MATHEMATICS ANALYSIS AND APPROACHES

INTERNAL ASSESSMENT

To what extent can companies use a mathematical approach to optimize their machine productivity?

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Content

Introduction

Germany is experiencing the problem of inflation in 2022. The people are facing this difficulty with a shift in their lifestyle, meaning that financial adaption is required based on the current economic situation. In combination with the Sars Covid-19 pandemic, specifically smaller companies are having a rough time keeping their heads above water. I will explore this issue in this internal assessment, as I am interested in the field of economics and business and curious about how businesses keep themselves in the industry. As a person living in Germany, I am focused on domestic companies tackling their production issues. Many factors, such as the higher prices, which are set by the manufacturer, are not randomly chosen but depended on the cost expenditures. A small company in Germany, which is focusing on producing spare parts for machinery, offered to lay out variables for calculation. Focused on two products, buttonhole knives and gears, the purpose of this investigation will be to calculate the machine productivity of a saw, router and drill, in order to propose a cost optimization to match the prices of inflation that changed in 2022. The mathematical topics of linear regression, machine productivity, linear optimization and primal simplex algorithm will be used in this mathematical exploration. The gained knowledge of using mathematics to aid and optimize the performance of a business will be useful in my following career studies.

Energy Price Development
Linear regression

The *Statisches Bundesamt* of Germany has published the line graph "Preisindex für Energieprodukte in Deutschland", showing the price index for energy products from March 2018 until October 2022 (Appendix 1). It has been disclosed that from October 2021 until October 2022, the price of energy price ascended by 43 percent[1] and with the inflation, it is definite that it will increase further.

It is said that "business and organizational leaders can make better decisions by using linear regression techniques".[2] Linear regression is being used in predictive analysis[3], in order to forecast trends, such as the development of energy prices in this case. Specifically, it is a multivariate linear regression model, as eight independent variables, the price of energy, are already known. It is clear that the variables have a positive correlation, as they can be measured at a continuous level. As for establishing linear regressions, homoscedasticity is vital, as it is "a condition in which the variance of the residual, or error term, in a regression model, is constant"[4]. For that, significant outliers are not respected. With the help of the Graph "Preisindex für Energieprodukte in Deutschland" (Appendix 1), the variables can be taken to sketch a linear graph.

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[1] www.aroundhome.de. (n.d.). *Entwicklung der Energiepreise 2023: Gas, Öl & Strom | Aroundhome*. [online] Available at: https://www.aroundhome.de/energieeffizientes-wohnen/energiekosten-sparen/entwicklung-energiepreise/ [Accessed 5 Jan. 2023].

[2] IBM (2022). *About Linear Regression | IBM*. [online] www.ibm.com. Available at: https://www.ibm.com/topics/linear-regression.

[3] Statistics Solutions (2013). *What is Linear Regression?* [online] Statistics Solutions. Available at: https://www.statisticssolutions.com/free-resources/directory-of-statistical-analyses/what-is-linear-regression/.

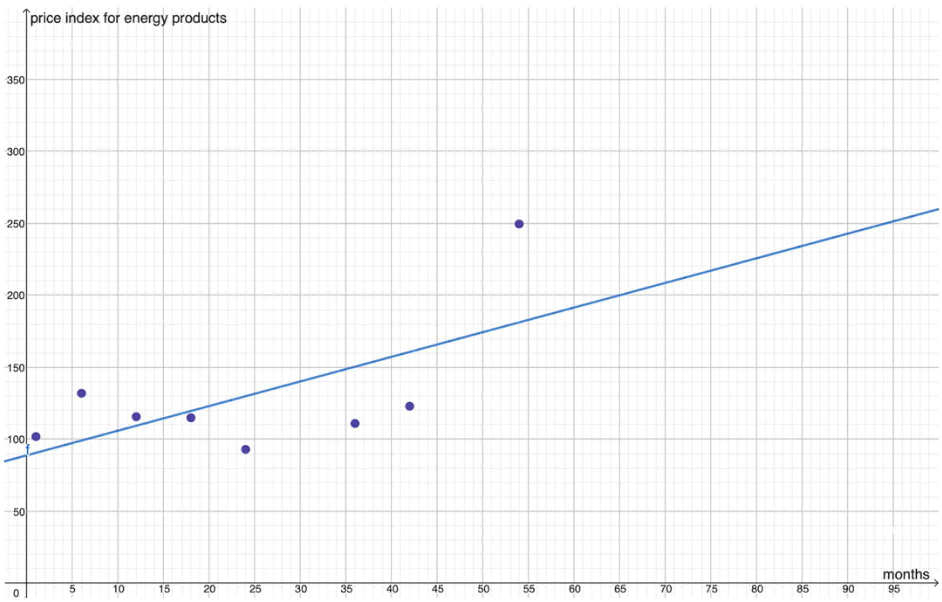
[4] Kenton, W. (2021). *Homoskedastic*. [online] Investopedia. Available at: https://www.investopedia.com/terms/h/homoskedastic.asp.

Table 1: Price Index

X (month starting from March 2018)	Y (price index for energy products)
1	101.9
6	132
12	115.7
18	115
24	93
30	(70)
36	111
42	123
48	(270.8)
54	249.6

It is noticeable that the price index in October 2020 (30) and March 2022 (48) are significant outliers, resulting in not being considered in the linear regression. With the help of the *GeoGebra Calculator Suite,* the graph will be plotted.

Figure 1: Linear regression of the price index for energy products



It can be detected that in the 65th month since October 2020, meaning August 2023, the energy price will predictively rise to 200 Euros. This ultimately means that the costs of the production of buttonhole knives and gears must be adapted, in order to maintain a profit in the future. An approach to enhancing the profit with constraints is to improve the machine's productivity.

Optimization

Machine Productivity

As shown that material costs are steadily increasing, the company has to spend more money. But they can save money during production, by optimizing the machine productivity. Machine productivity measures the rate at which outputs are prepared for sale to consumers and clients by comparing outputs to inputs.[5] The Machine Productivity (MP) is calculated by the total parts created divided by the machine's hours. Data collected in the "Data Table of butthole knives and gears" (Appendix 2) give away the variables to calculate.

Formula used:

$$MP = \frac{\text{total parts created}}{\text{the machine's hours}}$$

Calculation:

Total parts created = 30 buttonhole knives in an hour and 60 gears in an hour

$$= 85 \times 15 = 1275$$

The machine´s hours = 15 hours in a day

$$MP = \frac{1275}{15} = \underline{85 \text{ a day}}$$

[5] Bither, B. (n.d.). *How to Measure and Optimize Machine Productivity*. [online] www.machinemetrics.com. Available at: https://www.machinemetrics.com/blog/machine-productivity [Accessed 5 Jan. 2023].

In the long term, the machine has to perform with higher machine productivity, in order to maintain profit with the economical price shifts. Furthermore, the maximum contribution margin of the products on the machine can be calculated, keeping track of what the machines are capable of. This can be done with linear optimization.

Linear optimization

The linear optimization is a mathematical method used for problem-solving in the areas of production and logistics. It aims to maximize the linear function by taking account of the constraints and eventually giving the optimal solution.

General theory:

Nonnegative (n) decision variables make the standard linear optimization task

$$x_1,\ldots,\ x_n,$$

which must be determined so that the linear objective function:

$$Z(x_1,\ldots,x_n)=c_{11}x_1+\ldots+c_nx_n$$

takes a maximum value (Z) „and linear constraint relations (NB) in the form

$$1.\text{NB} = a_{11}\ x_1+\ \ldots+a_{1n}\ x_n \leq b_1$$

are satisfied", where b is the limit. For calculating, the variables (a), (b) and (c) are known numbers.

Products

In order to start calculating the optimum machine productivity, the number of products produced must be determined. In this case, x_1 is stating the quantity produced and sold by the buttonhole knives and x_2 the quantity produced and sold by the product 2, which are gears. As the numbers are given:

$$x_1 = 30$$

$$x_2 = 55$$

Non-negative conditions

$$x_1 \geq 0$$

$$x_2 \geq 0$$

Representation of the permissible solution range x

Variables

Table 2: Variables of the buttonhole knives (x_1) and gears (x_2), corresponding to the machines

Machine	Product x_1 in processing time in h/ amount	Product x_2 in processing time in h/ amount	Available machine capacity in h/week
M1 (saw)	$\frac{1}{12}$	$\frac{1}{10}$	≤15
M2 (router)	$\frac{1}{5}$	$\frac{3}{4}$	≤15
M3 (drill)	$\frac{1}{4}$	$\frac{1}{5}$	≤15

Target Function

The target function is

$$Z(x_1, x_2) = 30x_1 + 55x_2$$

and aimed to maximize it. Requirements are that it is in its normal form and non-negative. The constraints are build up by the general theory and are for:

Machine 1 (M1) - restriction $\frac{1}{12}\ x_1 + \frac{1}{10}\ x_2 \leq 15$

Machine 2 (M2) - restriction $\frac{1}{5}\ x_1 + \frac{3}{4}\ x_2 \leq 15$

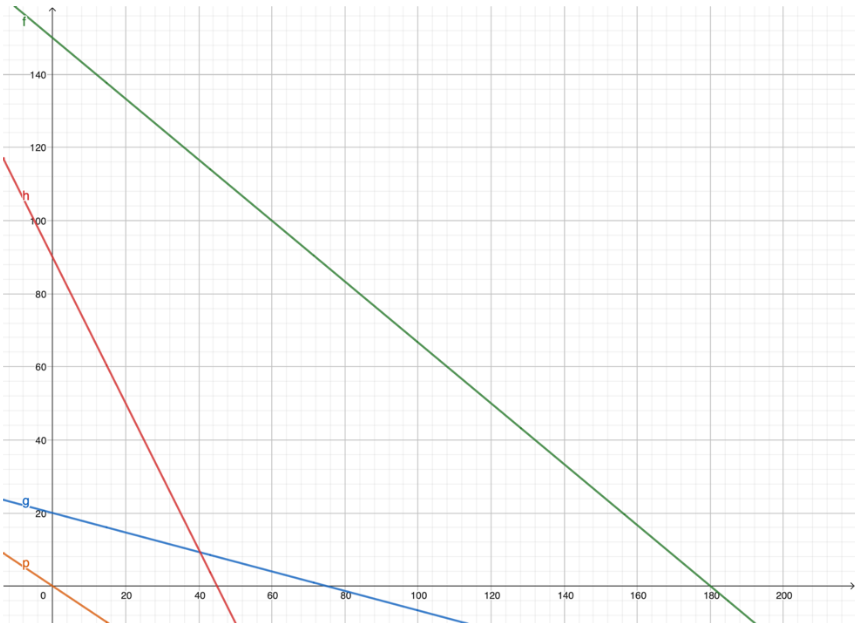
Machine 3 (M3) – restriction $\frac{1}{4}\ x_1 + \frac{1}{5}\ x_2 \leq 15$

Creating a graphic solution

Step 1: Diagram

A diagram is created, by converting the constraints and both products into a target function, in order to set the solution space.

Figure 2: Visualizing the solution space by creating a graphic solutions with all constraints and functions given



The intersections of the graphs in the first quadrant will set out the points of the limit.

Table 3: Key points of the four graphs

Points	X- Axis	Y- Axis
P1	0	0
P2	45	0
P3	40	9.23
P4	0	20

Step 2: The maximum function

The maximum function has to be found by inserting the key points (P) in the target function (Z).

Table 4: Determine the maximum function

Key points	Coordinates	Target function	Solution
P1	(0,0)	Z1 (0,0)	0
P2	(45,0)	Z2 (45,0)	1800
P3	(40,9.23)	Z3 (40,9.23)	2153,8
P4	(0,20)	Z4 (0,20)	1200

The key point P3 is the maximum function, as the solution 2153,8 is the largest variable. Having this as the base, the primal simplex algorithm is being applied.

Primal Simplex Algorithm

The primal simplex algorithm is being used to find an optimization procedure of linear optimization. This algorithm will be the last main step to help find a solution to the problem of inflation and price optimization.

Step 1: Creating an inequality form

Each machine will be assigned a variable. The saw will be (Y_1), the router (Y_2) and the drill (Y_3).

Table 5: The inequality form

Machine	Product x_1 in processing time in h/ME	Product x_2 in processing time in h/ME	Available machine capacity in h/week
Y_1	$\frac{1}{12}x_1$	$\frac{1}{10}x_2$	≤ 15
Y_2	$\frac{1}{5}x_1$	$\frac{3}{4}x_2$	≤ 15
Y_3	$\frac{1}{4}x_1$	$\frac{1}{5}x_2$	≤ 15

Step 2: Extended equation form by inserting slack variables

The decision variables x_1, x_2 and the slack variables Y_1, Y_2 and Y_3 are non-negative. The slack variables depict the available capacity under each constraint.[6]

Table 6: Extended equation form by inserting slack variables

BV	x_1(ME)	x_2 (ME)	Y_1 [M1/h]	Y_2 [M2/h]	Y_3 [M3/h]	z	b	$\frac{b}{Y}$
Y_1	$\frac{1}{12}$	$\frac{1}{10}$	1	0	0	0	15	150
Y_2	$\frac{1}{5}$	$\frac{3}{4}$	0	1	0	0	15	20
Y_3	$\frac{1}{4}$	$\frac{1}{5}$	0	0	1	0	15	75
Z	- 30	- 55	0	0	0	0	0	

[6] Simplexalgorithmus, Grundlagen, Produktionsprogrammplanung, B., Graphische, Htw, L., Prof, B. and Hartl, F. (n.d.). *Lineare Optimierung Teil 1*. [online] Available at: https://www.htw-berlin.de/fileadmin/HTW/Zentral/LZW/02-LO_Teil1.pdf.

Step 3: Evaluation of the output table

Choice of a pivot column

The solution is not optimal, because the Z-row is negative. Thereupon, there needs to be another iteration. To determine the pivot colum, the smallest variable of the optimal function row (Z) is taken. In this case, it is -55 and therefore x_2 is the pivot column.

Choice of the pivot row

The smallest quotient is determined in the pivot column by dividing (b) by (Y), where Y is the pivot column. The element which is in the pivot column and as well as in the pivot row is in this case $\frac{3}{4}$ (marked in red). This will be the pivot row and give us the variable for further calculations.

Calculation of the new base solution

A unit vector must be reached. Therefore, the 1 is set at the same position as the pivot element. Then each element of the pivot row must be divided by $\frac{3}{4}$.

The columns of the other pivot line are to receive the number 0 accordingly. Thereupon from the first table Y_1 minus the difference of Y_2 to Y_1 must be divided with Y_2. Then one receives the variables to Y_1. Then, to obtain Y_3, the difference between Y_3 and Y_2 must again be divided by Y_3 from the first table Y_2 minus Y_2. Then one gets the variables to Y_3. Finally, the Z row is also calculated.

Table 7: Extension to table 5 with the consideration of the calculation of the new base solution

BV	x_1(ME)	x_2 (ME)	Y_1 [M1/ h]	Y_2 [M2/ h]	Y_3 [M3/ h]	z	b	$\frac{b}{Y}$	operation
Y1	$-\frac{1}{60}$	0	0	$-\frac{2}{15}$	0	0	13	-780	$(Y_1) - \frac{2}{15}(Y_2)$
Y2	$\frac{4}{15}$	1	0	$\frac{4}{3}$	0	0	20	75	$\frac{(Y_2)}{\frac{3}{4}}$
Y3	$\frac{1}{20}$	0	0	$-\frac{4}{15}$	0	0	11	220	(Y_3) - $\frac{4}{15}(Y_2)$
Z	25	0	0	$\frac{220}{3}$	0	0	1200		(Y_3) + 220/ 3 (Y_2)

No further iteration must be performed, since the variables are not in the negative range anymore.

Results

Now the objective function no longer has a negative variable. The (b) column indicates the objective function value, which in this case is 1200. To check whether our result agrees with the objective function, we take the two variables, x_1 and x_2, which we insert into the function. B =20 (Y2) and b= 11(Y3) are now inserted into

$$Z = 30x_1 + 55x_2.$$

Insertion:

$$1205 = 30 \times 20 + 55 \times 11$$

Conclusion

By the linear regression, it was found that the price of energy will steadily increase, as the graph is predicting the trend of energy prices. As buttonhole knives and gears are working on the three machines, a saw, a router and a drill, they are consuming a lot of energy. Therefore, the machine productivity needs to be enhanced, where the outcome variable was 85 on a total of 15-hour machine shifts. Following, the search procedure of the optimal admissible solution of a linear optimization problem was found. By using the primal simple algorithm, it was calculated that roughly 2000 was the maximum variable and capability of the procedure. A limitation was the result of the primal simplex algorithm. As these calculations need to be as precise as possible, the deviation of 5 is not exact enough. Although, this does not invalidate the outcome of this assessment. As the small company offered me to show their expenses and sales, in return I can lay out my calculations and they can aim to fully reach the greatest efficiency possible. In a time of rising living- expenses, a businessman has to know how to keep their company a profit and therefore, I try to already orient myself in this topic with the mathematical practices.

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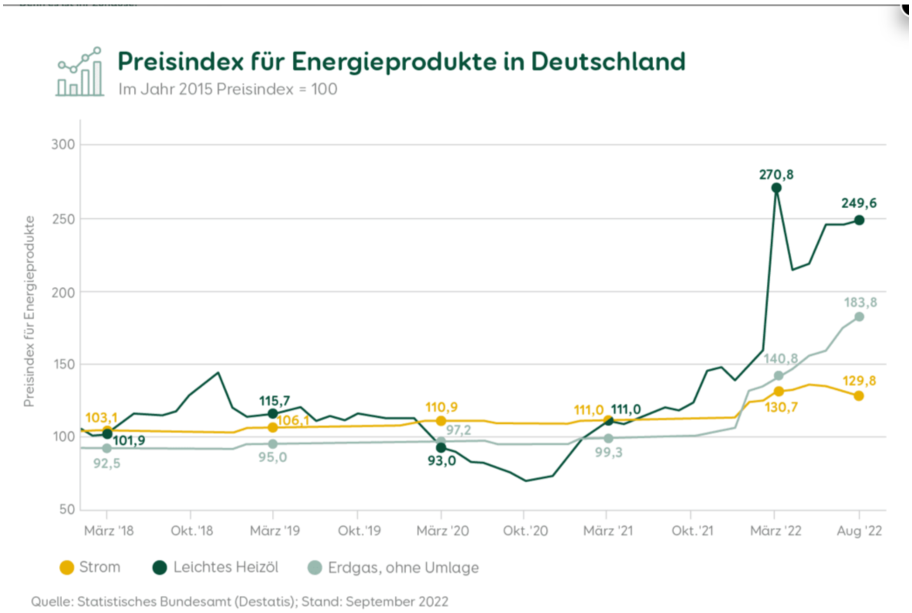
Appendix

Appendix 1:



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Appendix 2:

Kostenanalyse
Einfache Kostenberechnung 2015 und 2022 anhand von Maschinen-Messern und ausgehend von einem kleinen Messerhersteller, der in den vorhandenen, eigenen Räumlichkeiten nur 1 Produkt fertigt:

	2015	2022
Personalkosten, Chef / kfm. Mitarbeiterin	80.000	110.000
Lohnkosten, 5 Mitarbeiter	140.000	160.000
Lohnnebenkosten	40.000	50.000
Abschreibung / Maschinenpark, linear	10.000	10.000
Maschinenkosten / Betriebskosten / Einrichtk. 1-schicht, 2.000 h/Jahr inkl. Energie, Reparatur, Wartung, Betriebsmittel	60.000	90.000
Sonstige Hilfsmittel wie Werkzeuge, usw.	12.000	15.000
	342.000	435.000

Wenn der Markt für 1 Maschinenmesser einen Preis hergibt von	130,00	155,00
mit einem Materialanteil von Euro	30.00	40,00
und ein Bruttogewinn erzielt werden soll von	30,00	40,00
dann müssen von besagtem Messer gefertigt werden 342.000 / 435.000 : 70/75 *	4.885 St	5.800 St.

und der zu erzielende Umsatz muss betragen 635.050 / Jahr
899.000 p.a.

*Diff. von 130,00 ./. 30,00+30,00