<u>The Expectations of Gambling</u>

If there's one constant in this world that is universally loved, it would be money. Who wouldn't want to have all the money in this world? However, humans are imperfect beings. The human condition naturally dictates that if we want to achieve something, we would want to gain that particular thing in the easiest way possible. So, what would be the easiest method of gaining money without doing anything heinous or immoral? Well, a prime example would be gambling. Although, one could argue that gambling itself is an immoral act, but that's a debate for another time.

Theoretically, gambling could be one of the 'relaxing' ways to strike it rich. I mean, how hard could it be? Just play a couple games at the casino and you will feel like a million bucks afterwards. However, as the saying goes, looks can be deceiving. The next thing you know, you will be burning a hole in your pocket and hit rock bottom on your way home. The burning question that is on everyone's mind is quite simple – why? Why do people seem to lose more money rather than make money when gambling even though in theory, it seems rather simple to gamble in casinos? To answer that question, we will have to descend into the deep, beautiful world of mathematics.

It all started back in the 17th century when the famous mathematician Blaise Pascal sorted out the basics of probability in order to answer some tricky gambling questions. Say you have a single traditional die, roll it once, the chance of getting a 5 is 1 in 6. The same concept also applies to the roulette wheel found in casinos, where there is a 1 in 37 chance that 15 comes up. Here's where money comes into play. To a gambler, what really matters is not only the odds of winning, but also how much will they make if they win.

This leads to the idea of expectation – the expected fraction of the gambler's bet he expects to win or lose. Suppose we bet a dollar on red on roulette. We have an 18 in 37 chance of red in which case we win a \$1. On the contrary, there is also 19 in 37 chance of losing \$1. If we keep betting \$1 on red, on average we expect a loss of $\frac{18}{37} - \frac{19}{37}$ which is $-\frac{1}{37}$ of \$1, or $-\$0.03$. The implication of this is in the long run, we expect to have lost 3% of what we have bet. What if you bet that the number 15 comes up? If 15 comes up we will win \$35 and there's a 1 in 37 chance of that happening. On the opposite end, there's a 36 in 37 chance of losing our dollar. So to visualize it mathematically, our winning odds are $\$35 \times \frac{1}{37}$ while our losing odds are $\$1 \times \frac{36}{37}$. Therefore, our expectation, E would be $E = \$35 \times \frac{1}{37} - \$1 \times \frac{36}{37}$. Solving it further, $E = \frac{35}{37} - \frac{36}{37}$, which concludes in the same result as our first example, $E = -\frac{1}{37}$. As a matter of fact, no matter what you bet on the roulette, the expectation will always be $-\frac{1}{37}$.

Expectations can vary dramatically on gambling games, some casino games have close to 0% expectation down to -40% or so on some lotteries. The one common pattern here is that the expectation is pretty much guaranteed to be less than zero and negative means losing. Now that we've tackled the first question which is why does it seem far more likely to lose money rather than gain them in casino games, what can we do about it? A popular trick is to vary the size of your bet depending on whether you win or lose. One famous scheme is the Martingale strategy, which is also used in other branches involving risks like investing.

The essence of this strategy is this: following our first example, we will bet on red in roulette and we'll start by betting \$1. If red comes up, we win \$1 and we repeat the \$1 bet. If red does not come up, we lose \$1. To make up for our loss, we play again but with a doubled wager of \$2. If red comes up, we win \$2 and if we add up the \$1 loss from our previous game, the winnings amount to an overall gain of $1. Now that we have won, the next bet would go back to the initial bet of $1. What if red does not come up in our $2 bet? Well, we play again but with a doubled wager of $4. If red comes up, we win $4 and combined with the total loss of $3 from our two previous games, it will amount to a net profit of $1. After winning, we will go back to square one and bet $1 and repeat the bet. If we lost our $4 bet, we keep doubling our wager until we win our bet and gain a profit of $1. This process practically guarantees you always coming out on top in the long run.

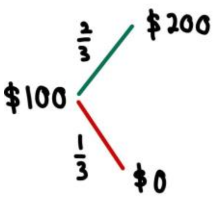
Let us ask another question: do bet varying schemes work? Looking at it from a mathematical perspective, it's quite tricky to answer these kind of probability questions, depending in a subtle way of our assumptions. For example, the martingale strategy definitely works if you happen to have infinite dollars to spare, but that defeats the whole purpose of gambling itself. With a finite amount of money in our pocket, what can we expect to happen? Suppose we make a sequence of bets with the same expectation for each bet, as in the setup we just looked at. The total amount we expect to win or lose is easy to calculate, it is just E multiplied with the number of bets. If the E is negative, then we'd be in trouble.

This brings us to the fundamental theorem of gambling, which states that if the expectation is negative for every individual bet, then no bet variation can make the expectation positive overall. The prospect of making profits from gambling is quite dire it seems unless we somehow find a game with positive expectation. Let's assume such a game does exist. How well can we do? If the mentioned game has the chances of winning are 2/3 and therefore chances of losing are 1/3. The game will also have similar winning system as in roulette in which you win or lose whatever amount you bet. The expectation of this game will be positive, precisely at 33%.

$$P(W) = \frac{2}{3}$$

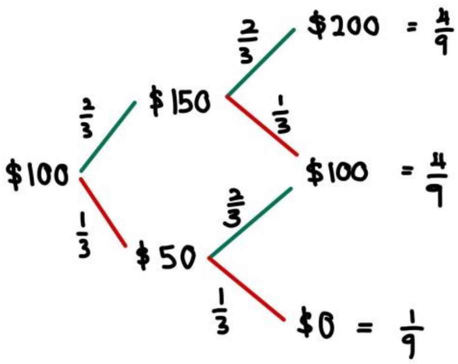
$$P(L) = \frac{1}{3}$$

$$E = 33\%$$



Now, let us play the game. Say, we start off with $100. How can we double that amount to get $200? If we bet all of our starting money, then obviously the chance of doubling it is 2/3. Here, we can visualize the game using a tree diagram to keep track of our bets' chances. Surprisingly, we can actually improve our chances if we bet $50 at a time and play until we lost all our money or we double our money. Let's draw another tree diagram to illustrate this.

If we place a bet of $50 initially, after one round, we will either have $150 or $50. Repeating the same bet of $50, we either have $0, $100, or $200. Reading off the tree, we see that the probability of doubling our initial money in the first two bets is 2/3 multiplied with 2/3 which is 4/9. Similarly, the probability to go back to square one which is $100 is 2/3 multiplied with 1/3 plus 1/3 multiplied with 2/3, which coincidentally, is also 4/9. If we're back at $100, we can keep on playing until we eventually have doubled our money or go bankrupt. Assume that D are the chances of doubling our money. In this particular case, D will equal to 4/9, which is the probability of doubling our money after two bets plus a second 4/9, the probability of being back where we started from multiplied with the probability to be able to double from this point on. This formula can be visualized as seen below.



$$D = \frac{4}{9} + \frac{4}{9} D$$

$$D = \frac{4}{5}$$

Now, let’s solve for D which gives that D is equal to 4/5 which is 80%, which is a vast improvement over the probability of going with just one bet of $100 which is 66%. If we repeat the trick and replace $50 with $25 instead, this results in about 94% chance of doubling our money. As a matter of fact, by making the bet size smaller, the probability of doubling our money will increase, eventually reaching to near certainty. Once we’ve doubled our money, we could go for more ambitious goals and quadruple, octuple, etc. our money using the same concept of doubling our money. This method is actually true no matter what probabilities we’re dealing with as long as the expectation of the game we play is positive. The conclusion of all of this is the second theorem of gambling – if the expectation is positive, then we can win as much as we like with as little risk as possible by betting small enough for long enough.

There is a glaring problem in this sea of optimism: no such game with such positive expectation exists yet. However, this does grants us a sneak peek into the confusing yet beautiful world of probability in mathematics. This theorem could also be applied to other aspects of life, such as investing where the risks and the rewards are taken into account before making every investment. To get back to the topic at hand, it would seem like gambling is a lost cause. In addition to the negative expectation of winning a bet in the casino, other negative factors such as the house edge are not yet taken into account, which further increases the amount of loss that you’d get from gambling. Perhaps gambling is not the best option for you if you’re looking to make a quick buck.