To what extent do chaos theory and fractal geometry have applications to and implications on human physical health?

Abstract

Recent research into the nature of dynamical systems has led to a myriad of results detailing the existence of chaos and fractal geometry in almost all subjects imaginable. This essay aims to explore the applications of these two mathematical concepts to human physical health and what implications they may have on understanding with emphasis on epidemiology, fractal physiology and the heart.

Introduction

The study of nonlinear dynamics, more commonly described by the term 'chaos theory', is a recently popularised science which is increasingly being found to exist in all walks of life. Since its official discovery by meteorologist Edward Lorenz in the 1960s, the field has exploded with an incredible new scale of applications (Reeves, 2014). Paired with the advancements in fractal geometry by Benoit Mandelbrot, which followed in the 1970s, the question of the applications to, and implications on health has since arisen, prompting a new string of research involving these two mathematical concepts and medicine (Horgan, 2009). In this essay, these applications will be explored, as well as the extent of their implications, including how chaos could shape the future of health.

Section 1 – An Introduction to Chaos Theory

1.1 The discovery of chaos theory.

Modern chaos theory was discovered in 1961, by Lorenz, whilst working on modelling weather systems at the Massachusetts Institute of Technology (Lorenz, 1963). Lorenz was striving to provide a model which could aid in longer-term weather prediction. As a result, he initially produced twelve differential equations, defining twelve dimensions or natural laws to be simulated (Gleick, 1998). Lorenz noticed that although there seemed to be near repetitions in his model, exact replications of previous trajectories never occurred. This corresponds with observations of the Earth's atmosphere; although there may be similar occurrences, no day of weather exactly matches another.

At one point, Lorenz started his simulation differently, instead using a rounded value for a variable of 0.506 in place of the exact 0.506127. As the new simulation ran, its trajectory began to drastically diverge from the original, despite the miniscule scope of the alteration (Figure 1). This is what Lorenz coined 'sensitivity to initial conditions', more colloquially known as 'the butterfly effect' due to him half-jokingly commenting that the flap of a butterfly's wings could cause a tornado halfway across the globe.

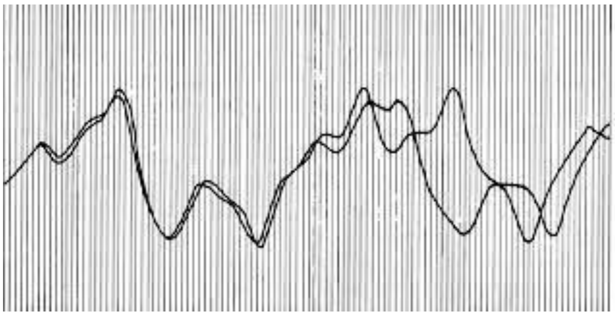


Figure 1 (Gleick, 1998)

Chaos has since been defined differently by many; therefore, it is often more accurate to consider it through its characteristics: sensitivity to initial conditions, complex dynamics, and determinism all of which are displayed in Lorenz's model (Kenkel & Walker 1996).

By considering these characteristics, chaos suggests that with complete knowledge of the exact initial conditions of a system, any future state could be determined. Unfortunately, although this may be true, the impossibility of measuring initial conditions precisely quashes this hypothesis for practical prediction (Lorenz, 1963). This can then lead to a common misconception – that chaotic systems are unstable. Instead, the opposite is true; no matter how much interference and noise a dynamical system is injected with, it will return to and remain on a chaotic trajectory, making such systems 'locally unpredictable [but] globally stable' (Gleick, 1998, p. 48).

1.2 Identifying and visualising chaos

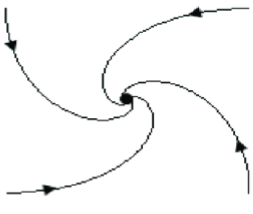
As the years progressed, discoveries of chaotic phenomena were continuously being made with a surge of new investigations after Gleick popularised the theory amongst those of all professions (Gleick, 1998). From nature to human interactions (Pool, 1989), chaos was discovered within multiple subjects.

Despite this, the existence of chaos is not easy to prove. To understand the proofs of chaos, the following section of this essay aims to provide insight into the various ways of identifying and visualising the chaos of dynamical systems, through introduction to phase space and the Lyapunov exponent.

Phase Space

After discovering the chaotic phenomenon, Lorenz created a visual representation of the events he had observed, by constructing a three-dimensional phase portrait in phase space, narrowing down his twelve dimensions to three variables. By doing this, each of the variables could represent an axis on a three-dimensional plane, with values dependent on time. Each point in a phase space portrays a possible instantaneous state of the system (Lorenz, 1963) which usually produces one of three, principal attractors, dependent on whether the nature of the system being plotted is random, systematic or chaotic (Philippe, 1993).

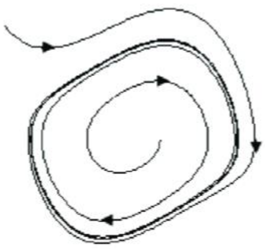
These shapes are called attractors due to the fact that if the system is knocked off course, it will return to the trajectory demonstrated in the phase portrait: a fixed point, periodic attractor or strange attractor.



Fixed point attractor

Figure 2 (Richardson, Dale & Marsh, 2014)

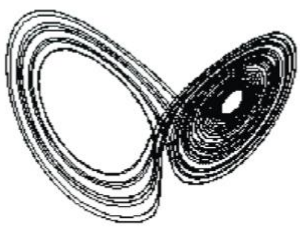
The simplest of the three attractors is the fixed-point attractor, represented by a single point, as demonstrated in Figure 2. For example, a marble being placed in a bowl, always coming to rest at its deepest point. The resting state of the bottom of the bowl would be the single point in phase space (Yale, n.d.).



Limit-cycle attractor

Figure 3 (Richardson, Dale & Marsh, 2014)

The second phase space portrait is the periodic attractor. In a diagram, this consists of a simple, closed shape: an ellipse, as seen in Figure 3. It represents periodic oscillation, in which the system repeats its motions indefinitely, such as in a frictionless pendulum – the lack of resistive forces means the pendulum swings, repeatedly producing the same coordinates in phase space.



Strange attractor

Figure 4 (Richardson, Dale & Marsh, 2014)

Finally, the strange attractor is the most obscure, yet the most important portrait when regarding chaos. These attractors are neither static nor periodic, instead forming a continuous trajectory which never crosses over the same path twice (Robertson & Combs, 2014). Strange attractors seemingly create a paradox; they form an infinite path within a finite area. Its path passes arbitrarily close arbitrarily often to any point through which it has previously passed, occupying just less than the number of dimensions it was plotted in (Lorenz, 1963). The most famous strange attractor (the Lorenz attractor) is depicted in Figure 4, resembling the image of a butterfly's wings, adding to the image of 'the butterfly effect'. Strange attractors are staples of chaos and are used in identifying chaos.

The Lyapunov Exponent

Another important concept in determining systems as chaotic, which can help in the construction of phase portraits, is the Lyapunov exponent. The calculated Lyapunov exponent of a system tells the investigator the nature of system in terms of the three types of attractor, by quantifying the sensitivity of initial conditions of a system and measuring the rate convergence or divergence of trajectories starting from nearby initial points (Sandri, 1996).

If the Lyapunov exponents calculated are always negative, then the system belongs on a fixed-point attractor, as the perturbation will always die down to zero. When the exponent is always negative or zero, the system should be represented by a periodic attractor. And when the Lyapunov exponents have negative, zero and positive values, the system can be plotted on a chaotic attractor (Guan, 2014). The positive value of the Lyapunov exponent shows that if there is a perturbation in a certain direction, it will grow exponentially. This shows a sensitivity to initial conditions which is, yet again, a defining characteristic of chaos.

Using such methods to identify chaos prove useful when applying chaos theory to a range of subjects, such as epidemiology, which will be covered in the following section.

Section 2 – Chaos in Epidemiology

2.1 Chaos in infectious disease

One of the greatest threats to human health has always been disease. In recent years, COVID-19 has infected conversations and flipped worlds upside down, yet knowledge and ability to predict such pandemics, despite advancing at an incredible rate, is still very limited.

There are many variations of epidemics that have been investigated and analysed and each have their own specific dynamics. Through the lens of chaos theory, the mechanics of disease can be seen under a new light. Chickenpox, and other periodically occurring diseases, plot elliptical attractors in phase space (Philippe, 1993). Other diseases are endemic, meaning that they occur consistently (CDC, 2012). Yet there are also sporadic diseases, occurring at irregular intervals and frequencies, such as measles (Philippe, 1993). It is these sporadic diseases which seem to display the most chaotic dynamics.

Like with all determination of chaotic dynamics, deciding whether a model of a disease shows a chaotic nature can be established through identifying certain characteristics:

aperiodicity, sensitivity to initial conditions, long term unpredictability, and determinism (Borah et al., 2022). In terms of disease, these would be translated respectively as follows: a lack of repeating patterns in the dynamics being investigated; a rapid divergence in trajectories with slight changes in initial conditions; propagation of the disease being fundamentally unexpected; the outcome, rate and spread of disease being defined by real changes in physical conditions.

Mathematical models can be a great aid in spotting these attributes, with the most popular being the Susceptible–Infectious–Recovered (SIR) model, first used by Kermack and McKendrick in 1927 (Momani et al., 2021). In general, these models have prominent use in epidemiology, helping to understand the growth and spread of infectious disease as well as assisting vaccination regimen (Sinha, 1997). However, when discoveries of chaos arise, it can become much more complex, either diminishing hopes of prediction or spurring on investigations to use chaos to our advantage.

2.2 Chaos in measles

One notable disease in which chaotic dynamics has been explored multiple times is measles. Due to the extensive data collected over the past 200 years for major cities such as New York, it has been considered to be one of the best contenders for the detection of chaotic fluctuations (Grenfell, 1992).

Evidence for chaotic dynamics in measles epidemics has been found throughout the US and UK, in a study based on observations of 80 major cities in the 'prevaccination' era of these countries. The dynamics of measles in the US were reported to result in chaotic patterns, due to exhibitions of unprompted shifts in periodicity, displaying the potential of the disease as a case study for chaos theory (Dalziel et al., 2016). Determining this hypothesised chaos in the data available was achieved through detecting and testing sensitivity to initial conditions through use of models like SIR models as well as calculating the Lyapunov Exponent for the major cities.

Creating the deterministic SIR models provided evidence needed for sensitivity to initial conditions, which arose mostly due to the seasonal fluctuations in transmission – changing around school terms, or due to migration of workers, for example. These slight changes in transmission of the disease led to significant variation in the complexity of measles dynamics, resulting in characteristically chaotic patterns. Interestingly, the US is reported to have a higher and more variable level of chaotic dynamics than the UK, with its sensitivity to initial conditions being greater than in UK cities, reducing the model's ability to be applied to measles epidemics elsewhere.

Further evidence for chaos came from the calculations of the Lyapunov Exponent, in which most US cities had a positive result, confirming the diverging of trajectories with arbitrarily close initial conditions.

Based on these investigations, measles is a clear example of the existence of chaos in the spread of infectious disease. Due to this, small perturbations in transmission rate can lead to rapid erosion of the capacity to forecast epidemic patterns, explaining the difficulty in predicting the outcomes of a disease and possibly reducing the efficacy of control measures such as vaccination (Dalziel et al., 2016, Olsen & Schaffer, 1990).

2.3 Chaos in COVID-19

Since explorations into the dynamics of measles epidemics, the question of the extent to which chaos theory can be applied to other communicable diseases has arisen. New knowledge regarding the origin and spread of the Bombay Plague Epidemic (in discovering multiple epizootics of rat were responsible instead of a single species of rat and flea), led to a new model, finding the progression of the disease to be chaotic (Mangiarotti, 2015). Other instances of diseases having undergone similar scrutiny and results include the Ebola virus (Borah et al., 2022), smallpox outbreaks, and most recently COVID-19.

Efforts to understand the COVID-19 pandemic have been never-ending in attempts to drastically reduce its impact. In order to do this, a multitude of chaotic mathematical models have been produced, hoping for an explanation as to why this pandemic has had such a colossal effect on the world. In this section, the evidence for COVID-19 being a chaotic pandemic will be laid out and discussed, before suggesting the implications that this may have on our understanding of the disease.

Searching for chaos in epidemiology means searching for its key characteristics. In the context of COVID-19, this has been approached in several different ways during the time in which the disease has impacted the world, using global and local data.

An example of a global investigation is the work published by Jones and Strigul in October 2020. The daily cases per country, data collected by Johns Hopkins University over the course of the pandemic, was used in this study. From this, Jones and Strigul designed a dynamic analysis tool to process and examine the data, comparing the courses plotted across countries and territories in search for chaotic features.

Unlike the SIR model, those recovering from COVID-19 were not taken into account, instead purely investigating the cumulative cases by date, still allowing sufficient analysis of infection rate. At first, the number of cases for each country was plotted - days since first recorded infection against the percentage of population infected. This is where the data of infected percentage of countries' populations showed wildly different trajectories, despite starting off with similar progressions. As seen in Figure 5 and 6, it would be impossible to predict the exact course of infection for a single country based on results of previous days, as there are clearly multiple trajectories which it could follow.

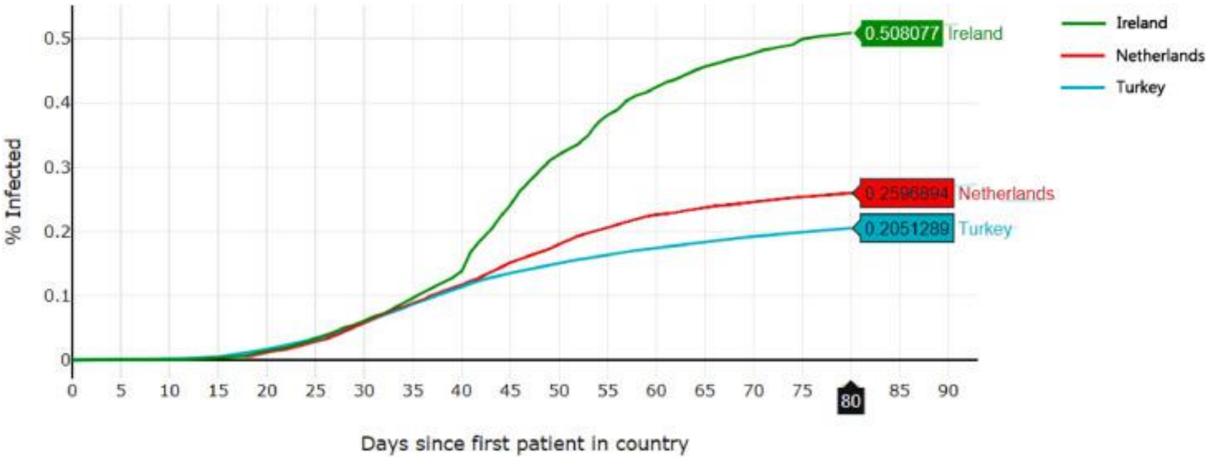


Figure 5 (Jones & Strigul, 2020)

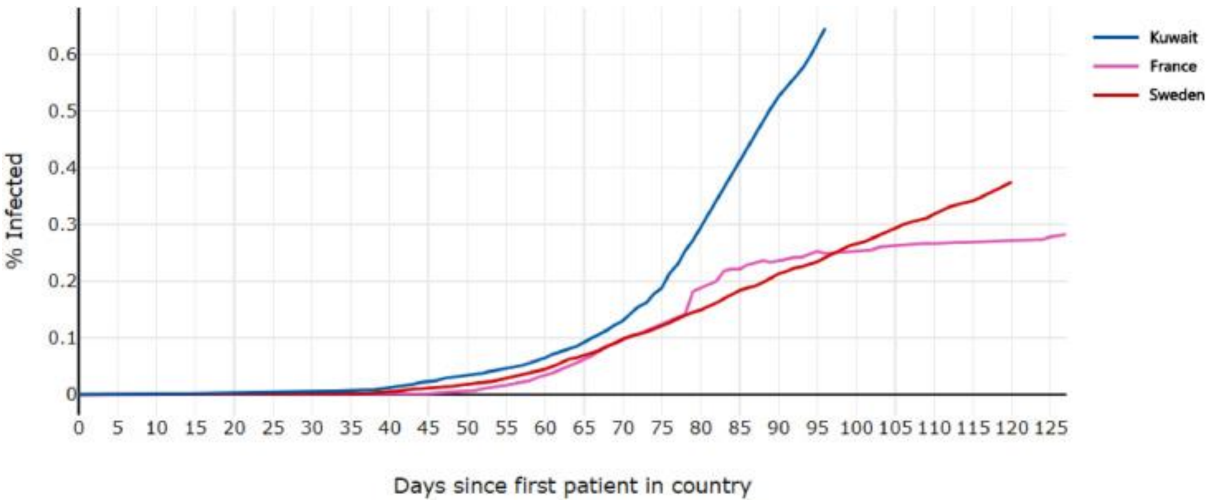


Figure 6 (Jones & Strigul, 2020)

These results not only display unpredictability in the data, but also sensitivity to initial conditions. To further support this observation, a plot of the rate of infection (first derivative) followed by plots of rate of change in spread (second derivative) were carried out. By measuring the second derivative, a quantitative analysis of the sensitivity observed was available, solidifying evidence suggesting sensitivity to initial conditions. This can be visually seen in Figure 7, an example of Italy's second derivative, which resembles the distinct time series of a classic chaotic system.

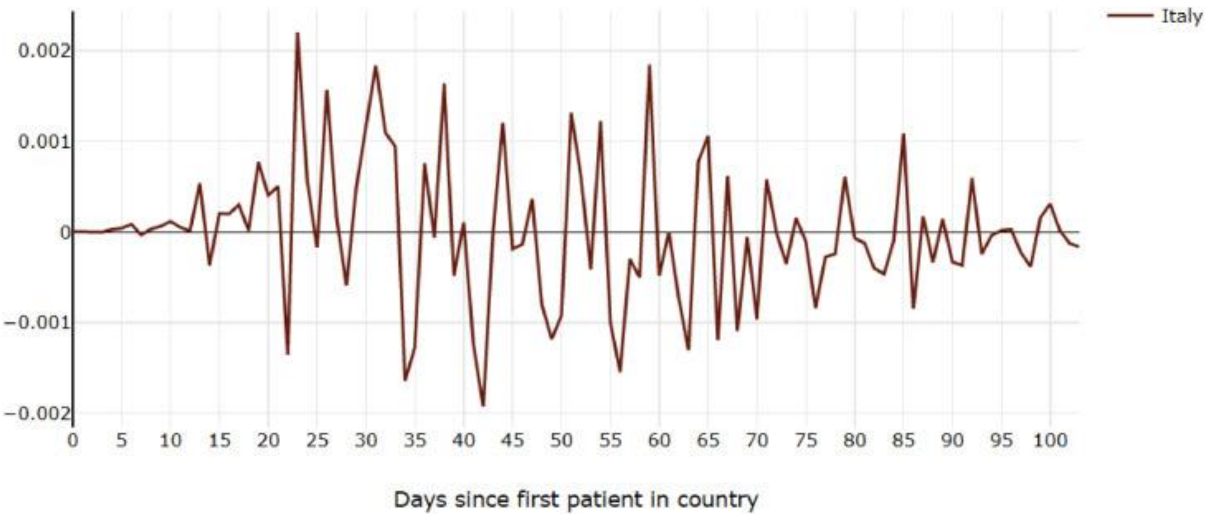


Figure 7 (Jones & Strigul, 2020)

However, comparison of the countries and territories of the world was not where the analysis of COVID-19 ended. In separate studies, the dynamics of the disease in Japan and the US – Sapkota et al. (2022 and 2021 respectively) - were investigated, mainly using a modern technique for detecting chaos, called the 0-1 test. This test has no need for construction of phase space yet still has the capability to distinguish between periodic and chaotic dynamics, with a result nearing one strongly suggesting the presence of chaos (Gottwald, 2016). In both of these studies, the dynamics of COVID-19 varied on a local scale, dependent on the prefecture in Japan and the state in the US. On this scale of analysis, 76.6% of Japanese prefectures and 35% of US states showed chaotic behaviour, showing that although not every instance returned with a positive result, chaotic dynamics still exist in the disease. Lyapunov exponents were also calculated to be positive, further supporting this.

Despite COVID-19 not meeting the requirements for chaotic dynamics for all scopes of study, the majority of conclusions (including additional analysis of phase portraits) have led to the disease being considered chaotic on the whole (Borah et al., 2022). Like measles, this discovery has several implications on approaches to combat infection rate, with the major influence being sensitivity to initial conditions.

Attempts to utilise knowledge of chaos in predictions of the disease have already been made using data regarding the emerging epidemic in China, Japan, South Korea and Italy, and applying it to predict the disease's course in sixteen other countries (Mangiarotti, 2020). Although in some cases the closest scenarios identified initially provided some resemblance across countries, in most situations, the differences accumulated, causing trajectories to diverge drastically and predictions to deteriorate, due to differences in control measures and the population's response from country to country.

Despite the sensitivity to initial conditions rendering predictions of COVID-19 courses practically useless, chaotic models have had other uses. In the case described above, it was concluded that the chaotic global modelling approach to COVID-19 could have been (and still can be) useful for decision makers in analysing the efficiency and efficacy of control measures, advising those in countries in which disease levels were still low on approaches to tackling infections.

This is supported by Sapkota et al. (2021) in writing that an improved understanding of the underlying dynamics of COVID-19 and effects on infection rate could be used to enhance the effectiveness of public health interventions, providing reassurance that chaos can be used to benefit human health.

Section 3 – Fractal Geometry in the Human Physiological System

Chaos is not only found in the patterns of the population of disease, but intrinsically in each individual, through the behaviour and structure of physiological systems of human bodies, mostly in the form of fractals. The works of chaos can be found in the structure of the circulatory system and the way a heart beats, right down to the composition of DNA (Kenkel & Walker, 1996). It is in this way that chaos makes up an essential part in the way the human body functions, impacting physical health in a way that nobody had thought of before Lorenz's fateful discovery.

After an introduction to fractal geometry, the following section of this essay will explore the applications of fractals in the human physiological system and what this may mean for the future of medicine. Through this, the surprising truth in the statement 'chaos is health' will be delved into.

3.1 Fractal Geometry

In 1975, Polish-born mathematician Benoit Mandelbrot introduced the term 'fractal' to mathematical understanding, transforming outlooks on Euclidean geometry (Gleick, 1998). Instead of concerning himself with traditional one-, two- and three-dimensional shapes, Mandelbrot created a branch of mathematics which looked at shapes with non-integer, fractional dimensions.

Due to its complexity, fractal geometry was not formally defined until several years after Mandelbrot's discovery. Therefore, it has been found that the best way to understand this concept is not through its calculations or complex shapes, but through the context and application of nature. In his 1982 book, 'The Fractal Geometry of Nature', Mandelbrot famously observed, "Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line" and contemplating this clearly shows how insufficient the world of Euclidean shapes is in the practical universe.

The coastline of Britain can be used to comprehend the basics of fractal geometry. From afar, the shape of Britain could be described as somewhat rectangular, however, this becomes further from the truth as the image is magnified. Rough edges of the coastline become harder to outline with Euclidean shapes, and complexity continues to increase upon further magnification. Wildly different results are produced if the path of a boat around the coastline is measured in comparison to the pathway of an ant around the same coastline, around rocky surfaces, yet they would both be valid outcomes.

With this aid of the coastline, the main characteristic of fractals becomes clearer: their ability to undergo infinite magnification and still behold intricate, self-similar patterns (The Colours of Infinity, 1995).

Computer generated fractals more accurately abide by defining attributes, with simple formulae producing self-similar patterns with infinite resolution, much like how complex deterministic systems can be described by just a couple of variables and differential equations.

Examples of such fractals include the Koch snowflake, which were discovered before Mandelbrot's findings (Stewart, 2009). The Koch snowflake arises from simple iterations, beginning with an equilateral triangle and replacing the middle third of each side with a 'wedge', creating a star-like shape the process of which can be seen in Figure 8 (Crawford, 2020). Through the repetition of these steps, the complex Koch snowflake forms (Figure 9).

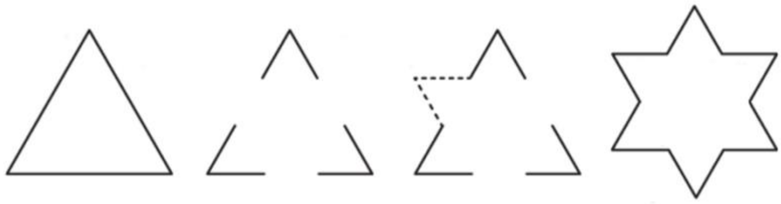


Figure 8 (Cruzan, 2019)

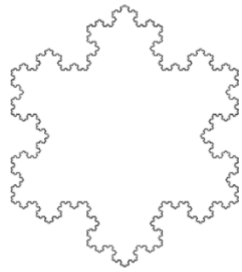


Figure 9 (Berg, 2023)

Unlike in Euclidean geometry, the Koch snowflake, and other fractals such as the Sierpiński triangle are far from limited to integer dimensions. Instead, the dimensions of fractals can be calculated to be fractional – non-integer numbers which describe how the shape takes up space. Both Britain's coastline and the Koch snowflake have dimensions between one and two, being approximately 1.25 (Mandelbrot, 1967) and 1.26 (Vanderbilt, n.d.) respectively.

The calculation of such dimensions can lead to quantifying the complexity of fractals in terms of how densely it takes up the space it's plotted in. A higher dimension leads to a higher element of 'roughness' and therefore a higher complexity (Husain et al., 2021).

Fractals appear at many points in chaos theory, often being described as 'remnants', or visual identities of chaos due to fractals resulting from chaotic processes (Goldberger et al., 1990). The strange attractor – the symbol of chaos theory – has also been discovered to exist in a fractional dimension, being an infinite pathway in a finite space. In this way, two great discoveries of the twentieth century become linked, uniting to inspire incredible research, including into how human health may be impacted by them.

3.1 - Fractals in Human Physiology

After finding so many natural examples of fractals, it is not a surprise that similar structures exist in the human body as well. The distinct fractal branching found in trees and rivers emerges in multiple human systems, including the circulatory, respiratory and nervous systems (Gleick, 1998). The human body may well be the place where fractals and chaos are most abundant and available to study.

The recurrence of such structures in so many places throughout the body has led to multiple researchers suspecting the existence of a simple internal code, repeated throughout development to create otherwise unrelated systems so similar (Goldberger & West, 1987). It has also been suggested that the processes resulting from this code are examples of deterministic chaos - another reason as to why the structures formed are fractal-like. In order for the body to have evolved to contain several instances of fractals, there must have been valuable reasons for it to do so, leading to investigation into the role fractals in human health.

One of the main purposes for the repeating self-similar organisation, from bronchi into bronchioles and villi into microvilli, is the optimisation for substance exchange. It is well-known that a large surface area to volume ratio is ideal for diffusion, osmosis and active transport improving efficiency, and a fractal structure provides this. Information processing in the neurons can also have this applied to it. Just like the coastline, consequent to their fractal dimensions, these self-similar structures increase surface area, benefitting function of the human body and therefore its health.

Furthermore, typical fractal structures provide security when faced with external damage or internal genetic alteration. They are robust and resistant to injury, due to their flexibility and adaptability in response to modification (Kenkel & Walker, 1996). This follows the same principle as chaotic systems, in which the trajectories of a strange attractor return to their stable paths, in spite of perturbations to the dynamical system.

In addition to providing stability to the structures of human physiology, revealing the fractal dimension of physiological compositions through fractal analysis can be used in identifying, and in some cases predicting the course of diseases. The lungs give plentiful examples of this, the most obvious being the bronchiole branching (Lennon et al., 2015). Yet beyond this, the composition of the lung tissue has a fractal dimension.

3.2 Fractal Analysis of the Lungs

The lungs have undergone numerous examples of intense fractal analysis in order to investigate how dimensions vary with health. An example of this is the analysis of silicon casts of human airways made from the autopsy material from people of three groups: nonfatal asthma, fatal asthma and no asthma (Boser et al., 2005). Apart from the clear abnormalities in the airways of the asthma patients, fractal analysis concluded that they also correlated with lower fractal dimensions (1.76), in comparison to no asthma (1.83).

As lower dimensions describe less complex structures, this confirmed the notion that higher levels of complexity are associated with better health. Continuing the research also led to comparisons of fractal dimensions between fatal (1.72) and nonfatal (1.76) cases of asthma in which fatal cases had a lower fractal dimension, supplying a means of possibly quantitatively defining severity of the disease.

There have been several other lung diseases which have recently been viewed through fractal analysis, including chronic obstructive pulmonary disease (COPD) as a result of emphysema and airway disease. In the case of Bodduluri et al. (2018), non-invasive CT scans were used to remodel the airways of over 8000 participants with COPD and carry out fractal analysis. Like in asthma, the results displaying lower fractal dimensions were in more severe cases of the disease and also showed higher risks of disease progression.

The exception to this pattern seems to be lung cancer. Cancer has great impacts on fractal dimensions of lung tissue but leads to a higher value dimension in comparison to healthy lungs (Lennon et al., 2015). It has also been hypothesised that aggressive cancer tumours have higher fractal dimension than non-aggressive tumours, although this requires further confirmation in larger studies.

The potential fractal analysis provided could bring notable implications to how diseases such as asthma, COPD and cancer are diagnosed and treated. Fractal analysis of the lungs not only identifies the presence of the disease but may also be used in classifying subtypes within it, especially in emphysema (Tanabe et al., 2020) and cancer (Itik & Banks, 2009), the beginnings of which have been shown in how dimension depends on asthma severity. This could be developed as an invaluable technique to predict the prognosis of lung disease, influencing approaches to treatment.

Section 4 – Chaos and Fractal Geometry in the Heart

Like the bronchiole tree and organisation of neurons, the structure of the circulatory system is one which displays typical features of fractal structures, with the blood vessels branching out into self-similar forms. This provides an optimal surface area for the exchange of nutrients and gases in and out of the bloodstream and is especially prevalent in the patterns of the coronary arteries surrounding the heart (Goldberger et al., 1990). Yet chaos is also found in the human heartbeat.

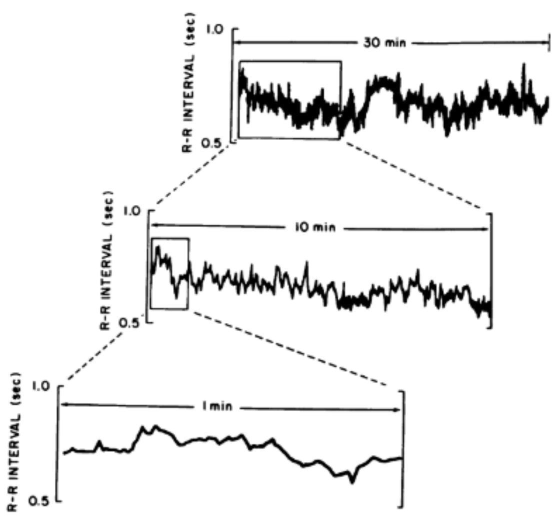


Figure 10 (Goldberger & West, 1987)

It is obvious that between periods of rest and activity, heart rate changes. The heart is required to beat faster in response to exercise, to provide sufficient resources to the muscle cells around the body (Kumar, 2012). Yet during intervals of rest, most picture the heart rate to be periodic, mapped out as a regular sine wave on a time series plot. In fact, the resting heart rate of healthy individuals is instead slightly irregular – chaotic, even (Firth, 1991).

Carrying out time series analysis of beat-to-beat heart rate fluctuations in healthy subjects revealed a highly erratic graph, with self-similar fluctuations over multiple time scales (Figure 10) (Goldberger & West, 1987). On first glance, this seems to express the idea that heartbeats vary randomly, yet the scaling in the time series graph gives the first hints of fractal qualities, therefore suggesting the presence of chaos. To confirm this, a phase space representation can be formed the result of which is a trajectory in phase space resembling a strange attractor (Figure 11) – a pathway which never repeats itself.

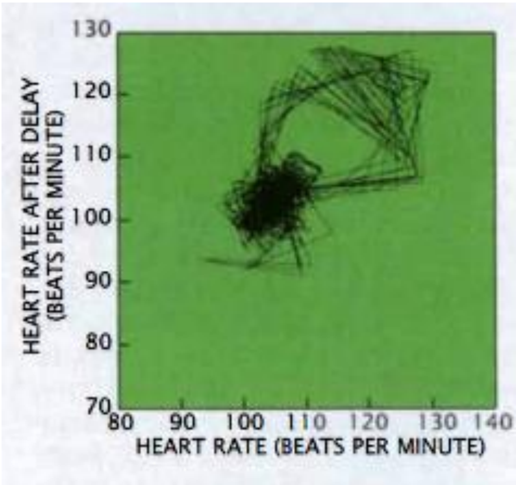


Figure 11 (Goldberger et al., 1990)

Since the discovery that a young, healthy heartbeat is a chaotic system, numerous academics have researched into what this may mean for the approach to heart health and disease. One of the main subsequent findings was the variation in the levels of chaos present dependent on age and in the lead up to cardiac arrest. Just like in the fractal structures in other areas of the physiological system, the heart conforms to the fact that complexity suggests health; heartbeats lose variability and become more regular with age and disease (Wu et al., 2009).

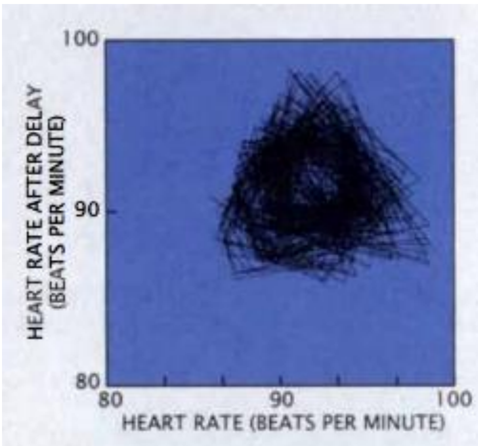


Figure 12 (Goldberger et al., 1990)

Analysis of the workings of the heart in this way, could have major implications on detecting cardiac arrest. In the minutes and even months leading up to cardiac arrest cases, the resting heartbeat becomes more regular, with phase portraits starting to evolve from strange, to periodic, to fixed point attractors (Sessa et al., 2018). This can be seen in Figure 12 – a 'noisy' limit cycle attractor produced by a heart eight days before sudden cardiac death - and Figure 13 – a point attractor produced by a heart thirteen hours before sudden cardiac arrest.

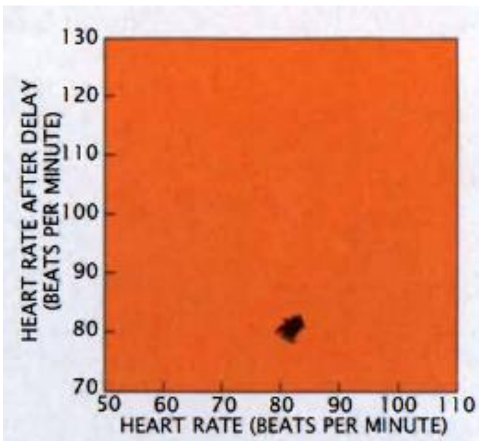


Figure 13 (Goldberger et al., 1990)

Being able to predict the oncoming of sudden cardiac arrest is the first step in prevention and treatment, and applying chaotic analysis in the future could identify such cases in hopes of counteracting it. Similar results of a loss in variability also occur when observing disorders in other major organs including epilepsy, Parkinson's and manic depression in brain waves, and certain cases of leukaemia in the number of white blood cells (Goldberger et al., 1990).

Regarding chaos and treatment of heart abnormalities, is experimental data carried out by Garfinkel et al. (1992) in which they utilised chaos and the heart's chaotic features to eradicate heartbeat arrhythmias (i.e., abnormal heart rhythms). It was believed that the identification of chaotic phenomena in the body such as the heart could lead to the development of new therapeutic strategies.

To confirm this, the study induced arrythmias in portions of rabbit heart using the chemical ouabain and used methods of chaos control to return the heartrate back to normal. The approach to chaos control used was a modified version of one developed by Ott, Gregbogi and Yorke (OGY) in which it was theorised it would be possible to stabilise a chaotic system around one of its periodic motions. Garfinkel et al. altered the OGY method, allowing application to systems in which the defining equations were unknown, and called it proportional perturbation feedback.

In eight of the eleven instances of chaos control, proportional perturbation feedback successfully returned the heart to a normal rate, reducing tachycardia as well as eliminating arrhythmias. The success of the method was confirmed as when removing the chaos control system, the ouabain induced arrhythmias returned. Having an instrument which could continuously use the heart's chaotic features to provide this level of chaos control could therefore allow heartbeats to permanently be controlled and prevent the onset or continuation of heart arrythmias.

At this point in time, it is uncertain whether the methods of chaos control could be applied to the in vivo heart, yet the knowledge that such methods exist could be enough to ignite the development of the current techniques and move towards treating arrhythmias in humans. These could then be applied to other chaotic systems, to control electrical waves in the brain for example, helping to suppress epileptic attacks (Stewart, 2009).

Conclusion

Chaos theory and fractal geometry play a big part in defining the state of human health, ranging from the external patterns of infection rate of a population as a whole, to the intrinsic chaotic and fractal workings of the organs in each individual. Throughout this essay, the links of chaos to epidemiology, specifically the disease measles and COVID-19, have been explored, alongside the multitude of examples of fractal geometry and chaos in human physiology, especially the lungs and heart.

There are plenty of applications of these two mathematical concepts within human health, providing many implications. Mastering how chaos theory links to certain diseases could allow researchers to re-evaluate approaches to predicting the course of epidemics and pandemics, taking sensitivity to initial conditions and the limits they provide into consideration. However, not all disease dynamics conform to chaos, and so the extent to which it can be used to define health in this aspect is limited. Alongside this, proving the existence of chaos in these instances is difficult, but efforts to utilise chaos in the prediction and understanding of COVID-19 have been successful and therefore, searching for chaos should still be pursued.

When introducing fractal geometry into chaos, applications and implications on human health (in terms of the individual) rise dramatically. There is a large extent in application to human physiology, from the microscopic scale of DNA to the macroscopic respiratory and circulatory systems. It seems that the influences of complex fractal geometry and chaos improve efficiency of these systems and contribute to good health overall.

Making an effort to understand chaos within health could lead to newly improved approaches in therapeutic strategies and diagnosis of disease, as seen in the Garfinkel et al. experiment on the heart. This then may lead to similar advancements in other chaotic organs including the brain, yet again providing insight into the extent to which chaos can have applications and implications on human health.

Overall, although chaos theory and fractal geometry are not applicable to every aspect of human health, their influence in the state of wellbeing of individuals and populations alike is undeniable, and they have the potential to provide new and exciting pathways in the future of medicine.

Word count: 5421

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