Unseen Mathematics in the Natural World

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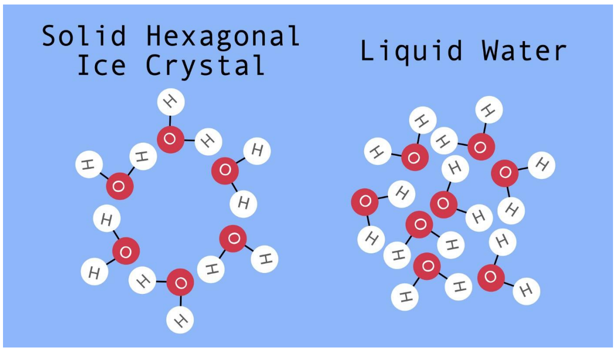
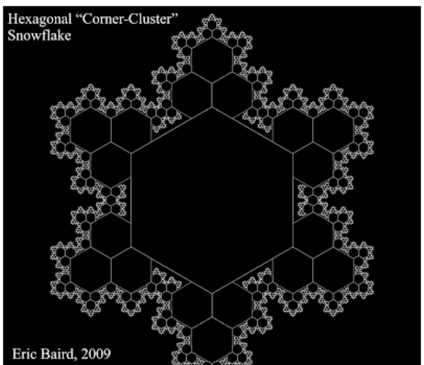
How can the two, seemingly discrete concepts of mathematics and nature, be related? In this essay, I will dissect a few of the many ways in which mathematics can be perceived in nature.

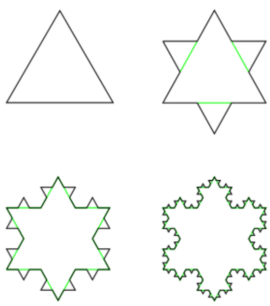
Honeycomb and Hexagons

William Kirby, the father of entomology, named bees "heaven-instructed mathematicians" as they cleverly use hexagons to create their honeycomb. Hexagons are highly efficient shapes because most of their area is open space, which allows bees to use a smaller amount of wax for a larger volume of honey. More complicated patterns will result in more wax being used, so less honey will be stored. Bees produce one ounce of wax from eight ounces of honey, so creating the least possible wax will save time and effort. As well as this, hexagons do not divide into miniature versions of themselves, therefore they are strong and sturdy. Hexagons are used by honey bees because they increase their chances of survival, ideal for the natural world.

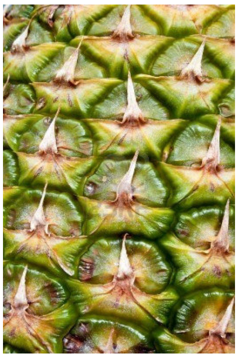
Another example of hexagons used in nature is snowflakes. Snowflakes have six sides and join together in a hexagonal structure. Their arrangement is one oxygen atom bonded to two hydrogen atoms inside one hexagon, with oxygen atoms being bonded to three hydrogen atoms in the whole snowflake. Their regular structure repeats to create a snowflake with six rotational lines of symmetry. The arrangement of the hydrogen and oxygen molecules are bonded efficiently, similarly to that of the honeycomb. Snowflakes are fractal structures, meaning they are made up of similar patterns that repeat at progressively smaller scales. This term was coined by the mathematician Benoit Mandelbrot in 1975, meaning "broken" or "fractured" to convey the chaotic essence of nature. Koch's snowflake has an infinite perimeter because the fractals infinitely repeat, allowing for limitless iterations. Each iteration multiplies the number of sides of Koch's equilateral triangle by four, and in a hexagonal snowflake this multiplies the number of sides by seven.

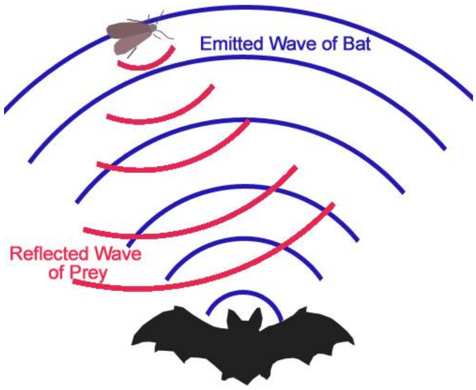






Furthermore, we see hexagons in animals themselves, such as on the shells of turtles. The shell is made up of tesselations of hexagons surrounded by a ring of pentagons. We can also see this structure in fruits, such as pineapples. In summary, hexagons are used throughout nature because of their highly efficient design.



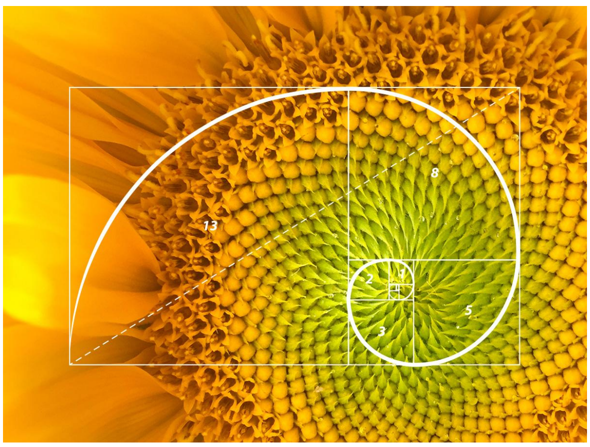


Echolocation

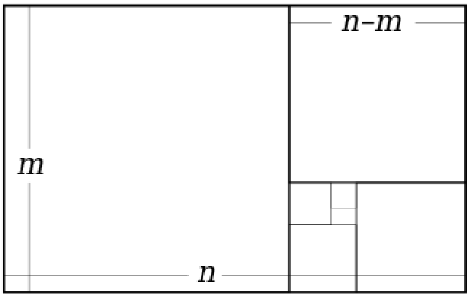
Echolocation is used by bats to navigate and find food when it is dark. Bats use sound pulses with frequencies of 9kHz to 200 kHz that are emitted through their mouth or nostrils. These sound waves are reflected by objects, producing echoe, meaning the prey of the bat is reflecting sound waves and these echoes can be picked up by the ears of the bat, determining the location of their prey. Echolocation is also used by dolphins, which is more efficient as sound waves travel 5 times in water than in air. Dolphins pick up the sound waves through their lower jaw and foreheads as these are filled with fatty tissues that channel the sound waves towards the ears and then the brain where the sound waves are interpreted.

You can determine the distance from an emitter to a receiver by taking away the initial time from the final time, which is the time difference, and then because the speed is equal to distance/time, we can calculate the distance by doing speed x time, then divide by 2 to find the length between the emitter of the wave and the receiver (e.g. a bat and a moth). Yet again, we see that this adaptation has increased the survival rate of bats, similar to the adaptation of the structure of a bees nest. In both scenarios, both have evolved to try and maximise their chances of survival (demonstrating Charles Darwin's theory of evolution and natural selection).

God's Fingerprint



The Golden Ratio is often referred to as God's Fingerprint as it is found everywhere in nature, from seashells to sunflowers. This ratio is best estimated by the fibonacci sequence, where each term is made through the sum of the two terms before it. The golden ratio is around 1.618 and represented by the greek symbol phi.

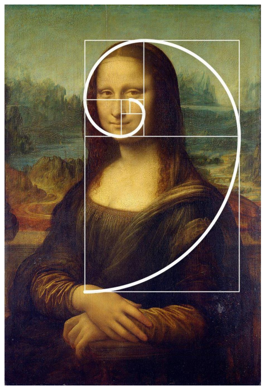


The best way we can represent this mathematical relationship is through the use of a rectangle. The longer side is n and the shorter side m. The 2nd largest rectangle is m-n because inside the rectangle the left square has equal sides. This process repeats itself with the rest of the squares.

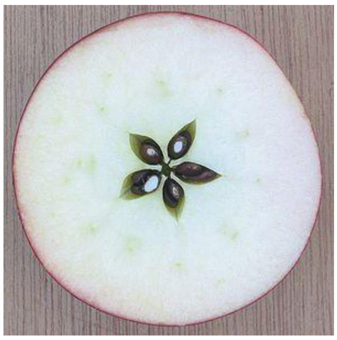
We see this in nature where the number of petals on a flower head will often be a fibonacci number. Also, the petals of a plant can be arranged in a 'golden angle'.
When one petal sprouts, another sprouts away from the first petal at a golden angle. This process can repeat itself endlessly (nearly!) because there is always space between petals where a new one can grow.

Examples of the golden angle in nature can be found in pinecones, flowers, hurricanes, spiderwebs, chameleon tails, and even tsunami waves. Humans, too, have utilised the beauty of the golden ratio in paintings, for example the famous Mona Lisa and Starry Night. By using this technique, artists make the paintings easier to process by the human eye as they look more perfect and

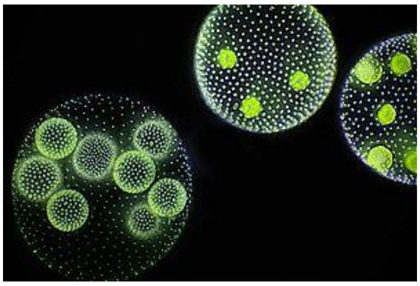
aesthetically pleasing, ironic since the golden ratio is a messy and irrational number. Adrian Bejan, a mechanical engineer in Duke University, argues that the reason we see the golden ratio in everyday objects is because as animals evolved and got 'smarter', we started noticing finer details. As there are two sides to every story like there are two sides to a coin, there are two main arguments about the golden ratio. Some people believe that it is true and found everywhere in nature, whereas some people say that we see the golden ratio everywhere because it is easier to store information that way if we see something of a known proportion.



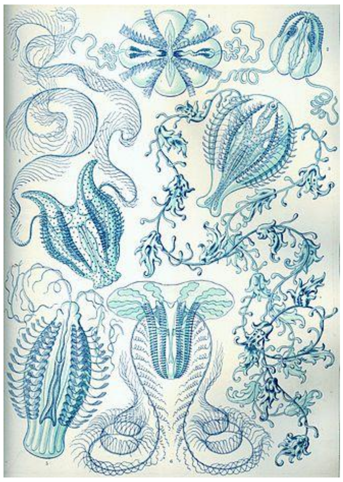
The Symmetry of Animals



Most animals have bilaterally symmetrical bodies, meaning they have one line of symmetry. 99% of animals have bilateral symmetry, and some plants too! Bilateral symmetry is important becauseit allows the development of organs such as eyes and the brain. Without symmetry we would look and function completely differently. On the other hand, plant can have many lines of symmetry (radial symmetry). Icosehdral symmetry occurs in organisms that have 60 subunits with 20 faces, each an equilateral triangle with 12 corners. We often see this in viruses as they use repetition of subunits.



Bilaterally symmetrical animals have an almost-perfect reflection when split in half, for example tigers or birds. Bilateral symmetry helps increases mobility of an animal as it increases the efficiency of rapid movement. Radial symmetry is when there is a repeating pattern around a centre, e.g. repeating something 5 times is called pentamerism. Organisms such as jellyfish and flowers have radial symmetry. Radial symmetry is especially crucial to jellyfish because it means they can attack from all angles, increasing its chance of survival. It can also detect food from any direction, once again increasing its chance of survival. Hexamerism is frequent in coral where there is 6-fold symmetry.



Spherical symmetry is when there is an infinite number of lines of symmetry. However, this type of symmetry cannot be found inside animals. Instead, it can be found in green alga such as Volvox. As wellas this, biradial symmetry is when organisms show features of both bilateral and radial symmetry. They can be divided into only two planes. They can be seen in ctenophores, more commonly known as comb jellies.

These symmetries can help organisms adapt to their environment.

Capture and Recapture

Capture and Recapture is a method often used to estimate the size of a population for a single species. This method is much faster than counting each and every individual organism, and is quite accurate when done correctly. How it works: A sample of animals, for example fish, are captured, and marked. They then are released back into their natural habitat. After a period of time, a sample of fish are taken again, and only a few are marked, the rest are not. By dividing the number of marked fish by the fraction of the total frogs divided by the marked ones.

Example question:

Poppy wants to estimate the number of frogs in a lake. She catched 50 frogs and marks them, then returns them into a lake. The next day, Poppy catched 400 frogs, and 10 are marked. Work out an estimate for the total number of frogs in the lake.

To work out the number of frogs in the lake, we do the first sample of marked frogs multiplied by the fraction of the total frogs divided by the marked ones. So,

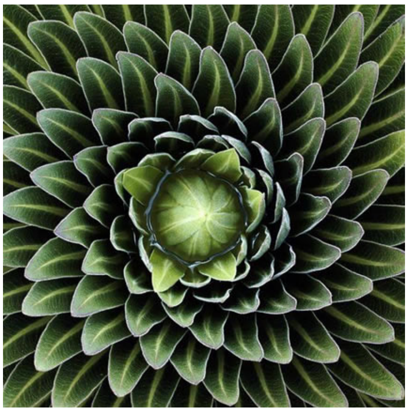
50 X (400/10)= 2000 frogs.

However, this method has many assumptions, and may be often unreliable. The Lincoln Index is when we estimate a population size by only two visits to a study area, where the following criteria are required:

- During the two visits, no organisms die
- No organisms are born
- No organisms move in or out of the study area
- No marks fall off the animals
- Organisms have an equal chance to be captured (random)
- Organisms will be randomly distributed after release

This type of closed system is extremely hard to mimic in reality, so many researchers take multiple repeats of this method to improve the accuracy of their result.

Conclusion



As we can see from all these examples, mathematics is truly a part of nature, through which we see remarkable patterns and intricate shapes. After writing this, I have a question that keeps popping up in my mind- is mathematics a fundamental part of nature, or is it man-made?

On one hand, before the arrival of human kind, plants and animals still had symmetry, still grew in proportion to the golden ratio and snowflakes were still made up of many iterations. Nothing has changed after we humans arrived, nature is still, virtually, the same. Mathematics in nature has helped animals and plants become more efficient and adapt to ttheir surroundings. However, I think that after the arrival of humans, we felt the urge to label and categorize things, put ideas and object into neat, and tidy boxes. Through the use of mathematics we could define and grasp the concept of nature, explaining things by numbers instead. We wanted answers and so we constructed them, and these answered seemed to correlate with that of nature. In my opinion, mathematics will always be a part of nature, whether or not humans choose to define it.

And as Galileo Galilei once said, “The book of nature is written in the language of mathematics”.