How to win with game theory

Game Theory is a field of economics that analyses the behaviour of consumers and firms as if they were players in a simple game in which their outcomes are affected by circumstances inflicted by themselves and the choices of other players. When we look at and aspect of game theory involving companies called non-zero-sum games, there are no universally acceptable solutions, nor predictable outcomes. In these situations, one company's gain does not directly or necessarily result in the other company's loss. Therefore, all the winnings and losses do not add up to zero (hence non-zero). This allows everyone to gain: a win-win game.

In this essay, I will explore how to utilise game theory to make the best decisions in certain situations for the company "Textbooks R Us" and ways to increase profits and consumers against their biggest rivals "Textbooks superstore". This illustrates the challenges many companies face and how the interesting analysis of game theory allows us to try to make the best strategic business decisions.

Prisoner's Dilemma

One of the most popular game theorems is named the prisoner's dilemma. This theorem represents a situation where two parties are separated and unable to communicate. They must each decide whether to cooperate with each other or not. The biggest reward is when both communicate; however, the biggest loss is when one cooperates, and the other doesn't.

The easiest way to explain this dilemma is through robbers, and to see if they really are "as thick as thieves". Two people, called Sam and Cat, rob a bank. There are no witnesses that can prove the case, the only way to prove it is if one of the robbers testifies. Each robber must cooperate with their accomplice or testify that the other person did it. If they cooperate, they both have a lesser charge with only one year in prison. If one testifies and the other does not, the one who testifies is free and the other receives 5 years in prison. If they both testify against each other, they each receive 3 years in prison.

Outcomes:

Outcome	**Sam remains silent**	**Sam Testifies**
Cat remains silent	(1,1)	(5,0)
Cat testifies	(0,5)	(3,3)

Each robber would likely pick to testify against the other one. Using Cat as an example, if she testifies, she faces zero or three years in prison versus facing one or five if she remains silent. If she remains silent, she is guaranteed to go to prison. This relates to the minimax

decision rule. This means a player considers the other player's choices and selects the strategy to minimize the worst-case scenario. If Cat uses the minimax decision rule she will testify as the worst outcome is three years in prison, not five.

How does this relate to the rivals between companies?

If Textbooks R Us cuts the price of its textbooks from £25 to £20, customers will go to their company so they can buy cheaper textbooks. Then, in retaliation, "Textbooks Superstore" will also cut their price to £20. The companies then have the same amount of consumers as when they started but are making a smaller profit.

Outcomes:

Outcome	**Textbooks R Us keeps prices**	**Textbooks R Us drops prices**
Textbooks Superstore keeps prices	(£25,000, £25,000)	(£22,000,0)
Textbooks Superstore drops prices	(0, £22,000)	(£20,000, £20,000)

Therefore, the best outcome for both companies is to keep their prices at the same price. However, if a company doesn't cooperate, they both lose profits, however, the company dropping their prices would win customers over the other company.

What should Textbooks R Us do?

If we were the economists for Textbooks R Us, what would we tell them to do? This all depends on the strategies of our rival company. We will have different decisions based on if they are going to cooperate and keep their prices or drop their price.

If we know that Textbooks Superstore is going to keep their prices at £25 a textbook, we will cooperate. Being able to both make the maximum profit leads to the best outcome for both companies and no one suffers a loss of profit.

However, if we find out they drop their prices down to £20, we will also do the same. Both having a lower profit is better than them having a higher profit and us having none.

There is also an option where we can cooperate with the rival company and promise to keep the prices high if they also cooperate. Although no contracts can be formed as this is collusion or forming a cartel. This means we will just have to replicate their choices to not take a large loss in profits, linking back to the minimax decision rule.

The companies cooperating leads to Nash Equilibrium and therefore both companies should be satisfied with the outcomes. If they both cooperate, they receive a high profit but no advantage over the rival company.

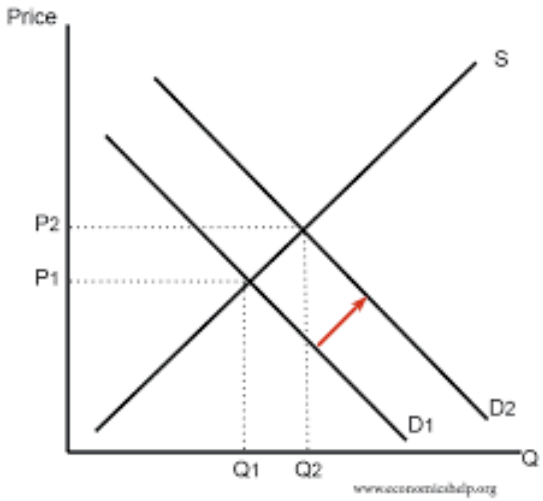
Cournot Duopoly

Cournot competition is an economic model illustrating rival competitions with identical products (in this case textbooks!) competing on the amount of output they produce, independently and at the same time.

For this example, Textbooks R Us and Textbooks Superstore are both oligopolies – which are markets with a small number of suppliers. This often leads to competition by stealing market share away from each other such as altering the number of goods produced and sold. Which links back to the ruthless, competitive textbook industry we are in.

Supply and demand

The Cournot duopoly is related to the laws of supply and demand. This is the relationship between the quantity of a product the producers which to sell, and the quantity consumers wish to buy; this is often used for determining the price at which these goods will be sold. This price is found in the equilibrium between supply and demand.



The graph above shows the correlation between the supply of a product and the demand, this allows companies to find their equilibrium prices at which to sell their products or services. If a company has a large demand and a small supply the company will increase the price/unit. On the other hand, if they have a small demand and a large supply the company will decrease the price/unit.

Back to Cournot Duopoly

To describe the Cournot duopoly, we will use algebra. Let's say we have two firms, the quantity(q(x)) of product they produce are labelled as:

Textbooks R US: q_1

Textbooks Superstore: q_2

Total quantity: $q_1 + q_2 = Q$

Since the firms are selling identical products the consumer demand function (P(x)) can be written as:

$$P(q_1, q_2)$$

The above is a function of the total quantity of the product that is on the market:

$$P(q_1, q_2) = P(q_1 + q_2) = P(Q)$$

To represent the cost function(C(x)):

$$C_1(q_1), C_2(q_2)$$

Now let's say there are n firms that are also in the textbook business. Let's take a random firm and label it k. The profit of this kth firm is I_k. This profit is the amount of money they make selling the product minus the cost of producing the product.

$$\pi_k = q_k P(Q) - C_k(q_k)$$

If the firms are assumed to be maximizing profit, the output of the firm is the derivative and setting it equal to zero (finding the maximum points of the graph).

$$\frac{d\pi_k}{dq_k} = P(Q) + q_k \frac{dP}{dQ} - \frac{dC_k}{dq_k} = 0$$

As this is a set of n equations with n unknowns, we will have to make some assumptions such that the demand for the product and the costs are linear.

To write the equations as linear models:

$$P(Q) = a - bQ$$

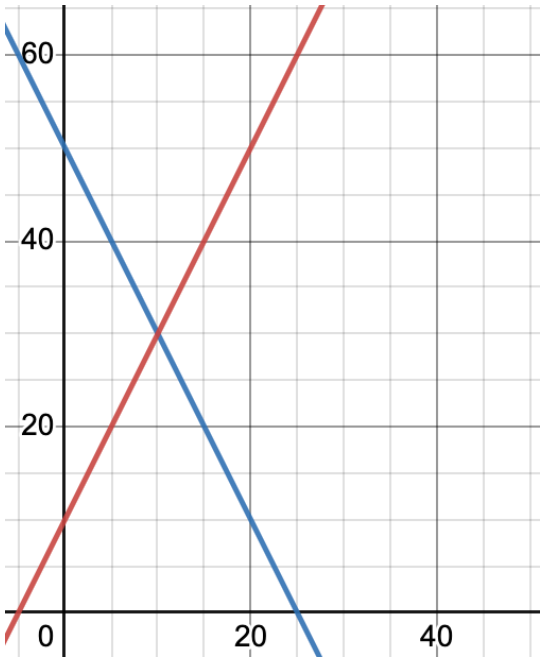
$$C_k(q_k) = C_k q_k$$

For example, let's say the maximum price for a textbook is £50 we would write the equation as:

$$P(Q) = 50 - 2q$$

Then the equation for cost as:

$$C = 10 + 2q$$



This graph shows the equation with the x-axis representing the quantity of the product and the y-axis representing the price.

This graph can show the supply side of supply and demand as it shows how the quantity of goods a company can produce affects the price of the goods.

The blue line represents the inverse demand function, and the red line represents the cost function.

To consider the uniform-pricing monopoly, Q = q and the profit function is:

$$\pi(Q) = (50 - 2Q)Q - 10 - 2Q$$

$$\pi(Q) = 48Q - 2Q^2 - 10$$

Finding the derivative of the equation and solving:

$$\frac{d\pi}{dQ} = 0$$

Q (total industry output) = 12

P (inverse demand) = 26

π(profit) = 278

CS (consumer surplus) = $\frac{12(50-26)}{2}$=144

TS (total surplus) = 278 + 144 = 422

Monopoly – Textbooks R US

Q	P	Π	CS	TS
12	26	278	144	422

Now consider the case of our two rival firms Textbooks R Us and Textbooks Superstore.

Let q_1 be the output of Textbooks R Us and q_2 be the output of Textbooks Superstore. Then $Q = q_1 + q_2$ and the profit functions are:

$$\pi_1(q_1, q_c) = q_1[50 - 2(q_1 + q_2) - 10 - 2q_1$$

$$\pi_2(q_1, q_2) = q_2[50 - 2(q_1 + q_2)] - 10 - 2q_2$$

Linking back to Nash equilibrium is a pair of output levels (q_1*, q_2*) such that:

$$\pi_1(q_1*, q_2*) \geq \pi_1(q_1, q_2*) \text{ for all } q_1 \geq 0$$

and

$$\pi_2(q_1*, q_2*) \geq \pi_1(q_1*, q_2) \text{ for all } q_2 \geq 0$$

This means that we fix q_2 at the value q_2* and consider the profit as a function of q_1 and vice versa. This allows us to compare them by considering π_1 and π_2 (individual profits) as functions of their firm while the other firm stays at a fixed value. However, a necessary condition is that the derivative is equal to zero (at a maximum value), so then we find the Nash equilibrium by solving the following system of two equations (the derivatives) with respect to the unknowns q_1 and q_2:

$$\begin{cases} \frac{d\pi_1}{dq_1}(q_1*, q_2*) = 50 - 4q_1 - 2q_2 - 2 = 0 \\ \frac{d\pi_2}{dq_2}(q_1*, q_2*) = 50 - 2q_1 - 4q_2 - 2 = 0 \end{cases}$$

Solving this gives the solutions:

$$q_1 = q_2 = 8$$

Q = 16

P = 18

$$\pi_1 = \pi_2 = 118$$

CS = $\frac{16(50-18)}{2}$=256

TS = 118 + 256 = 492

Comparing the monopoly to the duopoly:

Q	P	Π	CS	TS
12	26	278	144	422

q_1	q_2	Q	P	π_1	π_2	Total π	CS	TS
8	8	16	18	118	118	236	256	492

The competition in the rivalry against Textbooks superstore has led to an increase in consumer surplus and total surplus. A gain of 112 in consumer surplus greatly exceeds the loss in profit of 42.

This links back to the prisoners' dilemma. In this dilemma, we decided against dropping our prices due to a great loss in profits for the firm. The Cournot duopoly has shown the great increase we would receive in consumer surplus and how it is much larger than the loss in profits when competing against the rival firm. This proves our game theory decision of choosing the option with the minimum loss and, therefore, cooperating with Textbooks Superstore and keeping our prices the same.

Consumer surplus is when a consumer pays less than they are prepared to pay for the item. This means that due to the competition, the drops in price have led to people underpaying for the item then leading to a loss in profits. This shows the loss in profits that our company would have received if we decided to drop our prices in the dilemma we faced earlier.

Conclusion

In conclusion, in this essay we have found that when faced with a prisoner's dilemma of whether to drop prices and gain customers or to keep prices and therefore keep profits, its arguably better, in my opinion, to keep the high prices while the other firm keeps theirs high. This means that we keep the high profits for both firms while still retaining the same number of customers. This was then proven by the Cournot duopoly. This shows the competition of "who has the lower price" leads to a great loss in profit and an increase in consumer surplus. Thus, reaching the conclusion of keeping our prices high until the rival firm reduces theirs is the best way to win at game theory.