The natural number and its applications

Your chances of success at the lottery, the placement of cards after they're randomly shuffled and the scale of magnitude, can all be defined in terms of a shared constant- the natural number, or, *e* ≈ 2.71828.

Euler's number is inescapable; it's occurrence in our daily lives is so frequent it's almost unsettling, and in this essay I will describe its prominence in mathematical formulae, and its inherent disguised presence in our everyday lives and actions.

As the *natural number*, e's prevalence in mathematics and economics is infamous. Jacob Bernoulli would come across this notion in researching compound interest in 1683; he regarded that as the compounding periods occurred more frequently, i.e. instead of taking 100% interest after 1 year, 1/365 interest is paid daily throughout a year (where n=365), the final amounts came closer to reaching a specific limit, the closer n was to ∞. Therefore he could conclude that if1/∞ interest was taken instantaneously, the resultant amount would be *e*.

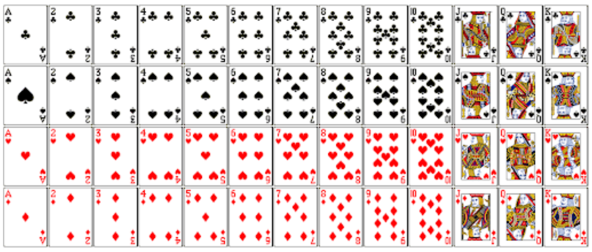
$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

But *e* had been known before that, with the research appendix on the natural logarithm by John Napier, in Mirifici Logarithmorum Canonis Descriptio: *A Description of the Wonderful Law of Logarithms*, in 1614. As it turned out, the base of Napier's logarithm, and Bernoulli's limit in fact equated to the same number, the irrational *e*. Since, discoveries of its applications in fields of maths and science have become innumerable.

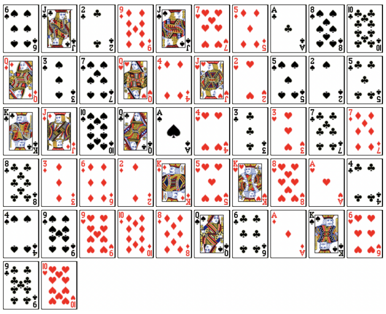
The function e^x also possesses an uncanny property; its rate of change (i.e. it's derivative) is always itself. In other terms, if a hypothetical plane's position P(t) equated e^t, where t=time in seconds, its position, velocity and acceleration are always all equal. Imagine this hypothetical plane at a position 350m from an origin, according to our formula, its velocity is 350ms^-1, its acceleration is 350ms^-2, and its further derivatives will follow this pattern.

The unavoidable appearance of e in probability becomes unnerving when you realise there really is a countless number of instances e could ever pop up in.

To use the first example of playing cards, imagine you open a fresh deck, and lay them out in their original order as below.



Now you reshuffle them, and lay them out on a table again, in their new random order. Although 'random', we can deduce that the probability that no one card remains in its original place is 1/e.



Continuing on the area of probability, let's use the example of the lottery, in which a million players have an equal chance of winning. This means that the odds of winning are 1/1,000,000. If someone played this lottery one million times, their chances of losing every time, stubborn they may be, comes to 1/e.

The role of e in the 'perfect identity' is yet another factor that deems it so important. Voted the most beautiful formula in mathematics, the perfect identity combines the most famous irrational numbers with flawless precision.

$$e^{i\pi}+1=0$$

But what's all the more significant is where this identity comes from, in other words, Euler's formula, where e is the base of the natural logarithm, i is the imaginary unit, and cos and sin are the trigonometric functions cosine and sine respectively:

$$e^{ix} = \cos x + i \sin x$$

This equation links an exponential function with e as the base to sinusoidal function, of which the a and applications are vast, from signal analysis to quantum mechanics and a simple mass on a spring for example. As these all involve some sort of oscillation we can expect to see Euler's formula.

Euler's formula is ubiquitous in mathematics, physics, and engineering. The physicist Richard Feynman called the equation "our jewel" and "the most remarkable formula in mathematics".

Euler's formula is so momentous as it incorporates *i*, or, the imaginary number (a number that when squared gives a negative result), allowing us to manipulate it. For example, if we substitute a constant like 1.1 in place of x, we can plot our findings on the complex plane (in which the real numbers go left-right, and the imaginary numbers go up-down), thus allowing us to visually interpret *i*, as well as relate it to an exponential function. Also, Euler's functions plotted on the complex plane produces a perfect circle.

Finally, it is important to mention e's role in explaining orders of magnitude; *e*^ln(10)x=10^x, up to the scale of the universe. Differences in order of magnitude can be measured on a base-10 logarithmic scale in "decades" (i.e. factors of ten). The use of e provides a generalised formula for the order of magnitude. For example, take ln base 10; an increase of one order of magnitude is the same as multiplying a quantity by 10. An increase of two orders of magnitude is the equivalent of multiplying by 100, or 102. In general, an increase of n orders of magnitude is the equivalent of multiplying a quantity by 10n. Therefore, 2315 is one order of magnitude larger than 231.5, which in turn is one order of magnitude greater than 23.15.

As values get smaller, a decrease of one order of magnitude is the same as multiplying a quantity by 0.1. A decrease of two orders of magnitude is the equivalent of multiplying by 0.01, or 10-2. In general, a decrease of n orders of magnitude is the equivalent of multiplying a quantity by 10-n. Thus, 23.15 is one order of magnitude smaller than 231.5, which in turn is one order of magnitude smaller than 2315.

This diagram illustrates the order of magnitude, from space time (quantum foam) at 10^-32m, to the whole universe at >10^27m.

To conclude, we have all encountered Euler’s number in more ways than one- from natural logarithms to the definition of the exponential function, as well as in ways we don't perhaps reaslie first-hand, such as with regard to probability of your seemingly random decision-making. Of course, there exists countless applications of the natural number, but these selected examples go to its vast applicability in our everyday lives and advanced mathematics.