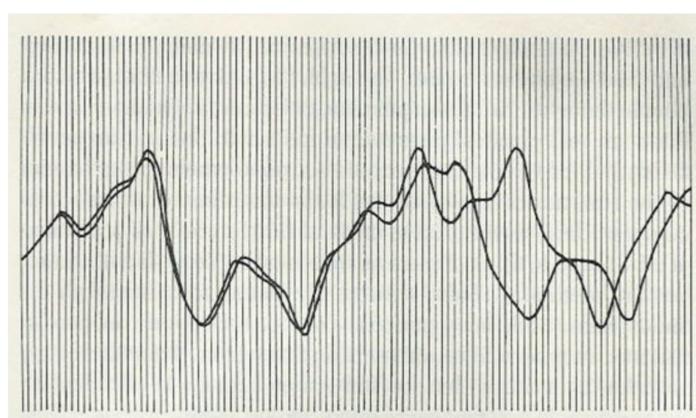
Overcoming chaos in our weather forecast by Emma Cliffe

It is indisputable that the mathematical field of modeling and data analysis is constantly advancing; constantly developing; constantly becoming capable of more and more mind-boggling things. It is also indisputable that one field where this growth is particularly observable by the general public is Meteorology. Given that our social plans, exercise regimes, and holiday packing, amongst other things, are usually arranged with the weather forecast in mind, the accuracy of the forecast is important (with inaccuracy usually resulting in population-wide grumbling and conversation-starters). Thankfully, this accuracy is also much more likely than in previous years, with the Met Office saying that its 'four-day forecasts are now as accurate as [its] one-day forecasts were 30 years ago.'

However, accuracy is still far from guaranteed.

Put simply, I am convinced that everyone has had the infuriating experience of seeing sunshine on the forecast, but waking up to a dreary, grey morning. The question, then, is that, given the complexities of maths algorithms, how does the forecast, predicted using models that are generated by supercomputers containing millions of CPUs and GPUs, churning through billions of data readings from thousands of weather stations and sources across the globe, sometimes get it so absurdly wrong? How can logical, data-driven models break down and fail, seemingly just to annoy people?

The answer lies in the concept of chaos. Or rather, Chaos theory - which is the fundamental concept of a tiny change or oversight, over time, having a massive impact. Many people know of this concept through the idea of 'The Butterfly Effect': named for, amongst other things, the paper that Edward Lorenz (credited as the conceptual father of Chaos Theory) presented in 1972: "Does The Flap of a Butterfly's Wings in Brazil Set Off a Tornado in Texas?". For the film nerds out there, it was also mentioned in Jurassic Park by Jeff Goldblum's character to explain the park's fundamental flaw: a tiny scientific oversight that led the dinosaurs to unexpectedly reproduce, and led to the violent failure of the park. So, next time you complain about the weather, be glad that chaos is just ruining your plans and not causing you to be eaten by a velociraptor.



Back to Edward Lorenz. He was a meteorologist and mathematician who worked in statistical weather forecasting: the field of creating forecast predictions by taking in data about initial conditions and using it to model variables like temperature, wind speed and pressure. In 1961, he was running one such model, using twelve weather-related variables to represent the initial conditions, and decided to re-run a specific section of it by entering in the conditions from part-way through the model. The subsequent prediction initially looked similar to its first iteration, but then started to differ wildly, until it hardly resembled its predecessor.

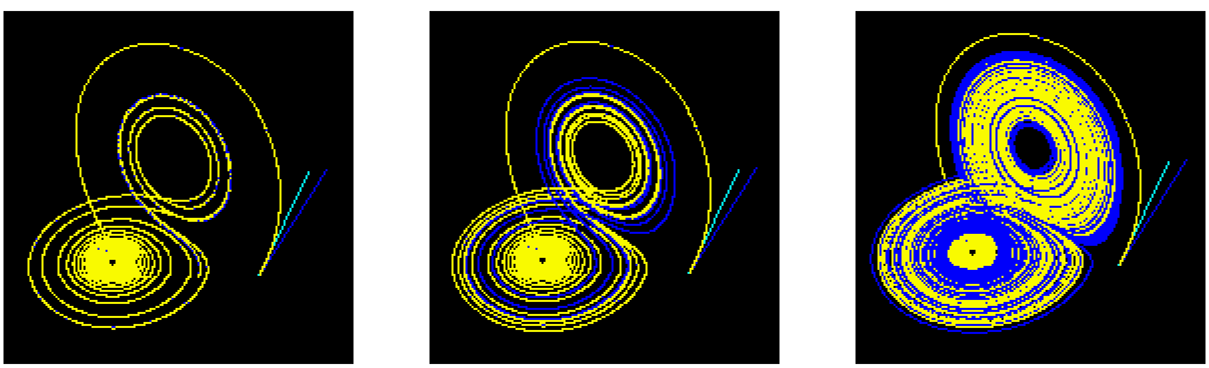
Lorenz realized that these results had occurred because the conditions of the variables that the computer had been going off in the original recording were saved to six decimal places, whereas the data that he had typed in in the second iteration had only been to three decimal places. So the value representing the airstream, .506127, had become .506, with a similar simplification occurring in all the other variables as well. The fact that changing values by such a tiny amount could alter the predictions so drastically forms the basis of Chaos Theory, and Lorenz's subsequent research into this field would lay the foundations for studies in so many sectors of our modern world: from population modelling to planetary orbits. Chaos, after all, is not only fascinating but universal.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y - x),$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y,$$

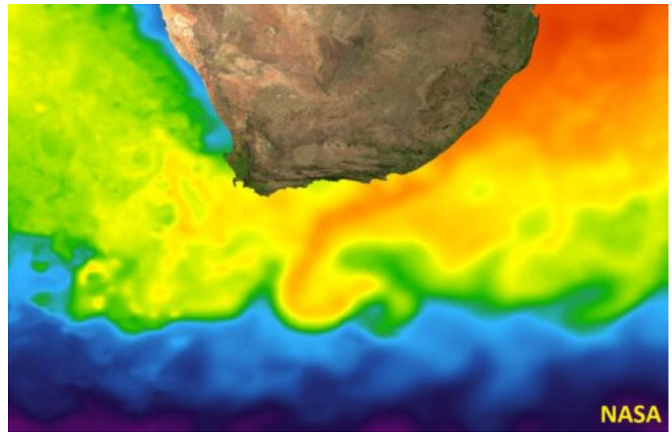
$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z.$$

A good example of chaos in work is the Lorenz System, developed by Lorenz in 1963. It is a non-linear, three dimensional, and deterministic (meaning that when you run it with exactly the same input, the output will always be the same) model for atmospheric convection based on a basic series of differential equations, with σ, ρ and ⍰ as the initial conditions. The Lorenz System is an example of a model that can show chaotic behavior. For example, when Lorenz used the values of $\rho = 28$, $\sigma = 10$ and $\beta = 8/3$, tiny changes in the initial point of the model caused the path it took to be completely different. In the below example (showing the model when t = 1, 2, and 3, from left to right), the initial point of the yellow and blue lines differs by only 10^{-5} on the x-axis, and yet as the t-value increases, their paths start to diverge more and more.



This simple example barely scratches the surface of chaos theory and all its fascinating aspects. Nonetheless, the fact that the Lorenz System, which has chaotic properties, was developed for atmospheric convection, in conjunction with the fact that chaos theory was first explored through the lens of meteorological models, showcases just how integral chaos *is* to Meteorology and forecast predictions. In other words, it tells us that, when we complain about how unpredictable the weather can be, we should be pointing the finger of blame at chaos.

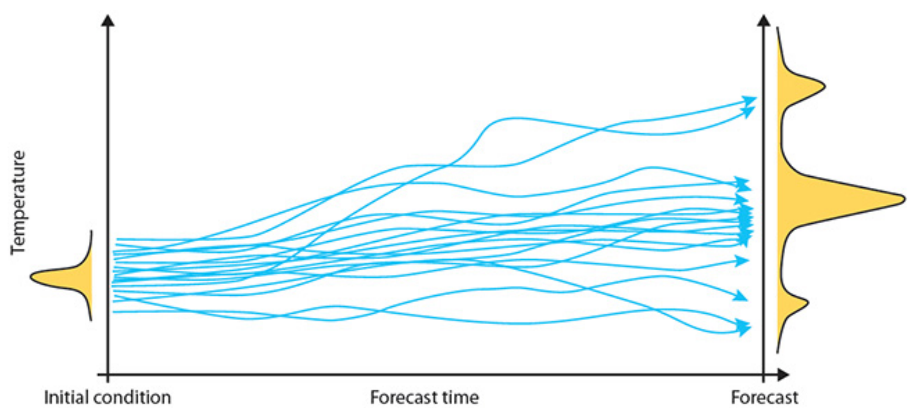
Unless we develop the ability to gain a full understanding of the atmosphere in real-time, analyzing every cloud, every temperature change, every gust of wind as it appears, we will remain unable to feed completely accurate initial conditions into any weather forecast modeling program. And, as chaos theory shows, this can lead to more predictions days in advance being very different from what actually happens. This can lead to a day of rain when a day of sunshine was expected.



On the other hand, it is important to note that, in the short-term, models can actually be quite accurate. After all, from the models showing chaotic properties we can see that, initially, the different paths tend to align quite well. So forecasting models running a single prediction (known as deterministic forecasting) for the *near* future can work very effectively. Furthermore, the advantage of deterministic forecasting is that it can show predictions for temperature and all the other weather variables in high resolution, since all the computer processing power possessed by those running the models can be focused on creating that one forecast, with no distractions or other models taking up processing space.

However, the obvious disadvantage is that, if the initial conditions fed into the deterministic algorithm are even remotely off (which, as we have already discussed, is almost inevitable), it is almost guaranteed to develop an increasing degree of inaccuracy as it tries to make predictions further and further in advance.

So how have mathematicians reduced this risk of this happening?



Answer: By using another modeling technique called 'Ensemble forecasting'. This idea, which was pushed by Edward Epstein in the 1960s, and is universally used for long-term predictions today, involves running a predictive weather model multiple times with slightly different starting conditions, and calculating what is most likely to happen based off of the 'ensemble spread' of possible outcomes that the model creates. This method is designed to account for the likelihood of errors that may abound as a result of both flaws in the model itself (due to simplifications of atmospheric processes, potential biases, as well as a range of other factors), as well as the potential inaccuracy of the data about the atmospheric conditions that, as we have already discussed, is almost inevitable.

Here is an example. According to the Met Office, the variant of MOGREPS (Met Office Global and Regional Ensemble Prediction System) that they use for global predictions, known as MOGREPS-G, runs predictive models on a 6 hour cycle (at 00, 06, 12, and 18). The system works by running a model using the set of data on weather conditions that it has (the control member), then making a series of perturbations (small changes) to this set and runs 17 more iterations using different perturbations. It then combines these 18 forecasts with the forecasts from the previous cycle to achieve an ensemble spread with 36 members.

There is evidence showing why ensemble forecasting is so useful for long-term forecasts, with one study suggesting that "the high resolution control member is the best member of the ensemble ONLY ~ 5 to 7% of the time for all lead times (forecast hours)."

Once the ensemble spread has been collected, the forecast is then determined by calculating the mean, standard deviation, σ, and spread of these ensemble predictions that are generated. Meteorologists calculate which outcome is most likely to occur using a number of logic-driven assumptions, going off what has happened in previous circumstances when there were similar readings, and in many cases, exchanging data and predictions with other companies responsible for predicting weather forecasts - companies that might use entirely different models, algorithms and systems - in order to form the most accurate idea of what the weather will do.

Most meteorology centres use a combination of deterministic and ensemble models, with the deterministic models mapping the forecast for the next few days, and the ensemble predictions taking precedence after that, up until the (standard) two week cut-off point that is commonly used. With our current technology and mathematical algorithms, the degree of accuracy beyond two weeks is generally considered to be too small, at less than 50%.

However, there's no reason to see why this won't change in the future. After all, meteorology and the art of weather forecasting is driven by maths, and maths, both now and through all of human history, is an ever-growing and improving field; a fire fueled by humanity's hunger for knowledge. We once thought that there was no order to chaos, but now, thanks to the efforts of Lorenz and other mathematicians, we know otherwise. Furthermore, it is evident that the rapid growth of fields like machine learning and AI will help us in our effort to fully understand this order in the future. To, in other words, overcome chaos.

The thing that interests me most about maths is about how it both *is* and *has been* present in every sector and epoch of history. It links every aspect of the past, present and future of our universe together, and our understanding and use of it, like the universe, is ever-increasing.

After all, I doubt that centuries ago, mathematicians could have imagined that maths would be used to tell millions of people whether they should pack an umbrella or suncream for their summer vacations.

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