TOES, TIME & TEN: A JOURNEY THROUGH NUMBER BASES

How many puppies are in this image? Using our system, we would express this as 11, but this isn't the only way. We use the decimal system – base 10 – where we have 10 digits (0-9 respectively), to express any number, from infinitesimally small to extraordinarily large.

From young ages, we are taught to count with our fingers, to keep track of how many items there are using fingers as placeholders. Eventually, we reach a point where we count in our heads and can leave our fingers for stroking the puppies, but it still remains in our processes and notation that we work in groups of 10. Roman numerals are one example of this, with their measurements focussed on multiples of 5 and 10, the numbers of our fingers, proving this has been used throughout history. Nevertheless, this isn't the sole way we calculate. Number bases work in any integer greater than 0 and other bases are prevalent in many aspects of the world. Here, I hope to show you where they can be found through toes, time and technology and ponder: is base 10 the most practical base, and will it continue to be the base which is predominantly used into the future?

Firstly, how do we convert between bases? The easiest way to show this is through a table:

10,000 (10^4)	1,000 (10^3)	100 (10^2)	10 (10^1)	1 (10^0)	1/10 (10^{-1})
	1	7	2	9	

This table shows denary place value, with each header signalling the ones, tens, hundreds and so on. Each row gives 10 to a power. This means that the 9 numerals can represent every number, and their place with regards to the other digits signifies each number. The example shown is 1729 (Ramanujan's number). The table explains there is 1 thousand, 7 hundreds, 2 tens and 9 ones in 4 digits.

So, how do we translate this to another number base?
Take base n:

n^4	n^3	n^2	n^1	n^0	n^{-1}

The same structure continues, where each column represents n to a different power.

Across the world, alternative number bases are used in plenty of different languages.

"Arithmetic is being able to count to 20 without having to take off your shoes" (Mickey Mouse)
Mickey Mouse gives reason to working in base 20 – we have 10 toes as well, why not use them?

Below is 1729_{10} in base 20, or vigesimal. In 469_{20}, there are four 400s, six 20s, and 9 ones in base 20.

160,000	8,000	400	20	1	1/20
		4	6	9	

Let's begin our worldwide voyage, our discovery of bases:
Travelling from the UK, we cross the English Channel, and across its ultramarine depths, we arrive in France. The English Channel is the busiest shipping area in the world, so you can expect it will be bustling here. Cacophonous noises emerge, accompanied by the subtle yet invasive scent of the area. Bulky boxes are shifted to and fro by mechanical giants, mathematically managed for efficiency and safety. They are moving 80 crates, or in French 'quatre-vingts', literally meaning 'four 20s'. This is a display of a base 20 system in action. From numbers 70 to 99, they are expressed in base 20, however the rest is all in base 10. French learners can be perplexed, counting here becomes a chore until the different system is integrated into their thought process and saying 'quatre-vingt-onze'(four-twenty-eleven) as 91 becomes a no-brainer. However, many modern varieties of French have stopped using the vigesimal system, in places such as Canada, Belgium and Switzerland.

Now, flying across the globe to south Asia, (approximately) to latitude 27.5 and longitude 90.4, we arrive in Bhutan. It is covered in lush, verdant forests, spanning for miles – forests cover 71% of the total geographical area of the country (making it a carbon negative country). As a result, forestry is key here. The trees sway as people maintain the forest, communicating. While many languages are spoken in Bhutan, the national language of Bhutan, Dzongkha, uses a full base 20 system. There are numerals for the powers of 20, 400, 8000 and 160,000. However, Dzongkha is becoming an endangered language, as there is often a lack of mainstream exposure to the language, for example, most TV programmes are in English or Hindi.

Moreover, travelling further south east, we arrive in New Guinea, in Oceania. New Guinea is the second largest island, in which several languages are spoken. Telefol and other Central New Guinea languages use a base 27, or septemvigesimal, system based on body parts. 1 would translate to the left pinky, 14 would be the nose, and 27 would be the right pinky.

While in English, we use the decimal system, we also use other systems based on numbers divisible by 6, often unnoticeably. For example, you might buy a dozen eggs, or half a dozen. Furthermore, why are there 60 seconds in a minute, or 360 degrees in a circle?

In base 12, duodecimal, 1729_{10} looks like 1001_{12}:

20736	**1728**	**144**	**12**	**1**	**1/12**
	1	0	0	1	

And in base 60, sexagesimal, it looks like Sn

12960000	**216000**	**3600**	**60**	**1**	**1/60**
			S	n	

While no definitive answer, there are a variety of plausible reasons for the integration of bases divisible by 6.This image below shows how to count in base 12 – try it out yourself. Using your thumb, you can count on each of the segments of your hand as shown. There is evidence that people in Mesopotamia counted with this method.

Another suggestion is that 60 is a superior highly composite number* (meaning it is a positive integer with the highest ratio of factors to some positive power of itself than any other number) with 12 factors: 1, 2, 3, 4, 5, 6, 10, 12, 15, 2, 30 and 60 meaning it is highly divisible for its size; 12 is also a superior highly composite number. However, other numbers exist with many divisors.

* Ramanujan wrote a paper entitled Highly Composite Numbers in 1915 – hence why I used 1729 in explanations!

Perhaps the Sumerian weights and measures lead to the development of the system, as 60 is easily divided in halves, quarters and fifths, but alternatively it could be the number system which caused the measure system to be like this. Others argue that the Sumerian's stargazing, and discover that the year was 365 days, led to the usage of base 60 (sexagesimal). It was decided that they rounded it down to 360, giving reason for base 60. 360/12 = 30, evidencing 12 30-day months, and the circular star maps could reason for the 360 degrees in a circle. Nonetheless, nothing is certain.

Leaping into the future, we arrive into a world of technology full of computers which understand base 2, binary. In computers, there are millions of tiny transistors, all neatly arranged. They store data by using the amount of current to determine if the value is 0 or 1. Generally, it is 0 with low voltage or off, whereas it is 1 with a higher voltage and on. Additionally, 1 and 0 represent "true" and "false" in Boolean Algebra, permitting logical expressions. Yet, binary numbers can be very long, for example: 1729_{10} in binary is 11011000001_2 so there is a risk of incorrect input.

1024	**512**	**256**	**128**	**64**	**32**	**16**	**8**	**4**	**2**	**1**	**1/2**
1	1	0	1	1	0	0	0	0	0	1	

Hence, base 16, hexadecimal, is useful because larger numbers can be represented with fewer digits, is easier to understand than binary, and is simpler to write and check. For example: 1729_{10} is $6C1_{16}$, where there are six 256s, twelve 16s and one 1.

65536	**4096**	**256**	**16**	**1**	**1/16**
		6	C	1	

Due to 2^4 being 16, it is easy to convert between number bases, as 4 binary digits (bits) are one hexadecimal digit. There are 8 bits in a byte, too, so 256 different numbers can be represented with 2 hexadecimal digits.

So, where does that leave us? How will society advance in its use of number bases; is base 10 truly the best base for us to use? If we change the foremost base, it may give us another perspective on maths, although it may be unnecessary. One argument is that duodecimal is better due to the number of factors it has opposed to 10; the factors of 10 are only 1, 2, 5 and 10, whereas 12 has 1, 2, 3, 4, 6 and 12. Being a superior highly composite number, it means that fractions are easier to express, such as 1/3 being expressed as 0.4_{12}, more precise than 0.333… There is a method of finger counting for it too, as previously explained. Technically 60 has more factors, but it may be too large and unwieldy. Another argument is that we will progress to a base which is a power of 2, such as 16, as society becomes progressively more integrated with technology. On the other hand, base 10 may remain the dominant base, as seen with how base 20 is vanishing in French and with Dzongkha – counting using one's fingers is intuitive, as seen through its use throughout history.

Additionally, modern society has been established using a decimal system – it may be easiest to continue with the regular base. The future of mathematics is indeed undecided.