THE HODGE CONJECTURE

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The sixth Millennium Problem pretty much sums up why a lot of people don't like maths. It is by far the hardest to explain in any terms, never mind simple ones, it is incredibly far out of reality and everyday experiences and mathematicians can't agree on what the actual problem is – never mind how to go about trying to find a solution. *I just want to emphasise this last point:* depending on which mathematician you ask to define the problem; you will most likely get a different version of what it actually is. *The official statement on the Clay Institute website sums it up perfectly: "The answer to this conjecture determines how much of the solution set of a system of algebraic equations can be defined in terms of further algebraic equations."* So if we have a solution to a set of equations, is that solution also the solution to another different set of equations. Solution, solution, solution.

Not to be deterred I will try my best to explain the Hodge conjecture to you – just maybe lower your expectations a little. Instead of that 150-year aged Scotch whiskey you're used to, on this occasion you might have to settle for a blue WKD ***(Jamie Vardy would be pleased at least).*** *Before we get to the problem let's start with the man that proposed it:* William Hodge. Hodge was born in Scotland in 1903, he studied at Edinburgh, then Cambridge where he was appointed to a chair position which he held until his death in 1970. He won some awards, including being knighted by the Queen in 1959, and his conjecture was announced in 1950. That's pretty much it, the full life story. It would be fair to say that we know as much about the man as we do about his conjecture...

I've probably stalled long enough, now onto the conjecture itself. In as simple terms as possible, the Hodge conjecture asks whether complicated mathematical things can be built from simpler ones. Not so dissimilar to seeing an entire working city built from Lego and realising that it is in fact all just made from little simple square blocks. Hodge is basically asking whether maths is the same as Lego. The Lego Movie brought the toy back to life so maybe Hollywood should do a Maths Movie to bring the subject into the 21st century. Mila Kunis and Brad Pitt in a race against time to work out the area of a circle before the world blows up... I'd watch it.

To explain the Hodge conjecture in a little more detail I need to give you another quick history lesson. In the first half of the 1900's mathematicians began to realise that highly complicated mathematical objects could be approximated very well by sticking together smaller and more simple building blocks. The idea being let's take something we don't understand and try and build it using something we do. You can imagine the following scenario: an archaeologist realises that his evolution tree is missing a dinosaur – there's no way the T-rex evolved into a chicken, there must be something in-between. The archaeologist has no idea what this dinosaur looks like, but can start using the information he has about other dinosaurs, for example other fossils and skeletons that were from the same time period to build up a picture of what the missing dinosaur might have looked like…

An artist's impression: It's only a guess and almost certainly isn't Big Bird, but after more searching and digging for fossils they might find something pretty similar to that guess. This is what mathematicians do. We use what we know to get a better understanding of the things that we don't know much about.

Now with this idea in mind, we can finally come back to the Hodge conjecture. It asks whether one important class of mathematical objects *(called projective algebraic varieties if you must know)*, which are made up of pieces called Hodge cycles, are in fact themselves made up of smaller pieces called algebraic cycles. In the Lego scenario, the problem is asking whether the Lego blocks making up the city are in fact made up of even smaller and simpler pieces. I did warn you this one gets a little tricky but hopefully now that we've got to the end *"everything is awesome!!!"*

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