about maps, specifically, I would like to focus on the London underground.

5 London Underground map:

Remembers Euler's rule from early, let me make a slight tweak, every measurement(length and angles etc) are all irrelevant, say no to trig!!!What is important

though is how points are connected, we see this in the map of the London underground. You see the London underground map isn't a physically representative/ accurate map, but you see this is where say, a graph representing the distance someone has travelled differs and where the beauty of the London Underground map really shines, you see, applying what Euler realised about the bridges of Königsberg, we can see that every point, well in this case, every station, apart from start to finish must have an even number of lines in and out of the station, this is why a round trip is possible as each station only has 2 lines, one coming in, and the other coming out. If you didn't understand Euler's rule at face value, think of and relate it to the London underground, just without the being squeezed next to man playing music a volume at which should concern his ears and may affect his long term hearing, oh yeah, forgot to mention his awful taste in music!!!

6 Euler's Brilliance

Euler was one of the most influential men of his time he is one of the most recognisable figures in Mathematics, in his later years how ever, most notably, his eyesight began to gave way and by 1771, Euler went completely blind, this didn't seem to bother him for as he put it (upon losing the use of his right eye) "Now I will have less distraction." Euler then suffered from a Brain haemorrhage and died on September 18, 1783. Remember at the start when i said that Leonhard Euler was one the most prolific of all time, well 50 years after his death, his work was still being published influencing almost every area of Mathematics. Euler turned any problem into a purely Mathematical one, a good one issued by Paris Academy of Science in 1727 where they asked on the optimal way to arrange Masks on a ship? he turned practical problem into a mathematical problem and as far as I can see, he has done this for The 7 Bridges of Königsberg and spawned one of the most influential branches of mathematics, Graph Theory.