The Controversial History of Calculus

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1 What is Calculus?

In simple terms Calculus can be split up into two parts: Differential Calculus and Integral Calculus. Differential calculus deals with changing gradients and the idea of a gradient at a point. Integral Calculus deals with the area under a curve. However this does not capture all of Calculus, which is, to be technical, the study of continuous change.

One of the difficulties in really defining calculus simply is the generality in which it must be described, due to its many applications across the sciences and other branches of maths and since its original 'conception' many thousands of years ago many problems have arisen, of which I will discuss some of them in this essay.

2 The Beginnings of a Mathematical Revolution

Calculus, like many other important mathematical discoveries, was not just the work of a few people over a generation. Some of the earliest evidence we have about the discovery of calculus comes from approximately 1800 BC in Ancient Egypt. However, this evidence only shows the beginnings of the most rudimentary parts of Integral Calculus and it is not for another 1500 years until more known discoveries are made.

Many important ideas in Maths and the sciences have their roots in the ideas of the philosophers and polymaths of Ancient Greece. Calculus is no exception, since in Ancient Greece both the idea of infinitesimals and the beginnings of the idea of a limit were began forming. The idea of infinitesimals was used often by mathematicians in Ancient Greece, however there were many who had their doubts, including Zeno of Elea, the famous paradox creator, who found a paradox seemingly inherent in the concept.

The idea of an infinitesimal and the paradoxes that can arise from them is one of the most controversial and contentious parts of Calculus, continuing right up until this day.

Even though there were many small but fairly significant developments across the world in Calculus from about 300 BC to the 17th Century, it wasn't until Newton and Leibniz that calculus was truly formulated.

Both of them originally considered the use of infinitesimals in their ideas, however they split off in their thinking due to their differences of opinion of infinitesimals. Newton, realising that they were unwieldy tried to get away from them without a great deal of success whilst Leibniz wholeheartedly embraced them. However, the reason their names have become so synonymous with calculus whilst all the other people involved in its development up to that point are not, is that both Leibniz and Newton created notation for and completed the missing link between Integral and Differential Calculus: The Fundamental Theorem of Calculus.

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F(x) is the antiderivative of f(x).

3 The Controversies

One of the greatest controversies surrounding Calculus is that of the infinitesimal. At first glance they seem very intuitive and look like they work, however it is very difficult to define them rigorously, as they can often lead to paradoxes, but on the other hand they seemed to be intrinsically linked to Calculus and even though they didn't really work themselves, Calculus worked. This persisted until the 19th Century when the idea of a limit arose to take over the place of an infinitesimal as the premier way to define Calculus, due to it being more rigorous and precise. From this new basis, many other mathematical discoveries could be made with the rigour required of them to be accepted by both the mathematicians and philosophers of the time.

However, during the 20th Century a mathematician by the name of Abraham Robinson developed a rigorous framework in which both infinitesimal and and infinite numbers could be incorporated back into modern mathematics. This was given the name of Non-standard Analysis, as opposed to Real Analysis and Complex Analysis which were already very important branches of mathematics. As of now the debate between these two competing ideas has yet to be resolved, however the idea of limits has such a grip that it is unlikely that infinitesimals will retake their position as the premier way to define Calculus.

However, there have been many implications from Non-standard Analysis such as Smooth Infinitesimal Analysis which arrives from Topos Logic and a set of 4 axioms including the Kock-Lawvere Axiom, which in particular states:

Let $D = \{d \in R | d^2 = 0\}$. Then for all functions f from all D to R, and all $d \in D$, there is a unique $a \in R$ such that $f(d) = f(a) + d * a$

R is defined in the other axioms as an extension of the real number line to include infinitesimals.

Another of the controversies and the most famous, is that surrounding whether Newton or Leibniz came first in their ideas. This particular controversy originates from Newton himself after he claimed that Leibniz had plagiarised his work. However, it is now believed that they both independently developed it at around the same time, clearing Leibniz, whose notation has become the more standard of the two.