# Predicting the Future with the Monte Carlo Simulation

Named after the gambling destination in Monaco, the Monte Carlo simulation has helped to model problems in various areas of science and technology. It is a simulation used to predict the probability of outcomes when one of the variables is random. It is often used in gambling, as the outcomes of rolling a dice or choosing cards is unpredictable.

## The method

At first, it may seem impossible to predict the probability of an event with random variables involved. However, the Monte Carlo simulation acknowledges that the probability cannot be firmly determined, and therefore works by randomly inputting values to replace the random variable, obtaining an approximate value or range of values. The Law of Large Numbers states that the larger the sample, the closer the average to its expected value. This is how the Monte Carlo simulation works – random values are substituted into an equation, which contains the known variables, such as historical data, and the process is repeated, increasing the number of samples. Over time, the simulation gives a rough estimate or a range of values.

## Applications

This simulation is used in a variety of fields, from theoretical mathematics to biology. It is used in particle physics, as the behaviour of fundamental particles is a random variable. The Monte Carlo simulation is also commonly used in engineering to analyse variations in circuits or to predict the wind energy output of a wind turbine. In finance, this method is a useful tool to analyse risk and predict stock prices. In biology, it helps to compare patterns in genomes and proteins.

## Finding $\pi$

Here is a simple example of the Monte Carlo simulation being used in mathematics. Imagine a square and a circular container on a plane surface. The side length of the square and the radius of the circle are both equal to x, so the area of the base of the square container is equal to $x^2$ and the area of the base of the circular container is equal to $\pi x^2$. It is therefore clear that $\frac{Area\ of\ circle}{Area\ of\ square} = \pi$.


This can also be shown using the Monte Carlo simulation. Imagine that marbles are now being randomly dropped so that they land on random points on the plane. Meanwhile, the number of marbles that landed in each container is being counted. To find the ratio of the area of the circle to the area of the square, the number of marbles in the circular container is divided by the number of marbles in the square container. At first, the number may be very different to π: if 2 marbles landed in the square and 10 landed in the circle, $\frac{Area\ of\ circle}{Area\ of\ square} = 5$.

However, the simulation evolves as more marbles are randomly added, and on the graph, we can see that $\frac{Area\ of\ circle}{Area\ of\ square}$ gets closer to π. For example, when there are 153 marbles in the circle and 49 marbles in the square, $\frac{Area\ of\ circle}{Area\ of\ square} = 3.12 \approx \pi$.

As the sample size increases, the quotient of areas gets closer to its true value. This reflects the concept of the Monte Carlo simulation well, as it shows that by sampling random points, an approximation of the true value can be obtained.

<u>Predicting stock prices</u>

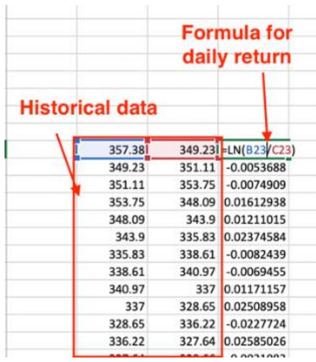
The Monte Carlo simulation is used in finance as a tool to estimate the next day's price of a stock. Two factors are considered: drift and volatility. The drift is the long-term movement of the price. The volatility reflects the short-term changes that occur due to unpredictable factors. Historical data is used to calculate drift, while the random input represents volatility.

Calculating the drift involves looking at the average, the standard deviation and the variance of the historical data. The variance is the difference between a number in the data set and the mean, so it is a measurement of the spread between numbers in a data set. The standard deviation helps to determine volatility, and is the square root of variance. It is depicted by the symbol σ.

The price of the VOO ETF that tracks the S&P500 index is estimated in this example using the Monte Carlo simulation. This can be done in Excel:

1. Using the historical price data, generate a series of daily returns using the natural logarithm.

$$Daily\ Return = \ln\left(\frac{Day's\ Price}{Previous\ Day's\ Price}\right)$$




| | | |
|---|---|---|
| 357.38 | 349.23 | =LN(B23/C23) |
| 349.23 | 351.11 | -0.0053688 |
| 351.11 | 353.75 | -0.0074909 |
| 353.75 | 348.09 | 0.01612938 |
| 348.09 | 343.9 | 0.01211015 |
| 343.9 | 335.83 | 0.02374584 |
| 335.83 | 338.61 | -0.0082439 |
| 338.61 | 340.97 | -0.0069455 |
| 340.97 | 337 | 0.01171157 |
| 337 | 328.65 | 0.02508958 |
| 328.65 | 336.22 | -0.0227724 |
| 336.22 | 327.64 | 0.02585026 |

2. Use the resulting data set to work out the average, variance and standard deviation. Then, use the average and variance to calculate the drift.

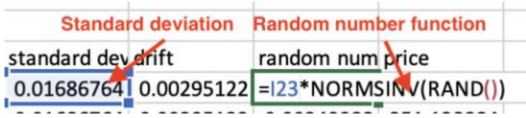
$$Drift = Average\ Daily\ Return - \frac{Variance}{2}$$


| average | variance | standard dev | drift |
|---|---|---|---|
| 0.00309348 | 0.00028452 | 0.01686764 | =G23-(H23/2) |

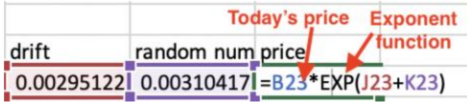
3. Next, a random input needs to be obtained. Since it is linked to volatility, it would involve the standard deviation.

$$Random\ Value = Random\ Number \times \sigma$$

4. Tomorrow’s price is calculated, using the variables calculated above.




$$Tomorrow's\ Price = Today's\ Price \times e^{(Drift+Random\ Value)}$$




In this example, today’s price is $357.38. The predicted price for this random sample is therefore $368.58. This does not mean that this is tomorrow’s price – this process needs to be repeated thousands of times for a more accurate value. After repeating this simulation with other random numbers, a bell curve is produced. The most likely result will be in the middle of the curve.

The Monte Carlo simulation will not work out the exact result. In the example with stocks, it assumes that the market is perfectly efficient. In real life, there are many other factors to consider. However, this simulation helps people estimate and assess different probabilities, and is used to solve problems in a variety of industries.

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