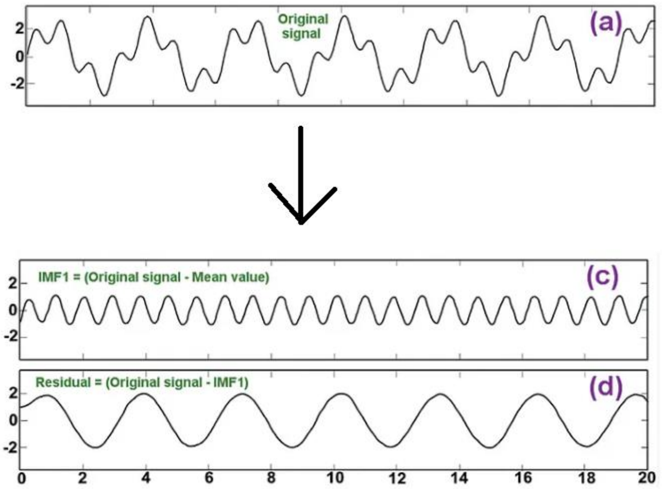
A Fourier transformation is a mathematical function that decomposes a wave into individual components, using these individual components we can identify characteristics and behaviours of individual waves, signals that are being decomposed are made up of cosine and sine waves which superpose constructively and destructively in order to form the original wave.

The Fourier transformation is names after a French mathematician called Joseph Fourier, who first introduced the concept for Fourier transformations by presenting a periodic function as a sum of sinusoidal functions . This became developed later and became an important concept that is used within the Fourier transforms today.

If you have a pure pitched sound and make a pressure vs time graph you will get a graph that has a very consistent structure with consistent oscillations at its equilibrium. Now imagine adding a different pure sound with a different structure, this will create a more complex structure that has an irregular shape.



As you can see, the original signal has been split up into two fundamental waves. The Fourier function runs as a function of time and describes the frequencies present in the original function. For instance, we can take the sound of a chord and we can decompose it into individual pitches with fundamental frequencies and with different intensities by inputting the audio into a Fourier algorithm to produce complex valued amplitudes of the frequencies found in the signal.

The Fourier transform is defined as follows:

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

In this equation we can see that the function of time f(t) is converted into a function of frequency(w), and essentially what's happening here is that we decompose a function into a series of sinusoidal waves that have different frequencies and amplitudes, and these are represented on a complex exponential function which have the form e^(-iωt).

The Fourier transform has many practical applications in science, engineering, and mathematics. It is used in fields such as signal processing, image analysis, acoustics, and quantum mechanics, among others. Some specific applications of the Fourier transform include:

- Audio and music signal processing, where the Fourier transform is used to analyse audio and music signals to identify their frequency components and extract features such as pitch, rhythm, and timbre.
- Image analysis and processing, where the Fourier transform is used to analyse and process images to identify their frequency components and extract features such as edges and textures.
- Quantum mechanics where the Fourier transform is used in quantum mechanics to represent wave functions and calculate probabilities of different states.
- The analysis of DNA sequences, the detection of gravitational waves, and the study of the brain's neural activity. These applications demonstrate the broad and diverse reach of Fourier transformations in modern science and technology.

In Audio and music signal processing, different types of filters can be designed to remove unwanted frequencies or enhance desired ones, such as removing background noise or boosting bass frequencies in a music track. Another application is in spectral analysis, which involves analysing the frequency content of a sound signal. This can be useful in tasks such as identifying the pitch of a musical note or detecting the presence of specific harmonics in a sound signal

In Quantum mechanics, Fourier transforms are used to describe the behaviour of particles, for example, we can transform a wave function form position space to momentum space, which allows us to measure how fast the particle of moving, so the Fourier transform allows us to express the wave function in terms of momentum, this is useful as it allowed us to calculate the probability finding a particle with a particular momentum, which is helpful to understanding the behaviour of quantum systems and predicting their properties.

In DNA sequences the Fourier transform is used to identify patterns and structures within the sequences .As DNA is a long complex sequence of nucleotides which are the building blocks of DNA made from amino acids.

In image processing, the Fourier transform is typically applied to two-dimensional images. The Fourier transform of an image represents the frequency content of the image, and it can be used to filter out high or low frequencies, depending on the application. For example, a high-pass filter can be used to sharpen an image by removing low-frequency components, while a low-pass filter can be used to smooth an image by removing high-frequency components.

What's amazing as well is that we can use the inverse of a Fourier transform to find what a wave would look like if it was composed of many different constitutional waves, this is the equation for an inverse Fourier transform

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$$

We can see here in the equation that we integrate the product of the frequency domain signal and the complex exponential function of e^(iωt).In practice, the inverse Fourier transform is often computed numerically using fast a Fourier transform (FFT) algorithm.

The FFT (fast Fourier transform) algorithms are essentially a widely used algorithm used for computing the discrete Fourier transform (DFT) of a sequence of complex numbers. The idea behind the FFT is that we use the ideas of symmetry and its periodic properties, which reduces the number of calculations and computations needed to calculate DFT. How the FFT works is that typically a sequence of discrete-time samples of a continuous-time signal is divided by the FFT algorithm into smaller sub sequences and then combining the Fourier coefficients(which are just the complex numbers that are the result from applying the DFT to that sub sequence)to obtain the DFT of the original sequence. The main difference between FT (Fourier Transform) and FFT , is that FT considers a continuous signal , while FFT takes a discrete signal as an input.

In conclusion , Fourier transforms are considered incredibly useful and helpful for many areas of maths and physics and the world , being able to distinguish the individual fundamental frequencies between some incredibly complex waves provides us the ability to expand our knowledge greatly within multiple areas simultaneously. It allows us to analyse and manipulate signals and data within the frequency domain , which is essential for many applications and ideas that require reinforcement.