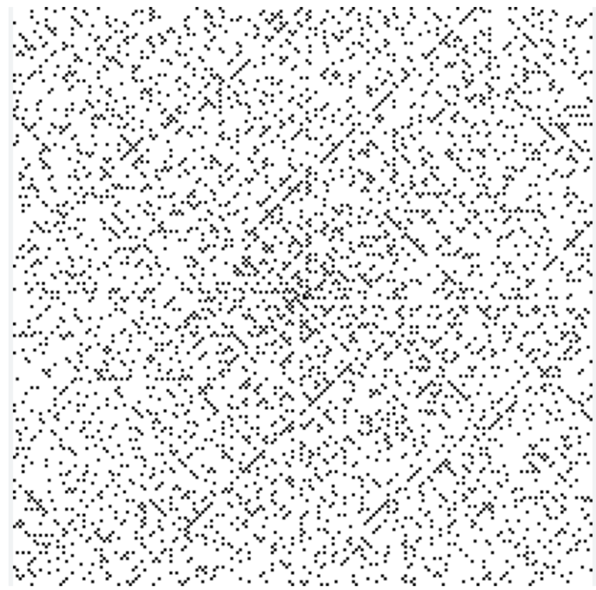
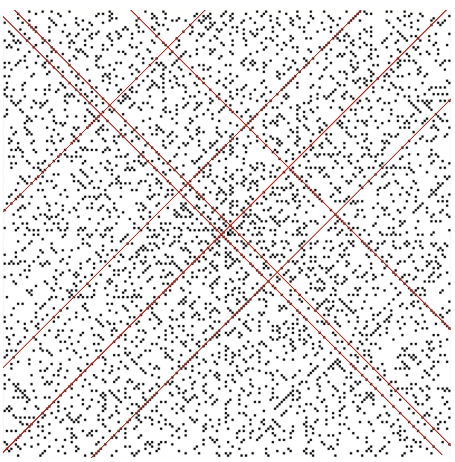
Chaos at the centre of Mathematics

At the very centre of mathematics, the pursuit of order, lies absolute chaos. The greatest problem in the mathematical world, the problem of the primes.

Prime numbers are the building blocks of mathematics and many conjectures have been constructed around them. The most notable include the Riemann hypothesis, the goldbach conjecture, the collatz conjecture and the twin prime conjecture.

People have struggled with primes for generations and so it is sometimes best to take a step back and try a different angle. Mathematicians have analysed the primes from a visual perspective using 'pictures' such as the Ulam spiral and the Sacks spiral. These both show that there are certain lines which have a high density of primes compared to others. However, another way is to not look at the primes themselves but to look at the gaps between them. These gaps are so fundamental that a recent Fields Medalist, number theorist Dr James Maynard, has investigated this showing how primes can often appear in clusters and have large gaps between the next prime. This 'negative space' is also the basis of the twin prime conjecture which asks whether there is an infinite number of primes that are separated by one (p and p + 2 are both prime).





I have always looked for patterns in whatever it is I did when I was younger. Whether it was always taking an even number of steps when walking or taking the same amount of steps on different coloured pavestones. I always hunted for patterns. And if there was no obvious pattern, which was usually the case, I would make one up. I believe that this is similar to others hunt for a pattern within the primes. Some have been successful whereas others have fabricated their patterns to fit smaller primes but they often break down very soon because the primes are seemingly random.

I myself have also had an experience in this fabrication of a pattern when I began looking in binary:

We always work in base 10. Why? Because we have 10 fingers? Convention? Many people have different ideas on the matter but the answer is not important. The fact is, we do. And so if people have been looking in base 10 for a pattern in the primes why not go to base 2 or base 3 or base 4. There is no good reason and so I did some experimenting of my own. Of course I never really had any hope that this would bear anything groundbreaking as mathematicians have seemingly tried every avenue when it comes to primes and so I was sure that someone would have thought to look in a different base. However, one reason why mathematicians encourage the exploration of ancient problems such as the Riemann hypothesis is because of what is discovered on the way, even if it is a dead end for the initial goal. I believe that true mathematics is about creating links and pathways that connect different areas of the mathematical cosmos so that others can not only see these paths but walk them themselves. These pathways should be permanent and everlasting. They are built from proof. And this is why we constantly strive to prove everything. So that there is no doubt in our minds that the mathematical foundations that we build the beautiful tower of maths from will never crumble and collapse. And so, I went on a journey into the realm of base 2:

I started by writing out all of the numbers in binary up to 17 and circling the primes:

| | | | | | |
|---|---|---|---|---|---|
| 1 | = | 1 | 1001 | = | 9 |
| 10 | = | 2 | 1010 | = | 10 |
| 11 | = | 3 | 1011 | = | 11 |
| 100 | = | 4 | 1100 | = | 12 |
| 101 | = | 5 | 1101 | = | 13 |
| 110 | = | 6 | 1110 | = | 14 |
| 111 | = | 7 | 1111 | = | 15 |
| 1000 | = | 8 | 10000 | = | 16 |
| | | | 10001 | = | 17 |

Then, I wrote a list of the primes to look for a pattern. At first I saw the very simple pattern highlighted in blue which suggests that binary numbers 11, 111, 1111, 11111 etc. are all prime. However, I could quickly see that this was not true for the number 15 as it is divisible by 3 and 5. And so the pattern quickly broke down. I then looked at the green pattern which conjectures that the following numbers in the pattern 101, 1011, 10111, 101111 etc. are all prime. This pattern agreed with the conjecture for the first 4 numbers all the way up to 95 where it broke down.

primes: 10, 11, 101, 111, 1011, 1101, 10001

Conjecture for a pattern: primes follow the pattern:

101, 1011, 10111, 101111, 1011111

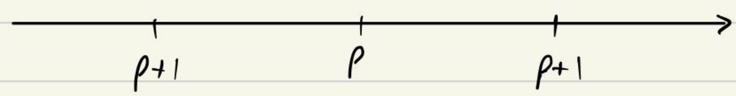
= 5, 11, 23, 47, 95

✓ ✓ ✓ ✓ ✗

This shows the deceptiveness quality of the primes as they seem to show a predictable pattern which can in fact work for many many numbers, but sooner or later, the randomness of the primes will take over and the pattern will not hold. There have been many examples of this. One notable example comes from Euler who said that all the integer values on the line: x^2 + x + 42 are prime up to the 40th number/term. This predictable pattern goes on and on up until 1601 until the pattern no longer holds. And so this is why I ask the question about whether we simply create these patterns or are they simply coincidences between 2 or 3 numbers. Surely a pattern that holds for the first 39 terms can be no coincidence and so this opens up a new question which asks what makes this pattern work compared to other patterns. The prime numbers are so erratic yet we are still able to spot <u>real patterns</u>. For example, every prime squared is one more than a multiple of 24 (excluding 2 and 3 as they are too small). And of course to extinguish any doubt we will prove it:

Is $p^2 - 1$ a multiple of 24, where $p$ is prime.

$p^2 - 1 = (p+1)(p-1)$




Not a mult. of:
2
or
3

$p+1$ and $p-1$ are both even.
Two consecutive evens $\Rightarrow$ one is a multiple of 4
$\Rightarrow$ multiplied together is a multiple of 8.
Three numbers $\therefore$ one of them is a multiple of 3.
Can't be $p$ as $p$ is prime
$\Rightarrow$ $(p+1)$ or $(p-1)$ is a multiple of 3.
$(p+1)(p-1)$ $[= p^2 - 1]$ is a multiple of 3 and 8
$\Rightarrow$ $p^2 - 1$ is a multiple of 24

Another pattern is with mersenne primes. Mersenne primes, along with fermat numbers, were ideas that were tested experimentally through rigorous calculations to generate primes. However, they did not work all of the time. Fermat numbers say that any number that is 2^(2^n) +1 will be prime; however this only holds true for values from n=1 to n=4. Mersenne primes were a better guess and said more simply that any number that is 2^n -1 will be prime. This again did not work for all values of n but it did work when n itself was prime.

Finally, I suppose that in the unlikely event that an applied mathematician stumbles across this, they may ask: So what? What use is understanding primes when they are an extremely abstract concept that has no applications in the real world. And now, this is the part where I show them how wrong they are. The infrastructure of our great cities rely, not on our understanding of primes but on our lack of it. RSA is the encryption method for almost all banking and secure information and it works through prime numbers. The reason that it is secure is because the calculations to crack the cipher would simply take too long even with computers. Quantum computers may change this but understanding primes is also likely to hold secrets that would pose a threat to our information.

The key that unlocks the exciting information of the primes is a proof. A proof to the tantalising hypothesis that has a million dollar prize attached to it. I am excited to be a part of this world as I look hopeful for the future as we all wait patiently for the proof for prime numbers. The proof that settles the chaos at the centre of mathematics. A proof of the Riemann hypothesis.