Whilst out on the water, you wouldn't expect that mathematics would affect you, or play an important role. You would simply hop aboard and the boat would sail off into the distance. However, sailing is only achieved through numerical means: the stability of the boat, the angle of the sails, the speed, height and direction of the tide and bearings to find your way. Fundamental ideas to consider whilst sailing are math related, so heading out into the sea without understanding these concepts and where the numbers have originated from as well as being able to use traditional navigational methods, is likely to be a bad idea, especially being the skipper of a vessel.

When sailing, the force of the wind is counterbalanced by the weight and buoyancy of the sailing yacht. The weight includes the crew weight, the weight of the keel and the equipment within the boat. For a monohull sailing yacht, there is a range of positive stability angles. At different angles the CG (centre of gravity) and CB (centre of buoyancy) do not change in their direction of force but the CB changes its placements and the CG stays in the same position, in the central point of the boat. The range of possible values that have a positive righting moment are between 0 and 130 degrees, with approximately 65 degrees being the best angle for stability and speed, and the AVS (angle of vanishing stability) at 130 degrees, after this, the boat cannot return to 0 degrees as the CB lies to the left of the CG, preventing rotation to the original state.This can be similar to the rotation in a dynamo but in comparison, the rotation cannot continue in a boat. The positive range values also create a parabola.

The stability is also based on the strength of the wind, and the point of sail. You cannot sail closer than 45 degrees to the wind, otherwise the boat slows down dramatically as the wind is hitting too forward at the bow. At different angles to the wind, your angle of sail changes, to maximise area of sail. As you sail close to the wind, it hits the yacht on the port or starboard bow and continues to travel until it hits the mast, where it splits. After splitting, the wind follows the shape of the mainsail, the outside of the curved main sail (the leeward side) gives the wind a further distance to travel therefore the wind accelerates, causing pressure to drop. The wind traveling across the windward side of the mainsail creates a high pressure side, as high pressure moves naturally towards the low pressure side, the resulting effect is lift, or the boat is 'sucked' forward. The faster the wind travels over the sails the greater the amount of lift. When the headsail is introduced it acts in the same way as the mainsail. Additionally, it creates a narrow point between the headsail and mainsail which is known as the 'slot'. The wind is forced through this gap and acts as a jet engine pushing the boat forward. So at closer angles to the wind, the sails are pulled tightly in and at larger angles, the sails are let outwards. Furthermore, when wind speed increases, the area of the sail needs to be reduced as the boat can only handle a certain amount of wind before the force overpowers the weight and buoyancy, therefore sails are reefed.

Another major determining factor that affects sailing once boat speed is established by adjusting sails and stability is the tide. The gravitational pull of the Moon and the Sun is the main cause of tides. The alignment of the Moon and the Sun can produce different types of tides, which produce different heights of tides. Both the Moon and Sun produce a complementary wake from their respective gravities on opposite sides of the Earth due to both of them now having more gravitational pull on the Earth than the oceans on that side. The weaker gravitational pull of the sun, which is about half that of the Moon, distorts the Earth's oceans, pulling them towards it. On a New Moon and Full Moon, the stronger gravitational pull exerted by the Moon combined with that of the Sun due to their straight line alignment distorts the Earth's oceans, creating higher tidal ranges, known as spring tides. On a first quarter and last quarter, the Moon is at 90 degrees to the Earth relative to the Sun. This now counteracts the gravitational pull between the Sun and the Moon, giving a smaller difference in tidal ranges, known as neap tides. In addition, there are daily tides as each day the Earth rotates, we experience two high and two low tides. The height of the tide is measured above CD (chart datum) which is the recorded depth. Tides can be recorded and tide tables can be produced for larger ports and indicating the times and heights of high and low water for every day of the year. Tide heights at specific times can be calculated with tidal curves. They use the low and high waters, as well as separate curves for springs and neaps, factored so that you can work out times of a certain height of tide or vice versa.

Although these factors are to be considered whilst sailing, traditional navigation is dependent on astro-navigation. This depends upon observing the angle between the horizon and the heavenly body of your choice. At present, there is a device called the sextant. It is called this because its calibrated arc is one sixth of a circle (60 degrees). By doubling the effect of its mirrors it is able to measure up to 120 degrees, representing a big leap forward from its predecessor the octant (45 degrees, doubled to 90 degrees). A sextant gives the correct altitude for the body and navigational tables work on the assumption that the observer and the sextant is at the centre of the terrestrial sphere, and not on the Earth’s surface. The Earth has a measurable size, at least in comparison to the Moon and the Sun, but this discrepancy leads to an error of parallax between what they are actually seeing (the terrestrial or visual horizon) and what the tables want them to see (the celestial horizon). When using a sextant, a few corrections must be made to reach the true altitude of the body. The dip is the correction applied because of the height of your eye enabling you to see beyond the theoretical terrestrial horizon. The refraction correction is necessary as light from a heavenly body is bent by the Earth’s atmosphere. Additionally when heavenly bodies are observed at altitudes below 10 degrees, the refraction produced by the Earth’s atmosphere begins to increase rapidly, producing unreliable results in practice. The semidiameter involves guessing the position of the centre of the body inaccurately by placing it on the upper or lower limb. The parallax, which is the displacement or difference in the apparent position of an object viewed along two different lines of sight, is also related to the two corrections mentioned above.

Obtaining a position using astro-navigation techniques, is then plotted on a chart using position lines, but these lines are theoretically circular. For such huge circles to be usable, only particular sections are drawn on a chart. A determining line gives the bearing of the body from your rough position (in degrees True) and then constructs position lines at right angles to it. The bearing line is called an Azimuth, defined as the horizontal direction of a celestial point from a terrestrial point. Using a compass however, measures in degrees Magnetic, as there is a true north as a fixed point on a globe and a magnetic north where the compass needle points as it aligns with Earth's magnetic field. However the magnetic north pole shifts and changes over time in response to the changes in Earth's magnetic core. This is where variation and deviation come into play. Magnetic bearings are affected by variation and compass bearings are affected by both variation and deviation. They are either west or east and depending on conversion, these errors are either added or subtracted.

Finally, defining our position on the globe is effectively by coordinates, latitude and longitude. In an east/west direction, we make use of the meridians of longitude. They are great circles which converge at the poles of Earth. Any great circle must have equal halves in the northern and southern hemispheres, and its points of greatest latitude are called vertices. At a vertex, a tangent to the adjacent parallel of latitude is made and is thus travelling briefly east-west. Positions are measured in terms of angular distance east or west of the datum meridian as we know as the Greenwich Meridian. For the fix in the north/south direction, we use the parallels of latitude, which defines the angular distance north or south of the equator. It is a great circle on a plane of right angles to the Earth's axis, halfway between two poles, creating a fix of degrees north/south and west/east. There is a subdivision of degrees of 60 minutes, then 60 seconds. By convention, 1 minute of latitude is equal to 1 nautical mile and 1 second of latitude is equal to 101 feet or 3 boat lengths for the average yacht.

A variety of different aspects of mathematics are involved in the concept of sailing and navigation. Not all of these are put into consideration whilst voyaging, but by understanding the theory behind the Earth, and how it has related to navigational methods used today, is significant for any skipper or for that matter, any mathematician.

Sources

Celestial Navigation *colour edition* by Tom Cunliffe
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RYA Day Skipper Shorebased Notes (theoretical course booklet)
www.rya.org.uk