A CONVOLUTED COLLISION: Maths In Geography

Physiography, Longitude and Latitude, Maps… is any of this what comes to mind when we think about Mathematics…?

THESIS

Thinking about the above question… does Mathematics typically play any role in anything remotely Geographical? Most people would presume not, but what if there is more of a connection there that meets the eye? Geography and Mathematics might seem like two completely different subjects, but they are more closely related than one might think. Defined, Geography is the study of the Earth's physical features, including its landforms, oceans, and climate. Meanwhile, Mathematics is the science that deals with the logic of numbers, shapes, patterns, quantities and arrangement. The word "Mathematics" actually comes from the Greek word "Mathema" meaning "science, knowledge or learning". Whereas, the word Geography is derived from the Greek word "Geo" (the Earth, in its broadest meaning) and graphos (to write about) making the literal meaning "to write about the Earth". A pioneer in mathematics and considered to be the founder of Geography, Eratosthenes (born in 276BC, in Cyrene, modern day Libya) is best known for being the first person known to calculate the circumference of the Earth, which he did by using the extensive survey results he could access in libraries; his calculation was remarkably accurate, thanks to his use of mathematics, of course! He led what would come to be known as the revolutionary world of Mathematical geography. In this article, you will be taken on a journey into the intertwined relationships between Mathematical principles and Geography.

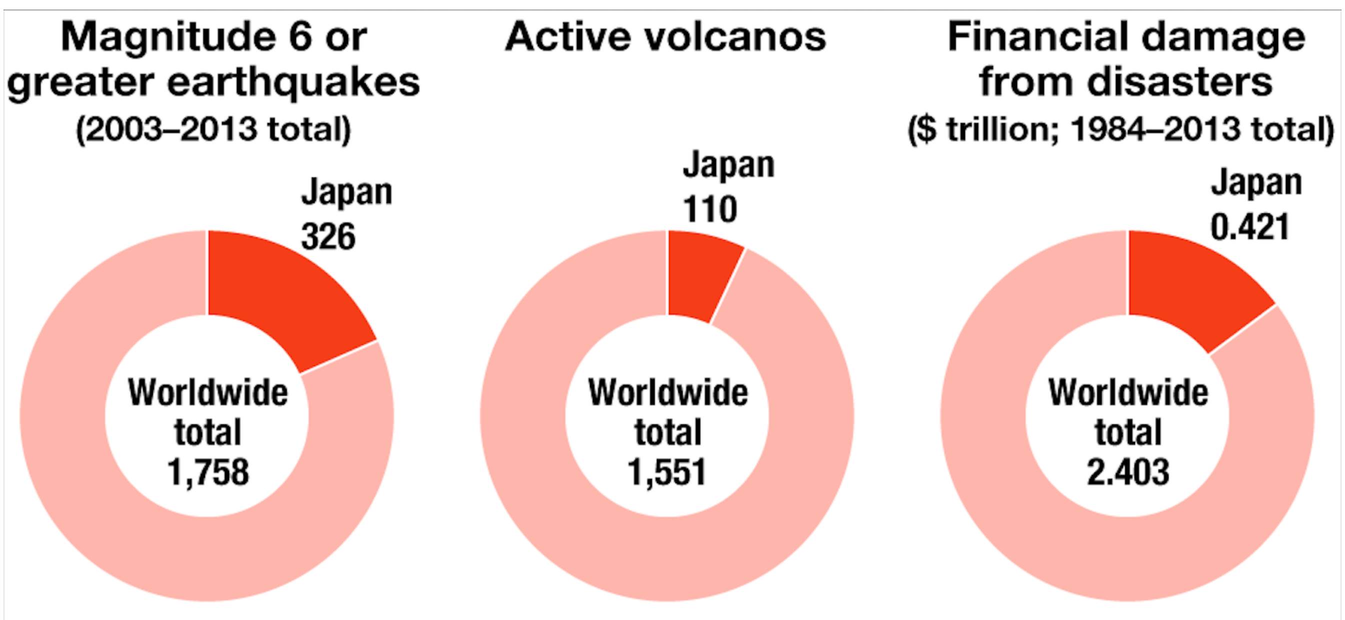
Written by Leena Makkawi

Statistics

Geography is a broad and dynamic topic covering all aspects of the Earth, so many statistics are looked at, which Geographers analyse to create trends (or patterns) so we can better understand this Green planet we live on. Geographers look at spacial patterns between factors, which involve statistical data. For example, how would light intensity affect the likelihood of drought? These correlating factors are represented through maths, in the form of percentages, pie charts, bar charts and more! Statistics become important when looking at things like the aforementioned weather patterns or pollution rates. They can range from a wide variety of factors that affect us on Earth. In the realm of Environmental Geography, let us take a look at statistical data relating to the frequency of natural disasters in Japan:

Japan only accounts for 0.28% of the world's land, and 1.9% of the world's population. Furthermore, it is the site of 18.5% of the world's earthquakes that have a magnitude of 6 or greater, these are considered to be **strong** earthquakes.

Figure 1 below is the finding of a 2014 Japanese white paper on disaster management, sourced from Nippon.com:



As shown in Figure 1, the quantitative data we are given in the form of pie charts and percentages (which are part of mathematics!) helps us, as Geographers, gauge how intense the impact of natural dangers is on Japan meaning that Mathematics is essential to us as Geographers because it conveys Geographical data in a comprehendible but concise way.

Latitude and Longitude

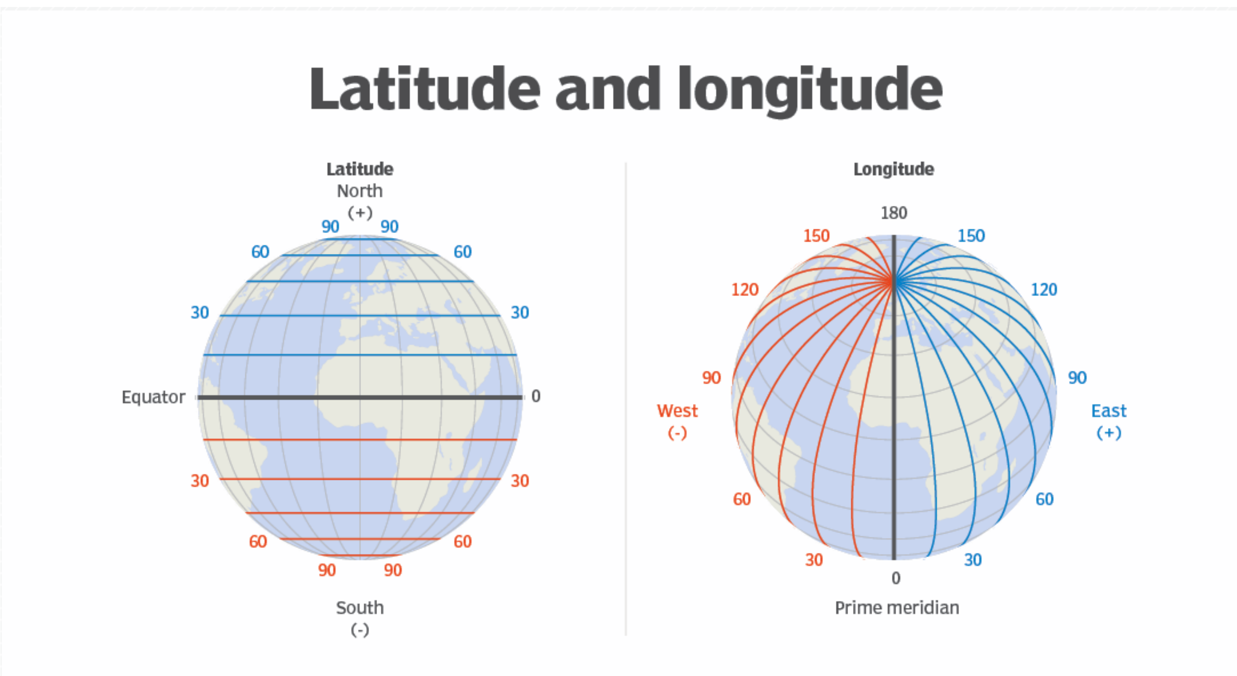
You often see grid lines on a map, these help us decide as Geographers where things are placed in terms of the geographic coordinates of Latitude and Longitude. These coordinates are very important because people who work as Navigators or Surveyors rely on this geographical system to pinpoint an exact location. Furthermore, when applied to real life, Longitude and Latitude are relied upon by GPS systems and regular maps that we use to find our way around Earth!

Latitude (Horizontals) are geographic coordinates that denote a point on the Earth based on its North-South location. Lines of Latitude (Horizontals) are also called parallels as they parallel the equator. The equator passes through many countries, including Ecuador, Colombia, Brazil, Gabon, the Republic of the Congo, Uganda, Kenya, Somalia, Maldives, and Indonesia.

Conversely,

Longitude (Verticals) are geographic coordinates that denote a point on the Earth based on its East—West location. Lines of Longitude are also known as Meridians because they measure east or west of the Greenwich Meridian which is located in Greenwich, England.

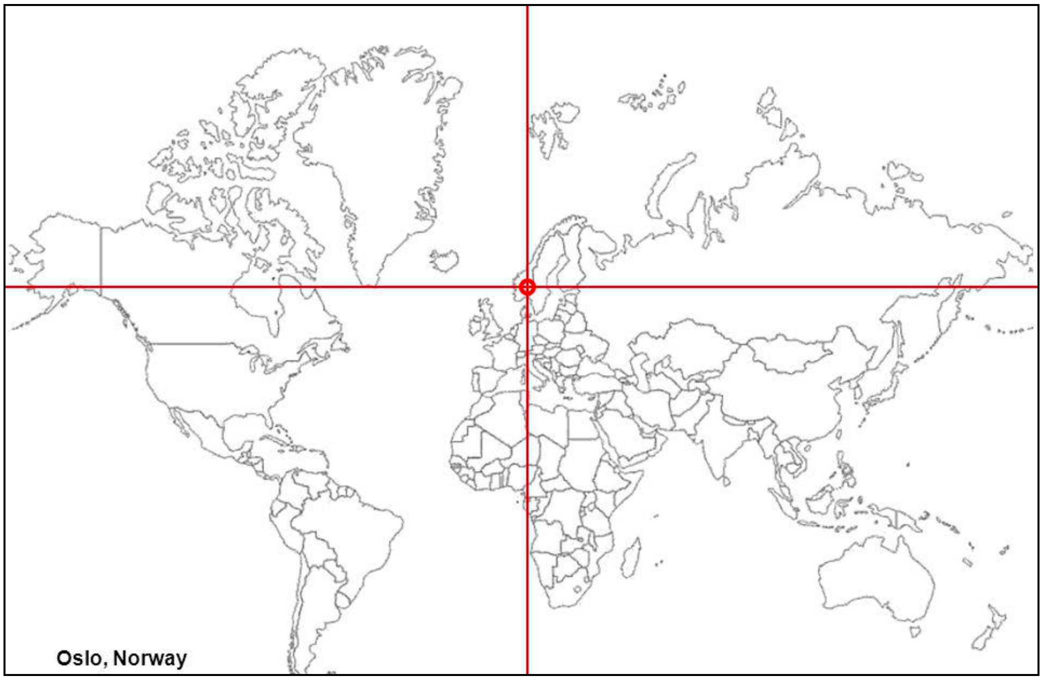
Note that in notation (no pun intended), we would express Latitude and Longitude in degrees (°) and minutes (′), below (Figure 2) illustrates Latitude and Longitude visually.



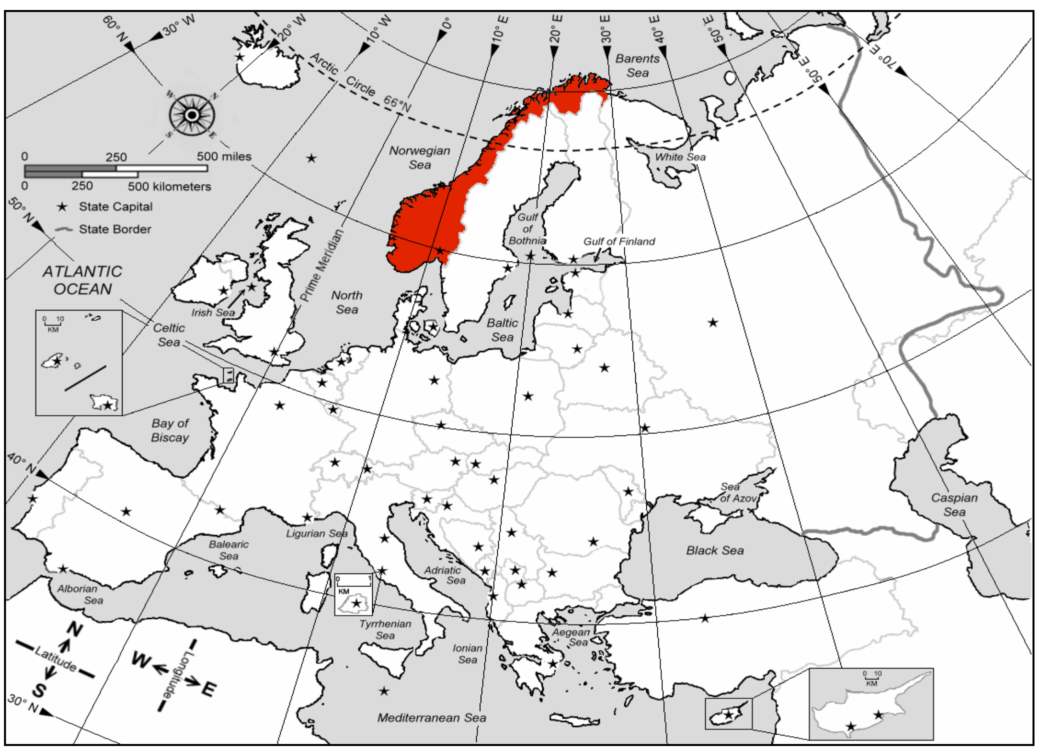
An example of a Latitude and Longitude location would be Oslo, Norway with the geographic coordinates of 59° and 57′ north of the Equator (Latitude) and 10° and 45′ east of the Greenwich meridian (Longitude).

This is illustrated by the diagrams, **Figure 3a** and **3b,** shown below indicating Oslo on a map:

a

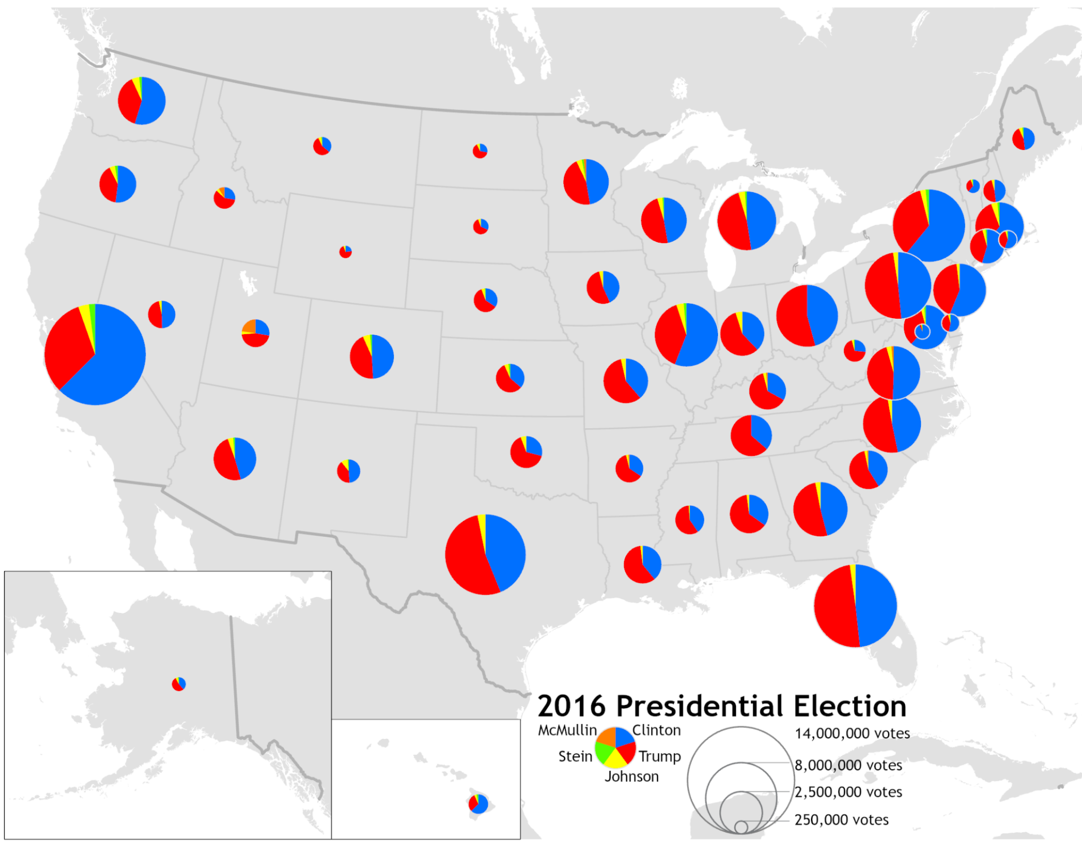


b



Maps that use mathematical data representation

Maps are the pinnacle of Geography, they compress so much of what we know as Geographers onto a single piece of paper, a profound realization indeed. Geographers study the inner workings of maps, namely, specific countries, geographical features such as Volcanoes, or even seismic activity (which is earthquake activity). There are so many ways to represent the data we discover on Maps, and many use mathematical data representation to make that clear. Lets us take a look at **Figure 4** below, a map, which represents data in unique mathematical ways.



As you can see here, we can see a map of the United States of America covered in pie charts and colours which represent data, furthermore, this specific map represents what were the results of the Presidential election that occurred in 2016. The circles range in size, and this is deliberate to proportionally represent the number of votes each state received, and the scale tells us approximately that, ranging from 250,000 to 14,000,000. In addition, each circle is a pie chart, with each colour in the circle representing a candidate. The visual representation of the map uses mathematical data representation and scale, proving that Mathematics is used throughout maps! When we measure how much each circle represents in terms of votes, and calculate the proportion of each colour in each circle we can see that the candidate Trump would have won this election.

A Mathematical Application of Latitude and Longitude - The Great-Circle distance

We can now look at how Longitude and Latitude (geographical coordinates) are applied to calculate distances between two given points on the earth, this is calculated using the Haversine formula! It is crucially important to navigation, and a special case in the more general “Law of Haversines” in spherical trigonometry.

The Haversine formula is a very accurate way of computing the shortest distance between two points on the surface of a sphere. This is known as the great-circle distance. The formula is shown below, it is a difficult formula to comprehend, I should add that this formula doesn’t compute any hills encountered!

Haversine formula:

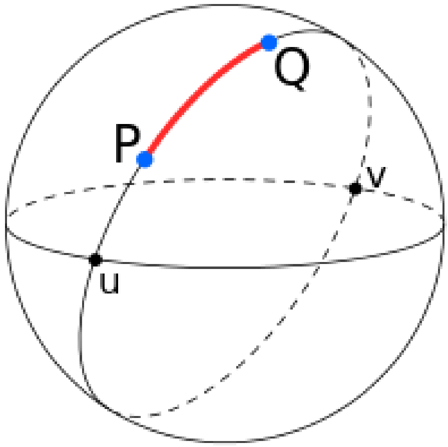
$$a = \sin^2(\Delta\varphi/2) + \cos\varphi_1 \cdot \cos\varphi_2 \cdot \sin^2(\Delta\lambda/2)$$

$$c = 2 \cdot \operatorname{atan2}(\sqrt{a}, \sqrt{(1-a)})$$

$$d = R \cdot c$$

where φ *is latitude,* λ *is longitude,* R *is earth’s radius (mean radius = 6,371km);*
note that angles need to be in radians to pass to trig functions!

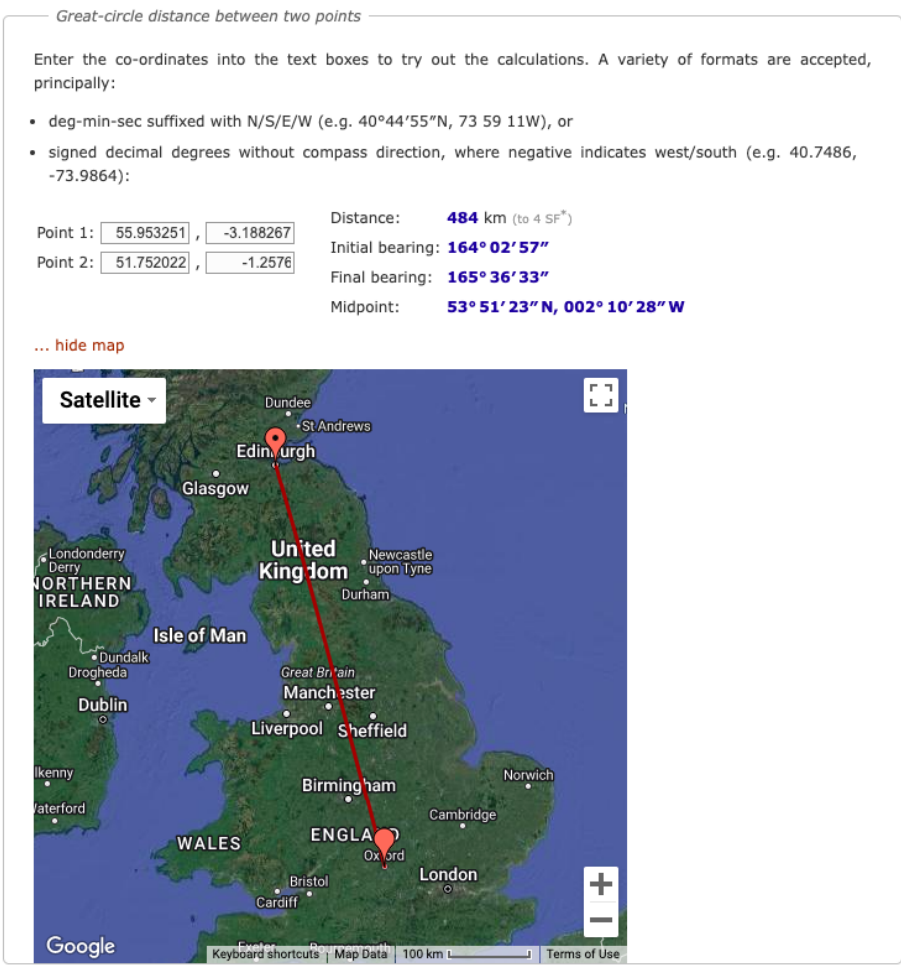
Underneath, on the next page, is **Figure 5**, a diagram illustrating great-circle distance (drawn in red) between two points on a sphere, P and Q. U and V are highlighted to show the opposite points of the sphere. The Earth is nearly spherical, so great-circle distance formulas give the distance between points on the surface of the Earth correct to within about 0.5%



When referring to the Haversine Formula, we can see that Longitude and Latitude are used. As we mentioned before, they are degrees that denote geographical coordinates on the Earth, however, to be used in the Haversine Formula, we must express them in terms of radians so they can transfer into trigonometric functions. We used the formula to achieve this:

Radians = Degrees x π/180. The Greek letter Pi (π) is used to convert degrees into radians!

Now that the technicalities are explained, let us plug in coordinates into the formula!
This is, of course, courtesy of this website which is included in the citations at the end: (http://www.movable-type.co.uk/scripts/latlong.html?from=48.86,-122.0992&to=48.8599,-122.1449). Below is **Figure 6**:



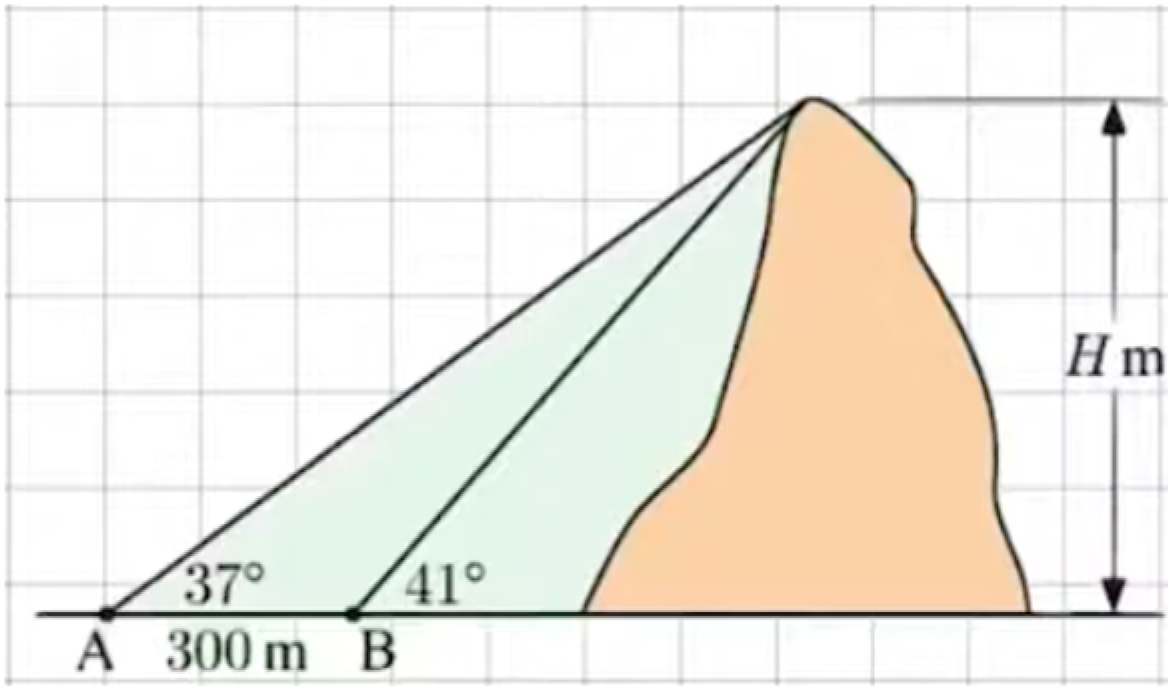
Here, we can see the plotted points of the Latitude and Longitude points of two cities, Oxford and Edinburgh, to calculate the Great-circle distance between them using the Haversine formula! Note that Point 1 is Edinburgh, and Point 2 is Oxford! The result is a distance of 484km, to 4 significant figures. If you were to research this to double check, you would find that it takes an average of 8h 34m to travel from Oxford to Edinburgh by train, over a distance of 483 km! That's quite spectacular indeed.

Physical Geography (Physiography)

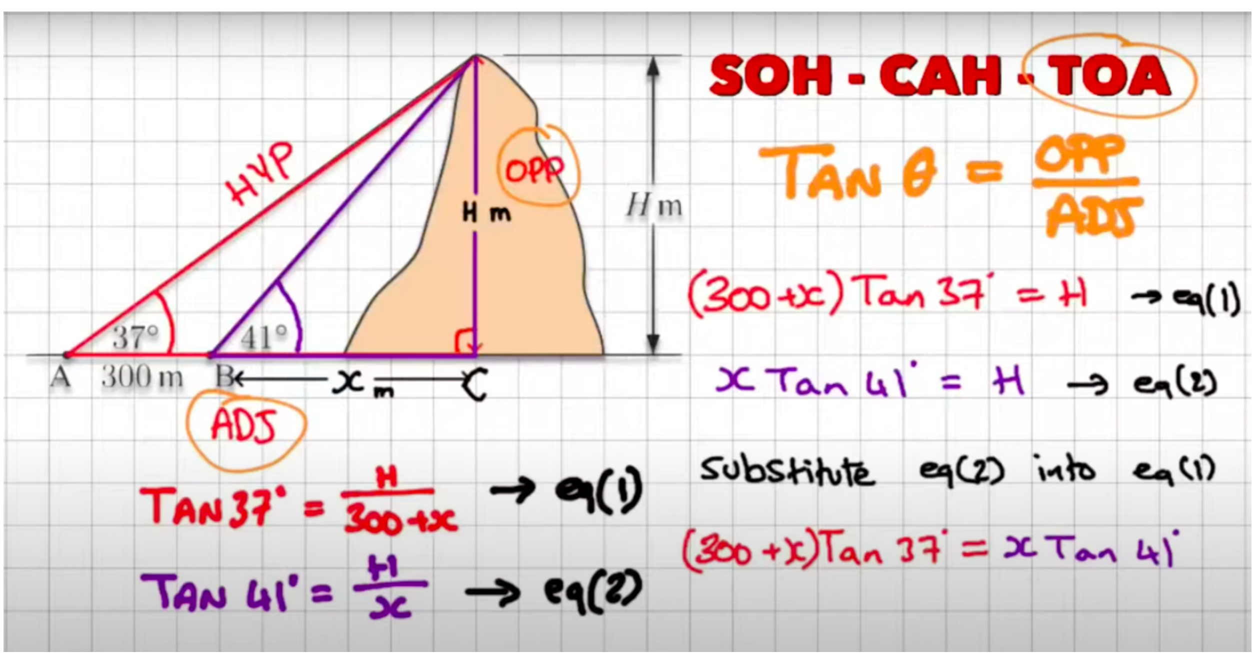
Physiography is another word for physical geography, which is used to know the processes and patterns occurring naturally in the Earth's environment. Mathematical calculations are necessary to find the gradient of hills, distances, heights and areas of places. An example of this would be

Surveyors in Geography, they use trigonometry to measure the distance between landmarks as seen here in **Figure** 7:

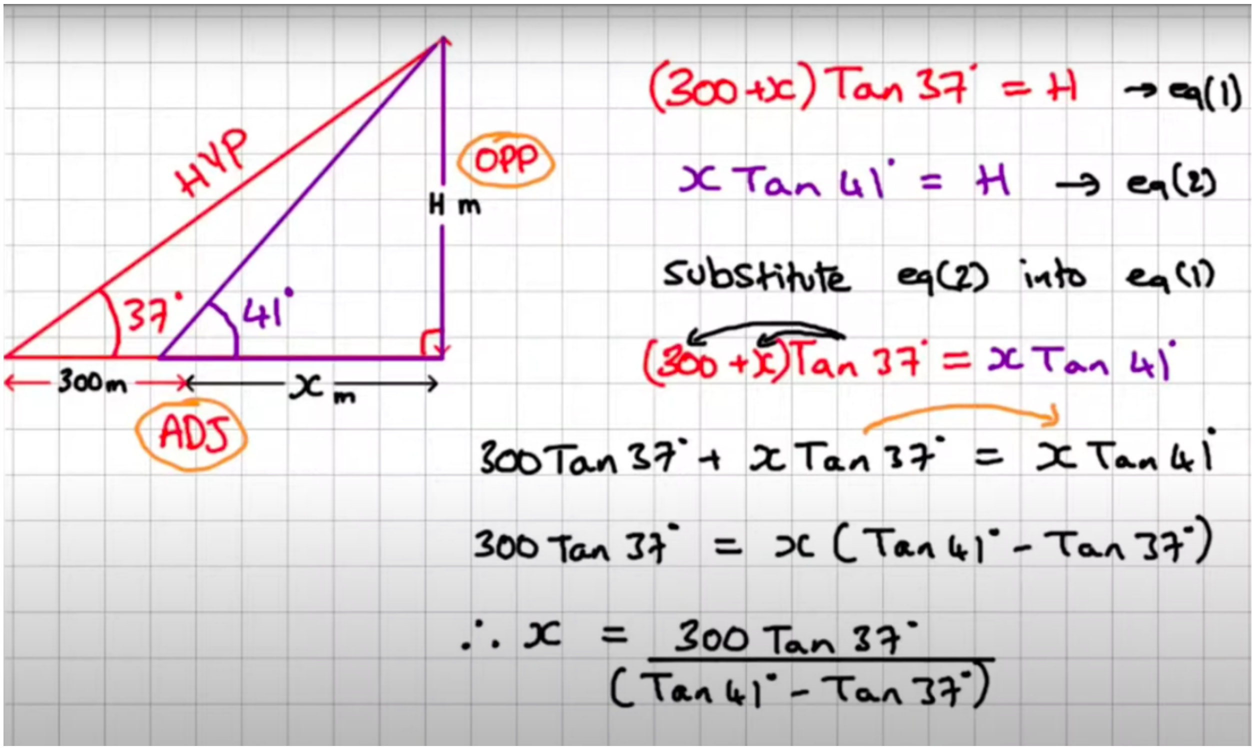
This can be applied if we want to measure the height of a mountain - Geographers can use trigonometry to receive accurate results: on the next page, is **Figure 8**, we have a mountain (coloured green and orange), for which we want to find H - or the vertical height of the mountain.



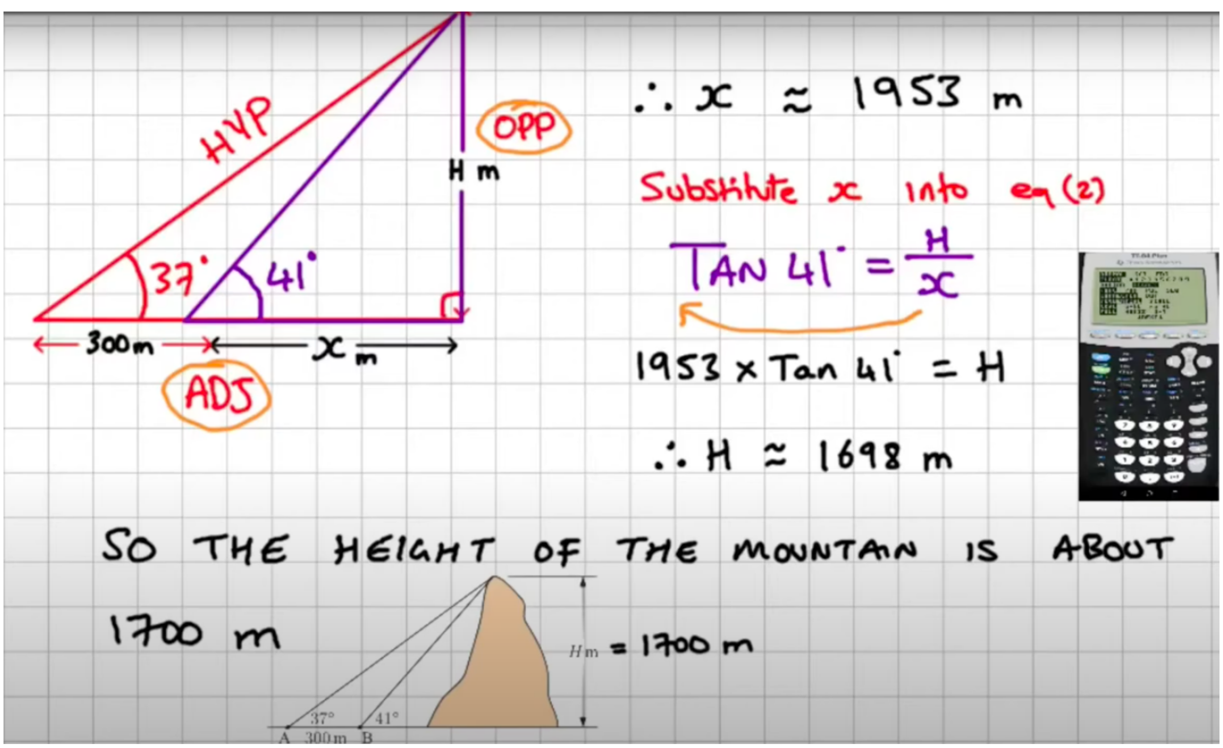
We create a right-angled triangle from the tip of the mountain, down to the base (in other words, draw a line with the vertical height of the mountain). We should then label the right-angled triangle with Hypotenuse (longest side), Opposite (the side opposite the two angles which is H), and Adjacent (the remaining side which is the base of the mountain). To solve this, we must find out the full length of the base of the mountain, meaning (300m + x) to figure out H (vertical height). We use the trigonometric function (Tan). The calculations are shown below in **Figure 9**!



The equations are solved simultaneously to leave us with the base height which is what the working out below in **Figure 10** shows:

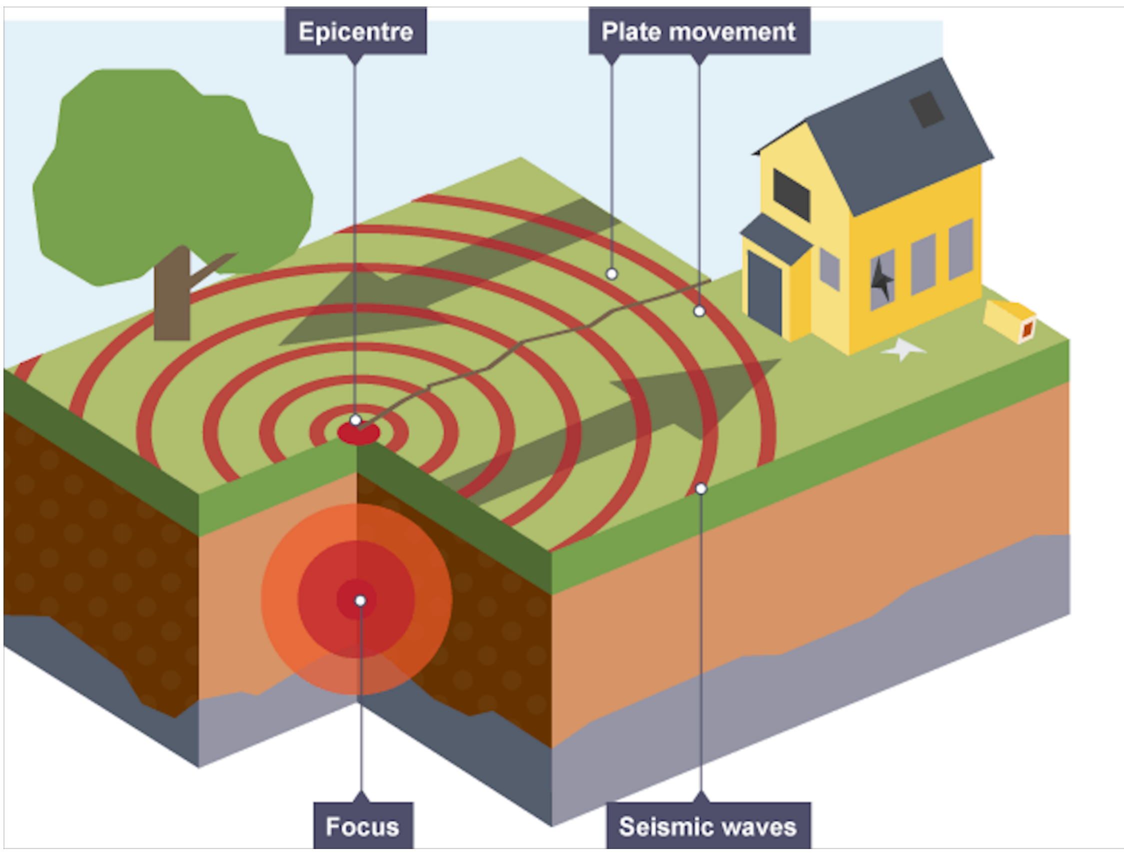


Finally, in **Figure 11**, using the value of x, we can then apply the function (Tan) one last time to get the value of H, using a calculator.



Given the complexities of trigonometry, Geographers must have a substantial understanding of these mathematical principles to find out values for geographical features.

Furthermore, another vital application of trigonometry in Geography would be that Seismologists (earthquake researchers) use trigonometry to accurately determine the epicentre of an earthquake, a more shallow epicentre in the ground means that the seismic waves an earthquake emit travel less to reach the surface, possibly leading to more damage to the impacted area, whether that be infrastructural damage (building damages/collapses) or environmental damage (habitat damages/destructions). This is demonstrated below in **Figure 12**, a diagram by BBC Bitesize:



In reading this article, I am optimistic that you enjoyed embarking on this journey with me to discover the convoluted collision of Maths in Geography! Both subjects are of great interest to me, and I hope that you as the reader are able to realize the nuances and complexities of both subjects which make them convoluted but rewarding to study. In future, we as the human race, will continue to discover more things which will advance the field of mathematical geography and the capacities it can reach, but most importantly, these two subjects help us learn more about our environment, and ourselves too!

Citations:

1. https://en.wikipedia.org/wiki/Great-circle_distance
2. http://www2.clarku.edu/faculty/djoyce/trig/apps.html#:~:text=As%20the%20earth%20is%20also,used%20it%20to%20his%20advantage.
3. Proportional symbol map. (2023, January 8). In *Wikipedia*. https://en.wikipedia.org/wiki/Proportional_symbol_map
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6. https://en.wikipedia.org/wiki/Eratosthenes
7. http://www.movable-type.co.uk/scripts/latlong.html?from=48.86,-122.0992&to=48.8599,-122.1449

Thank you for reading!