Golden Ratio: Applications in Design and Nature
Shanjeev Mathialagan

Why is this one of the most interesting numbers in human history? Despite being an irrational number with an infinite number of digits after the decimal point, it is frequently applied to design and repeatedly occurs in nature. This “golden” number is represented by the Greek letter Phi Φ and is more famously known as ‘The Golden Ratio’. Phi’s approximate value is 1.61803, but its exact value is written as $\frac{1+\sqrt{5}}{2}$.

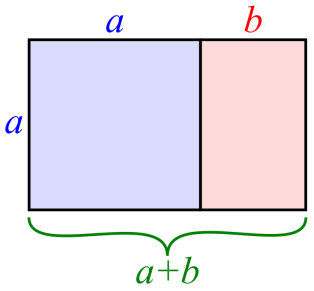
Mathematics

It was first studied by Ancient Greek mathematicians, because it regularly appeared in geometry. Euclid, regarded as the ‘father of geometry’, called it "the division in extreme and mean ratio”.

Simply, the Golden Ratio is when the ratio of two quantities is equal to the ratio of their sum to the larger quantity (with a being the larger quantity here):

$$\frac{a+b}{a} = \frac{a}{b}$$

In geometry, a golden rectangle is a rectangle which has its dimensions in the Golden Ratio. A golden rectangle that has long side *a* and short side *b* produces a similar golden rectangle when adjacent to a square with sides of length *a*. The larger golden rectangle now has the long side *a* + *b* and short side *a*[1]. This helps us to visualise the relationship between two quantities which are in the Golden Ratio.



Fibonacci Numbers:

There is a relationship between the Golden Ratio and the Fibonacci sequence (a sequence where each number is the sum of the two numbers in front of it). The ratio between any two consecutive Fibonacci numbers is very close to the Golden Ratio. For example, the ratio between 144 and 233 is 1.618056.

Nature

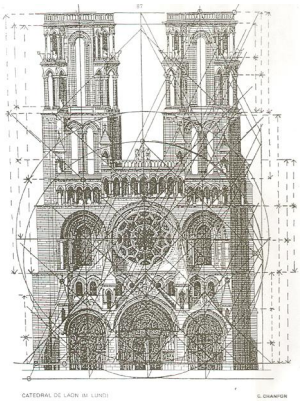
The Golden Ratio is even called the "divine proportion”, because of its frequency in the natural world. The number of petals in flowers follows the Fibonacci sequence and the Golden Ratio. For example, lilies have three petals and buttercups have five petals; both three and five are numbers in the Fibonacci sequence. Phi also plays a significant role in the arrangement of petals. It has been found that each petal is placed at 0.61803 per turn (out of a 360° circle) in order to maximise the exposure to sunlight to maximise the rate of photosynthesis, which is the process that produces food for the plant. 0.61803 is the reciprocal of Phi (1/1.61803...) to 5 decimal places. The reciprocal of a number is the inverse of the number, so we divide 1 by the number (in other words the number is now written to the power of -1). For example, the reciprocal of Phi is 1/Phi, also written as Phi^{-1}.

Curiously, it is claimed that Phi appears throughout the human form with many proportions of the body having a link to the Golden Ratio, such as in the face, body, and fingers. For example, the ratio of the forearm to hand is the Golden Ratio. Furthermore, DNA molecules, which carry the genetic code that programs our bodies, are based on ‘The Divine Proportion’. A DNA molecule measures 34 angstroms long by 21 angstroms wide (1 angstrom is a unit of length equal to 1×10^{-10} metre) for each full cycle of its double helix spiral. Naturally, 21 and

34 are successive numbers in the Fibonacci sequence and their ratio, 1.6190476 closely approximates Phi, 1.6180339.

Architecture

Throughout history, the ratio has been used by designers to create magnificent structures. Some well-known examples include the Great Pyramids of Giza in Egypt, the Parthenon in Greece and the Taj Mahal in India. The proportions of these constructions show that the architects deliberately applied the golden ratio to their designs, since it was believed to be more aesthetically pleasing. So is it really a coincidence that the named examples above are all Wonders of the World?



In his 1919 book Ad Quadratum, Frederik Macody Lund, a historian who studied the geometry of several Gothic structures, claims that the Notre-Dame of Laon has golden proportions - shown by the superimposed regulator lines on the illustration[3].

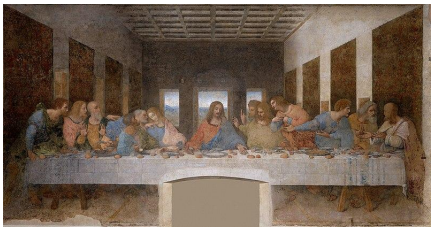
More modern applications of the Golden Ratio can be seen in the design of Toronto's CN Tower. For example, the ratio of the observation deck at 342 metres to its total height of 553.33 is 0.618, the reciprocal of Phi.

So why do architects keep using the principles of the golden rectangle? When applied to design, it is not only visually attractive but it is also one of the simplest ways to impart a sense of balance to a structure. It can be used for the various proportions that appear in a structure, ranging from its height, width, roof size, door and even window positions. By simply following the principles of the Golden Ratio, an architect can easily alter a building's proportions to make it larger or smaller to satisfy the needs of the client.

Art

"Without mathematics there is no art," said Luca Pacioli, an Italian mathematician in the 15th Century, who collaborated with Leonardo da Vinci.

The Golden Ratio is not just found in the design and beauty of nature; it is also present in art. Artists used the Golden Ratio in their work because it was thought to be attractive.



Leonardo da Vinci has used it elegantly to create visual harmony within his paintings. One famous example is "The Last Supper": the key dimensions of the room and table were based on the Golden Ratio, which was known as 'The Divine Proportion' in the Renaissance period[4].

Conclusion

Overall, the Golden Ratio is a mathematical ratio that can be found all around us from nature to architecture to art, which are just a few examples. The use of the Golden Ratio in these various distinct areas is not down to coincidence.

Although we will never find one size that suits all the needs in design and nature, the Golden Ratio, however, is a tangible, mathematical approach that will always help us create impressive designs and view them in nature.

Reference List

1. Golden Ratio. [online] Wikipedia. Available at: https://en.wikipedia.org/wiki/Golden_ratio [Accessed 10 April 2021]
2. Clayton, C. (2017). *Fibonacci Numbers & The Golden Ratio in Flowers.* Cazamic. Available at https://www.cazamic.com/science/fibonacci-numbers-the-golden-ratio-in-flowers/ [Accessed 10 April 2021]
3. List of works designed with the golden ratio. [online] Wikipedia. Available at: https://en.wikipedia.org/wiki/List_of_works_designed_with_the_golden_ratio [Accessed 10 April 2021]
4. Meisner, G. (2014). *Golden Ratio in Art Composition and Design.* Goldennumber. Available at: https://www.goldennumber.net/art-composition-design/ [Accessed 10 April 2021]

Images

1. Golden rectangle, https://commons.wikimedia.org/w/index.php?curid=15656641, Public Domain, By Ahecht (Original); Pbroks13 (Derivative work)
2. Buttercups, https://unsplash.com/photos/HLHaSGAuF0A, Unsplash License, By Ant Rozetsky
3. The Last Supper, https://commons.wikimedia.org/wiki/File:Leonardo_da_Vinci_(1452-1519)_-_The_Last_Supper_(1495-1498).jpg, Public Domain
4. Illustration of Notre-Dame of Laon cathedral, https://upload.wikimedia.org/wikipedia/commons/8/80/Laon_Cathedral%27s_regulator_lines.jpg, Public Domain, sourced from a book by Frederik Macody Lund

Further Reading?

1. The Golden Ratio: The Divine Beauty of Mathematics By Gary B. Meisner
2. Irrationality of the Golden Ratio: Numberphile Video: https://www.youtube.com/watch?v=sj8Sg8qnjOg
3. https://www.mathsisfun.com/numbers/golden-ratio.html
4. https://mathworld.wolfram.com/GoldenRatio.html