Mathematics; a gateway to the world of music

Mathematics underpins many things in life, music is no exception. Mathematics is essential to the music you listen to every day from Mozart's sonatas to Taylor Swift's new album. Math's is pivotal to chord structures, time signatures, construction of sequences and phrases and forms the necessary building blocks of all music. It contributes to the creation of harmony, pitch and melody.

Much of mathematics is built upon patterns for example Pythagoras's theorem, algebraic patterns and fractal geometry, and music is the same. A fractal is defined as a curve or geometric figure and can be modelled in several different ways. It is a mathematical shape and fractals are found in an array of different things from landscapes to artwork to music. They are often considered to be infinite patterns that can appear at any scale or magnification.

I first listened to Johann Sebastian Bach's work in my year 11 GCSE music class, which then inspired me to listen to more of his work. Whilst Bach is often regarded as one of the greatest composers of all time, to me it is underestimated just how much of his music was based upon mathematical techniques. Lots of his music is pattern based, exactly like mathematics, and was built upon sequences and repeated phrases. Fractals reflect this. In a study published in 2007, Brothers discovered a fractal structure within Bach's Cello Suite No.3. After studying this piece, I noticed that it was composed of a mixture of different length phrases filled with repetition. This essentially created a 'never ending' pattern throughout the piece: a fractal. This could even be several fractals as the music has several different patterns within it, for example some of the phrases are built upon patterns of ascending and descending scales of quavers and arpeggios (this occurs in the prelude) whereas in sections like the Allemande, the pattern still consists of variations of scales but there are lots of rhythmic differences. This also creates a sequence.

Further into Bach's Cello Suite No.3 the self- similar nature of fractals is shown. This is done through The Bourrée, especially in the first 16 bars. These bars are essentially composed of two shorter phrases and then one longer phrase. Both the first and second set of 8 bars follow this structure. This pattern is continued throughout the whole section of this piece. This is reflective of the nature of fractals. Therefore, this whole piece is underpinned by mathematics.

As well as influencing patterns and a self- similar nature in music, fractals form the basis of the western music: a melody. The Twelve-tone scale underpins every piece of music as it contains every note within music. This can be used with a number sequence, formed from a fractal, to create a melody. This is because fractals can be modelled in several different ways, the most common being the Koch snowflake which is a shape created by Helge Von Koch where he started with an equilateral triangle then proceeded to create a 'snowflake' image by removing the middle third of each line segment and replacing it with two lines instead. He continued to do this until he had formed what is known as the Koch snowflake which can continue its pattern for infinity. This same concept can then be applied to creating a number sequence that will then become fractals. For example, a fractal number sequence was created by Brain Hansen and Cheri Shakiban, two mathematicians in America in 2002, which to create we need to begin with a set of numbers, S0= {0,2,3}. These integers then become our initiator and from these we can generate an entire number sequence, this is done by adding the entire set of numbers to each of its elements. If we look at the first two generations of this sequence, we have:

S0	0									2									3									
S1	0			2			3			2			4			5			3			5			6		
S2	0	2	3	2	4	5	3	5	6	2	4	5	4	6	7	5	7	8	3	5	6	5	7	8	6	8	9

This fractal sequence has the ability to be continued into infinity. It has a pattern that is self-similar, and this means the music created with it will also have a self-similar nature and can be considered fractal music. The self-similarity of this number sequence can be demonstrated by removing every integer except the 3rd and the 9 it creates the previous sequence. Hence showing it is indeed an infinite sequence and is not only a fractal sequence but has multiple number sequences embedded within itself which all portray a fractal nature. In order to create a piece of music with this sequence, assign a note in the western twelve tone scale any number. Then generate a fractal number sequence using any numbers of your choice. Use the corresponding notes to create a melody in the order of the generated fractal number sequence. This can create musical phrases and sequences and whole sections of a piece; all from fractal geometry. This technique can be used and adapted in so many different ways and genres of music: it can be used to create Sonatas or even the catchy melody for a chorus from your favourite pop song.

As well as this, fractal music generators use fractal sequences like the one above, to produce never-ending sequences of numbers that can be converted into not only note pitches in the scale, but also note durations, rhythmic patterns (shown in Bach's cello suite) and even dynamics. The vast amount of application of fractal geometry indicates just how much of the world of music is influenced by mathematics whether purposeful or not.

Therefore, fractals create the essential melodies, techniques and patterns that make music such an influential part of people's lives and provide the basis of what makes music so amazing to listen to. This means that so much of what makes music what it is today is formed from mathematics techniques.

By Abigail Pearce-Smith