WHAT IS THE BLAST RADIUS *OF AN ATOMIC BOMB?*

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P*icture the scene:* you're a scientist working for the US military in the early 1940's and you've just been tasked with calculating the blast radius of this incredibly powerful new weapon called an *'atomic bomb'*. Apparently, the plan is to use it to attack the enemies of the United States, but you want to make sure that when it goes off any friendly soldiers are a safe distance away. How do you work out the size of the fireball?

One solution might be to do a series of experiments. Set off several bombs of different sizes, weights, strengths and measure the size of the blast to see how each property affects the distance the fireball travels. This is exactly what the US military did *(see images below for examples of the data collected)*.

These experiments led the scientists to conclude that were three major variables that have an effect on the radius of the explosion. *Number 1:* Time. The longer the time after the explosion, the further the fireball will have travelled. *Number 2:* Energy. Perhaps as expected, increasing the energy of the explosion leads to an increased fireball radius. *The third and final variable was a little less obvious:* air density. For a higher air density the resultant fireball is smaller. If you think of density as how *'thick'* the air feels, then a higher air density will slow down the fireball faster and therefore cause it to stop at a shorter distance.

Now, the exact relationship between these three variables, time t, energy E, density p, and the radius r of the fireball, was a closely guarded military secret. To be able to accurately predict how a 5% increase in the energy of a bomb will affect the radius of the explosion you need a lot of data. Which ultimately means carrying out a lot of experiments. That is, unless you happen to be a British mathematician named G. I. Taylor...

Taylor worked in the field of fluid mechanics – the study of the motion of liquids, gases and some solids such as ice, which behave like a fluid. On hearing of the destructive and dangerous experiments being conducted in the US, Taylor set out to solve the problem instead using maths. His ingenious approach was to use the method of scaling analysis. *For the three variables identified as having an important effect on the blast radius, we have the following units:*

Time = $[T]$,
Energy = $[M L^2 T^{-2}]$,
Density = $[M L^{-3}]$,

where T represents time in seconds, M represents mass in kilograms and L represents distance in metres. The quantity that we want to work out – *the radius of the explosion* – also has units of length L in metres. Taylor's idea was to simply multiply the units of the three variables together in such a way that he obtained an answer with units of length L. Since there is only one way to do this using the three given variables, the answer must tell you exactly how the fireball radius depends on these parameters! It may sound like magic, but let's give it a go and see how we get on.

To eliminate M, we must divide energy by density (this is the only way to do this):

$$\frac{Energy}{Density} = \frac{E}{p} = \frac{ML^2T^{-2}}{ML^{-3}} = L^5T^{-2}$$

Now to eliminate T we must multiply by time squared (again this is the only option without changing the two variables we have already used):

$$\frac{E}{p}t^2 = L^5T^{-2}T^2 = L^5$$

And finally, taking the whole equation to the power of 1/5 we get an answer with units equal to length L:

$$(\frac{Et^2}{p})^{1/5} = (L^5)^{1/5} = L$$

This gives the final result that can be used to calculate the radius r of the fireball created by an exploding atomic bomb:

$$r \sim E^{1/5}t^{2/5}p^{-1/5}$$

And that's it! At the time this equation was deemed top secret by the US military and the fact that Taylor was able to work it out by simply considering the units caused great embarrassment for our friends across the pond.

I love this story because it demonstrates the immense power of the technique of scaling analysis in mathematical modelling and in science in general. Units can often be seen as an afterthought or as a secondary part of a problem but as we've seen here they actually contain a lot of very important information that can be used to deduce the solution to an equation without the need to conduct any experiments or perform any in-depth calculations. This is a particularly important skill in higher level study of maths and science at university, as for many problems the equations will be too difficult for you to solve explicitly and you have to rely on techniques such as this to be able to gain some insight into the solution.

And if that doesn't do it, then I wish you the best of luck with those atomic bomb experiments...

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