Applied Mathematics: Symmetrical Saviour or Probable Punisher?

"“Obvious” is the most dangerous word in mathematics” according to the Scottish Mathematician, Eric Temple Bell.

What Bell was attempting to imply was that mathematics is not as simplistic as it seems to the human eye and that one must always endeavor to decompose and dissect every problem, analyze, and evaluate every solution possible and consult every equation in the cosmos before making acceptable proofs. However, such myopic views of mathematics are no less irrational than π. To me, it is evident that the right word Bell was looking for was “unnecessary.” Humanity has been quick to conclude that mathematics can only have realistic applications. Well, let us move away from the set of real numbers and consider the imaginary world. Just how far can maths go? Obviously, the series of solutions must diverge somewhere. Let us consider the extreme. Could mathematics aid in survival? Could we escape a life-threatening situation because of mathematics?

Exhibit A: The Hawk

Before we begin, a dissection of the upcoming formula is required.

$$\lambda(t) = \mu(t) + \sum_{i:\tau_i < t} \nu(t - \tau_i)$$

“Above is Hawkes`s Process where $\mu(t)$is the background rate of the process (standard rate of loss or decay), where τi are the points in time occurring prior to time t, and where ν is a function which is sometimes called the excitation function.”

Terrorist attacks are an unpredictable string of events which unfortunately terrorize civilians in proximity and their loved ones. However, a paper by Dr Stephen Tench and Dr Hannah Fry from University College London suggests that attacks do follow a pattern and can be predicted. Of course, there is no pure mathematics behind human nature and visceral violence but sometimes when we consider strings of similar events “while the individual man is an insoluble puzzle, in the aggregate he becomes a mathematical certainty." as stated by Sherlock Holmes.

Now let us consider where mathematics can therefore crop up. The model which the mathematicians used is called the Hawkes Process. The fundamental idea behind Hawkes process is that some events are not mutually exclusive and not independent but when a certain event happens, you are more likely to see similar events shortly afterwards. As time goes on, however, “the probability of a subsequent event occurring gradually fades away and returns to normal.” Initially, Alan Hawkes used such a theorem to describe the frequency of earthquakes. Tremors are not independent and so his equations implied that after one earthquake, the odds of there being more increases which could make sense from a geographical perspective. Since Hawkes developed the model in 1972, these equations have served a multitude of purposes involving epidemic travel analysis, electrical impulse motion, and email transmission between point I.e. data between nodes.

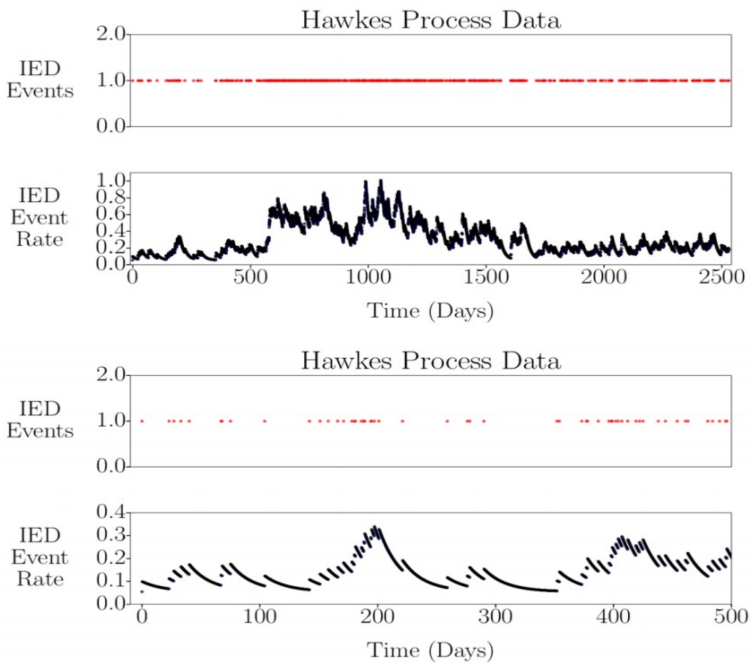
So how about terrorist attacks? If one gang shoots another gang member, they will be provoked to retaliate, hence increasing the probability of a second incident.

Dr Hannah Fry, one of the paper’s mathematicians, has also explained how your house burglarized does increase the chances that thieves will visit again. The criminals can pinpoint your location and the infrastructure, which means they are more likely to rob neighboring households.

In their paper, the mathematicians applied the same model to terrorism in Northern Ireland. 5,000 explosions of improved explosive devices (IEDs) went around Northern Ireland during “the Troubles” between 1970 and 1998.

The researchers used Hawkes Process to analyze when and where one group, “the Provisional Irish Republican Army (IRA), launched its terror attacks, how the British Security Forces responded, and how effective those responses were.”

The chart below shows the IED explosions follow a pattern with peaks and subsequent rises in probability.



Essentially, mathematics can reveal patterns in past terrorist activity or be used to design predictive models for any future terrorist attacks.

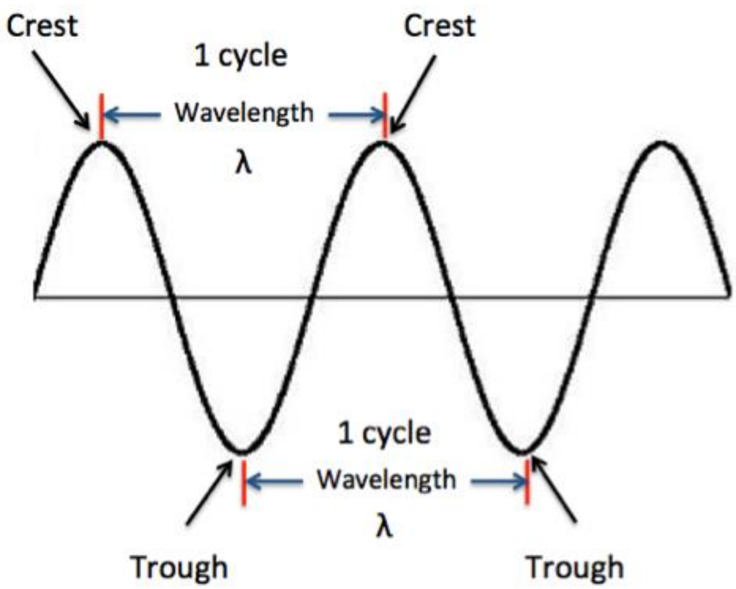
Research on counterterrorist operations also shows evidence that the death of Catholics during The Troubles, would cause the group to increase their IED attacks in retaliation. Operations carried out indiscriminately led to a backlash of violence. In contrast, counterinsurgency operations that were carried out in a discriminating, targeted way led to a lower level of violence than before.

After one terrorist attack, aftershocks can increase vulnerability for a prolonged time leading to authorities having to intervene and engage in strict defense based on attack probabilities. Mathematics is a symmetrical being with fairness on all sides as it has played a vital role in saving lives.

Exhibit B: The Wave

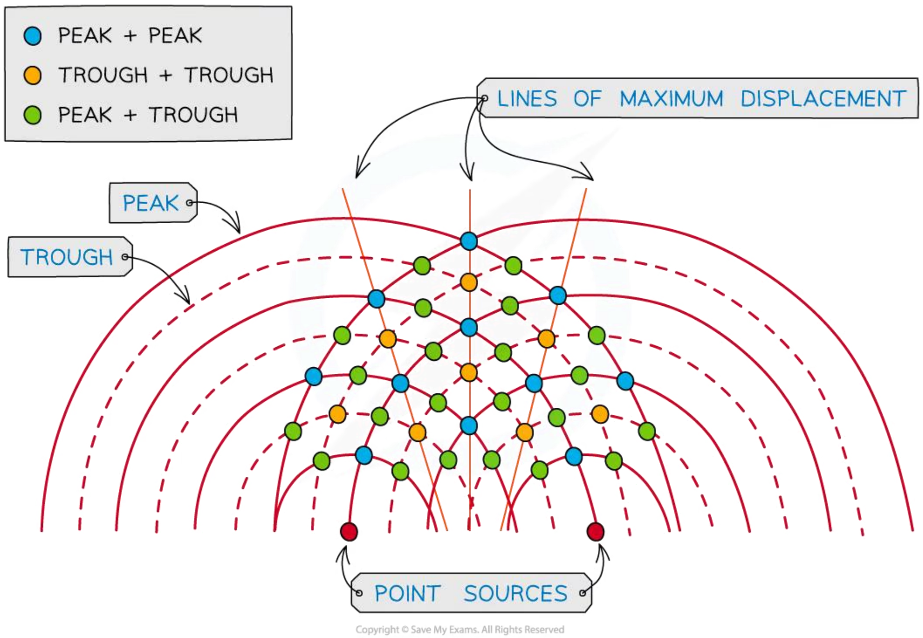
In her winning article from the 2022 Teddy Rocks Maths Competition, Zoe Burr discussed the music of maths and how tonality or dissonance tie accordingly. Mathematics has clearly played a role in sound and communication overall but imagine a world where sound was absent. Humans would fail to evade danger, fail to communicate, and fail to survive. We are all aware that waves are the prime focus of sound transmission but how far does mathematics play a role in this?

Firstly, let us consider the wave concepts of superposition and coherence. Superposition occurs at the instant where waves cross over each other leading to a combination of displacements that the wave is currently at. The effect of such superposition is called interference, which branches out into destructive and constructive interference. Constructive interference occurs when points on a wave have a phase difference (angle difference) of an even multiple of pi radians OR a wavelength away if we consider path difference. Destructive occurs when points on a wave have a phase difference of an odd multiple of pi radians. To get maximum interference, wave sources must produce coherent waves which have the same wavelength, same frequency, and constant phase difference.



Crest, Trough and Wavelength

Let us consider the diagram below. Interference can be investigated using sound waves where the source, in this example, would be an amplifier and a speaker. These two waves' sources should be coherent and if you walk across the room, you will be able to hear varying volumes of sound. Where peaks and peaks meet or troughs and troughs meet, we have constructive interference and the loudest volume whereas if peaks and troughs meet, the waves cancel out and the destructive interference leads to the lower volume of sound with the smallest amplitude (almost negligible).



Now for some more concrete mathematics. The mathematics of sound waves is a critical branch of mathematics that has many practical applications that help humanity to survive and thrive. Here are some of the key mathematical formulae and ideas that underpin the mathematics of sound waves and their applications:

1. Wave equation: The wave equation for sound waves is a partial differential equation that describes how sound waves propagate through a medium, such as air or water. The wave equation is given by:

$$\partial^2 p/\partial t^2 - c^2\nabla^2 p = 0$$

where p is the sound pressure, t is time, c is the speed of sound, and ∇^2 is the Laplacian operator. The wave equation is essential in many applications, such as acoustic design, noise control, and sound engineering I.e. audio analyzers needed by forensics when identifying criminals and modelling the sound waves using computational modelling.

2. Fourier analysis: Fourier analysis is a mathematical technique used to analyze frequency components of a sound wave. It is based on the idea that any complex waveform can be broken down into a sum of simple sine waves of different frequencies, known as Fourier components. Fourier analysis is used in many applications, such as audio signal processing, music production, and acoustic modeling.

$$\int_{-\infty}^{\infty} F(k)\, e^{2\pi i k x}\, dk$$

- F(k) represents the frequency-domain representation of the signal.

- X represents the time-domain signal being transformed.
- e^(-2pi*ikx) is the complex exponential function, with i representing the imaginary unit, k representing the angular frequency, and x representing time.

This equation describes the transformation of a continuous-time signal from the time domain to the frequency domain.

Fourier analysis is a powerful mathematical technique that can help save lives in many ways. For example, the process of ultrasound scanning, and MRI depends on raw data which can then be analyzed using the Fourier transform. Mathematicians and Physicians can then create internal images of the body to efficiently diagnose patients with a range of conditions and disorders. From a more engineering perspective, Fourier analysis is also used in signal processing to analyze and filter signals in many different applications, such as telecommunications, audio and video processing, and industrial automation. By identifying and filtering out noise and interference in these signals, engineers can improve the reliability and accuracy of many different systems, saving lives in critical applications, such as aviation and transportation.

3. <u>Sound intensity and sound pressure level</u>: Sound intensity is a measure of the amount of energy carried by a sound wave, while sound pressure level is a measure of the strength of the sound wave as perceived by the human ear. Intensity is mathematically defined as the rate of energy transferred per unit area at right angles to the direction of energy transfer. Both measures are expressed in decibels (dB) and are essential in many applications, such as noise control, acoustic design, and hearing protection.

4. <u>Doppler effect</u>: The Doppler effect is a fundamental physical phenomenon that occurs when there is relative motion between a wave source and an observer. The mathematics behind the Doppler effect is used in many different fields to help save lives, including:

A) <u>Meteorology</u>: The Doppler effect is used in meteorology to analyze the movement of weather systems, including storms and hurricanes. By analyzing the frequency shift of radar signals reflected off precipitation, meteorologists can track the movement and intensity of these weather systems and issue warnings to help people evacuate and take other necessary precautions.

B) <u>Traffic safety</u>: The Doppler effect is used in police radar guns to measure the speed of vehicles. By emitting radio waves and analyzing the frequency shift caused by the movement of the vehicle, law enforcement officers can enforce speed limits which reduce the risk of accidents.

C) <u>Aviation safety:</u> The Doppler effect is used in weather radar systems to detect turbulence and other hazards that can pose a risk to aircraft. By analyzing the frequency shift of radar signals reflected off precipitation, pilots and air traffic controllers can take evasive action and avoid dangerous weather conditions.

Overall, the mathematics of sound waves has many practical applications that help humanity to survive and thrive, such as noise control, acoustic design, music production, and medical imaging. By understanding the mathematical principles that underpin sound wave phenomena, scientists and engineers can design and optimize systems and devices that benefit society.

<u>Exhibit C: The Diamond</u>

In Season 2 of the famous Japanese Netflix drama “Alice in Borderland”, the viewers are introduced to a series of challenges encoded by each of the face cards. Shuntaro Chishiya, arguably the smartest character, is faced against the King of Diamonds whose game is called Beauty Contest.

Let`s quickly establish the rules:

- *“All players select a number between 0 and 100 in the given time.*
- *The average of the values will be multiplied by 0.8.*
- *The person who chooses the number closest to the calculated number wins. This constitutes one round.*
- *All losers will lose a point.*

4 players remaining:

- *If there are 2 people or more who choose the same number, the number they choose becomes invalid, meaning they will lose a point even if the number is closest to 4/5ths the average.*

3 players remaining:

- *If there is a person who chooses the exact correct number, the loser penalty is doubled.*

2 players remaining:

- *If someone chooses 0, the player who chooses 100 is the winner.*

Game Clear: *Be the last person remaining.*

Game Over: *Reach -10 points. (Referenced from the Alice in Borderland Wiki)”*

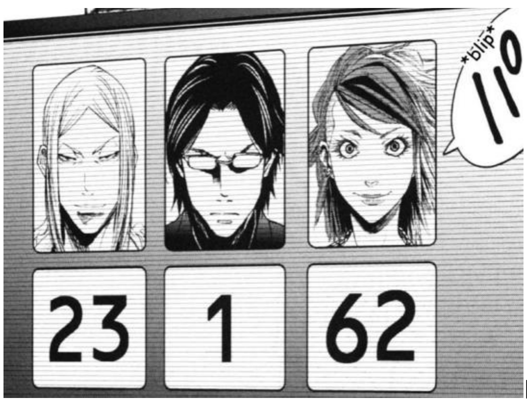
This Keynesian Beauty Contest is a game of “second order” thinking, which was proposed by economist John Maynard Keynes to explain how stock market prices can be influenced by the opinions of market participants. In the game, players are asked to choose a number between 0 and 100, and the winner is the player whose number is closest to two-thirds of the average of all the numbers selected. To win this game using mathematics, players need to understand the concept of common knowledge and apply game theory to their decision-making.

One example of a strategy is to assume that other players will choose numbers that are too high and try to choose a number that is lower than two-thirds of the average. For example, if players are asked to choose a number between 0 and 100, two-thirds of the average would be around 44. Therefore, a player might choose a number like 30 or 35, assuming other players choose higher numbers.

However, this strategy is not 100% guaranteeing of a win, as other players may also anticipate this strategy and choose numbers lower than two-thirds of the average. Therefore, players need to anticipate what other players might anticipate.

A way to do this is to use the concept of common knowledge, which refers to information that is known by everyone and known to be known by everyone. In the Keynesian Beauty Contest game, players need to consider not only what they think other players will choose, but also what they think other players think they will choose, and so on. This leads to a recursive process of thinking that requires a deep understanding of game theory.

Another strategy is to use mixed strategies, which involve choosing a number randomly according to a probability distribution. This can help players avoid being too predictable and increase their chances of winning. For example, a player might choose a number between 0 and 100 randomly according to a uniform distribution, or they might use a more complex probability distribution based on their analysis of other players' choices.



In conclusion, to win the Keynesian Beauty Contest game in Alice in Borderland, players need to understand the concept of common knowledge and apply game theory to their decision-making. They need to anticipate what other players might choose and use strategies like mixed strategies to avoid being too predictable. However, it is important to note that this game involves complex social dynamics and strategic interactions between players, so there is no guaranteed way to win solely using mathematics. So, mathematics could be the Saviour if luck is around the corner, but mathematics also is the punisher if luck is not feeling in the mood to be around you....

<u>Final Exhibit D: The Further Student</u>

As a student who has also accepted the intriguing challenge of studying further maths, I often how some of the topics apply to the world and more importantly how significant would they be to us.

1. Firstly, let us begin with **<u>complex numbers (root of –1 or I per say) and Argand diagrams</u>** which have played a crucial role in various fields of engineering, leading to technological advancements that have contributed to human survival and well-being.

Electrical engineering is one area where complex numbers are extensively used. One of the most important applications of complex numbers in electrical engineering is the analysis of AC circuits. AC circuits can be described by complex numbers in polar form, where the magnitude of the complex number represents the amplitude of the voltage or current, and the argument represents the phase angle.

This concept is known as phasor analysis, and it allows engineers to analyze and design circuits that operate on AC signals. It was first introduced by Charles Proteus Steinmetz, a German American mathematician and electrical engineer.

Control systems are another area where complex numbers are widely used. Control systems are used to regulate and stabilize the behavior of machines and processes, ensuring their reliability and safety. Complex numbers are used to model the behavior of control systems, and their properties are used to design controllers that maintain stability and performance. One important concept in control theory is the transfer function, which relates the input and output signals of a system in the frequency domain. This concept was first introduced by Harold Black, an American electrical engineer.

Argand diagrams have played a critical role in visualizing complex numbers, making them easier to understand and manipulate. An Argand diagram is a complex plane where the real part of a complex number is represented on the x-axis and the imaginary part is represented on the y-axis. The magnitude of a complex number is represented by its distance from the origin, and the argument is represented by the angle between the real axis and the complex number. The Argand diagram was introduced by Jean-Robert Argand, a Swiss mathematician.

2. **Volumes of revolution** have also played a significant role in the development of engineering and technology, which has undoubtedly contributed to our ability to survive and thrive as a species.

Volumes of revolution are a fundamental concept in calculus and are used in engineering and design to calculate the volumes of objects that are formed by revolving a 2D shape around a fixed axis. For example, the volume of a sphere can be calculated by revolving a circle around its diameter.

The formula for calculating the volume of a solid of revolution is given by the integral:

$$V = \pi \int_a^b y^2 \, dx$$

where V is the volume of the solid, y is the height of the 2D shape at a given x value, and a and b are the limits of integration. NOTE IN THIS EXAMPLE, WE ARE ROTATING ABOUT THE X-AXIS!

Volumes of revolution have many practical applications in engineering and design. For example, they can be used to calculate the volume of pipes, tanks, and other containers. They can also be used to design parts with specific shapes, such as turbines, propellers, and gears. Furthermore, the efficiency of internal combustion engines, for example, depends on the design of the cylinders and pistons. The volume of the combustion chamber can be calculated using volumes of revolution, allowing engineers to optimize the design for maximum efficiency.

Volumes of revolution were first introduced by Archimedes, a Greek mathematician and inventor, in his work on the calculation of the volume of a sphere and cylinder. They were later developed further by mathematicians such as Isaac Newton and Gottfried Leibniz, who developed calculus, the mathematical tool used to calculate volumes of revolution.

3. **The Poisson distribution** is a unique concept in probability theory and statistics, with a wide range of applications in STEM. While it may not directly contribute to human survival, the Poisson distribution has played a significant role in the development of many technologies and fields that have contributed to human well-being and advancement.

In probability theory, the Poisson distribution is a discrete probability distribution that describes the probability of a given number of events occurring in a fixed interval of time or space. It is often used to model the occurrence of rare events, such as accidents, natural disasters, or disease outbreaks.

The Poisson distribution is characterized by a single parameter, λ (lambda), which represents the average rate of occurrence of the events. The probability of observing k events in the interval is given by the Poisson probability mass function:

$$P(X = x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

where x is a non-negative integer, λ is the average rate of occurrence, e is the mathematical constant roughly equivalent to 2.71828, and x! is the factorial of k.

The Poisson distribution has many practical uses. For example, it can be used to model the number of defects in a manufacturing process, the number of calls to a call center, or even the number of traffic accidents in a city.

The Poisson distribution was first introduced by the French mathematician Siméon Denis Poisson in 1837, while studying the distribution of errors in astronomical observations. Poisson's work on distribution laid the foundation for probability theory. The Poisson distribution has been used in many fields, including epidemiology, ecology, physics, and finance. In epidemiology, it is used to model the spread of infectious diseases, and in ecology, it is used to model the distribution of species in a habitat. In finance, the Poisson distribution is used to model the occurrence of rare events, such as market crashes or defaults in credit portfolios. It is also used in the calculation of the value at risk, which is a measure of the potential loss that an investment portfolio may suffer under adverse market conditions.

Well, from a further perspective, mathematics might be an indirect Saviour. Not all its applications are directly targeted towards survival, but they end up doing so one way or another whether that be through transport, electricity, emergency services, construction or even healthcare too.

Conclusion

Unfortunately, unlike an infinite summation series, this essay (which now feels like a discussion) must converge and come to a definitive end. There is no shadow of doubt that mathematics has absolute power over the world and its exponential progression whether that be in healthcare, engineering or finance. Mathematics is always claimed to be a subject that is for intellectuals and needed to prosper in this society, but the truth is that everyone should appreciate mathematics for its universality. As we have seen, mathematics helps you and I observe and analyze the sound around us, allowed Chishiya to outsmart his opponents and survive, assisted characters in surviving the Hunger Games, helped researchers predict the unpredictable and so on. My point is that we would not be here without mathematics, which is why people like Dr Crawford and myself take such an interest in it. And so, to sign off, I truly hope you have learnt a lot about mathematics and learnt to appreciate such a vivid plane of numerical understanding.