The Order of Operations' Peculiar Abbreviations

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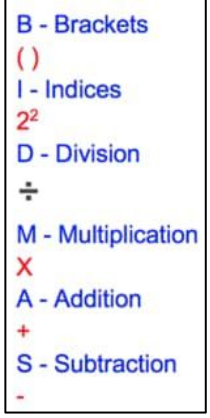
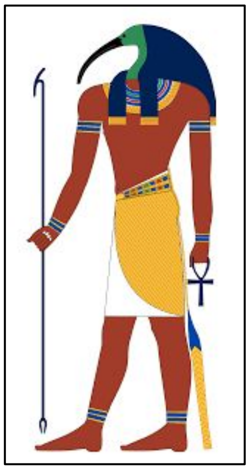
They taught you the BIDMAS at school - brackets, indices, divisions, multiplications, additions and then subtractions. Anybody worth reading this must know what this is and how it works; of course, this is the dictatorial Order of Operations. Challenging the BIDMAS would cost you a hefty fine - a mountainous, terrifying cross in your maths workbook, and, even worse, the shame of an amateur ‘silly’ mistake, the very thought of which gives one nightmares. Clearly, this cannot be right. Sure, mathematics is a land of order, but also a land of freedom and prosperity. You may be asking yourself, why is this the case? Why couldn’t the order be something else, like “BMAIDS”? Who is the omnipotent deity of mathematics that decides on these strangely specific rules and this particular order that we, as students and teachers, mathematicians alike are forced to accept? I will answer these questions in this very essay, so buckle up.



How the Order of Operations is taught at schools. Source: gcse.co.uk

When I say “this particular order”, what order am I even referencing? Based on mnemonics and acronyms - my favourite sources of statistics to do with national education - we can learn that the Order of Operations is in fact shortened to different things across the globe. In Canada and New Zealand, for example, it is taught as BEDMAS, which is understandable - indices are referred to as exponents, meaning the power to which a figure is raised. However, in the US, the PEMDAS (or GEMDAS) is used. This stands for Personally, I find some of the mnemonics they usd quite amusing, for example, “*People eat more donuts after school*” or “*Please excuse my dear Aunt Sally*”. Apart from doughnut consumption rate patterns and someone’s irrelevant relative, you’ll also notice is that the letters D and M (short for divisions and multiplications, respectively) have completely switched places. Therefore, one could assume that multiplications are to be calculated before divisions, which is different to how it should be done! Why would such puny forces as the geographical location of a school or the will of its government have the power to bend the rules of maths itself? Is the United States’ “math” allowed to be different from the mathematics of Australia, India, Nigeria, Pakistan and the United Kingdom? As you (hopefully) know, this is obviously not the case. Multiplications and Divisions are equal and interchangeable in the Order of Operations, meaning you can do them in any order, as are additions and subtractions, which come after multiplications and divisions. This makes our job of uncovering the reason behind <u>the</u> Order of Operations easier, if there only is one.


Thoth, the Ancient Egyptian god of magic, knowledge, the sciences and, wouldn't you know it, mathematics! Source: egyptianmuseum.org

To uncover this mystery, we must first investigate the Order’s history.

- Between the 15th and 16th centuries, the convention that multiplication precedes addition arose rather naturally and, what’s uncommon in the field of mathematics, nobody seemed to disagree. Scientists and mathematicians of the time adapted to this, slowly forming a seemingly-logical hierarchy in which multiplication is considered more potent than addition.
- In 1892, M. A. Bailey conveys his discouragement of the use of both divisions and multiplications in a single expression in his book, *Mental Arithmetic*. This is to avoid confusion on which operation to calculate first; if a division could not be avoided, it was to be expressed as a fraction. His view could be interpreted as considerate for future mathematicians, but also as lazy. If there’s one thing mathematicians should strive against, it is unnecessary laziness.

Laziness can lead to mistakes, mistakes lead to criticism, criticism leads to unavoidable shame. (This is a systematic algorithm of failure that can be applied to many things in both mathematics and life, which, along other things, shows how maths really is useful in "adult life".)

- In 1898, in the *Text-Book of Algebra*, G. E. Fisher and I. J. Schwatt write that *a÷b×c* is to be interpreted as *(a÷b)×c*. This puts division at a higher position than multiplication in the order, something that is not present in the modern Order of Operations.
- However, just nine years later, in *High School Algebra, Elementary Course* by Slaught and Lennes, the opposite is "*recommended*". It suggests that multiplications are to be calculated first, then divisions as they occur from the left to the right.
- In the 1910 book *First Course of Algebra* by Hawkes, Luby, and Touton, it says that divisions and multiplications should be calculated in the order they occur in a given expression, meaning that they are equal in the Order of Operations.
- In 1912, in the book *First Year Algebra*, Webster Wells and Walter W. Hart state that "*first, all multiplications and divisions in their order from left to right; then all additions and subtractions from left to right.*" This makes additions and subtractions equal, but lower than divisions and multiplications, which are also equal to each other.
- One year later, they state that "*operations under radical signs or within symbols of grouping shall be performed before all others; otherwise, all multiplications and divisions shall be performed first, proceeding from left to right, and afterwards all additions and subtractions, proceeding again from left to right.*" This finally introduces indices (powers and roots of powers, described as "*radical signs*") and symbols of grouping, which include parentheses ( ), brackets [ ] and braces { }. However, here the indices and grouping methods are seen as equal which is not the case in the modern Order of Operations.
- In 1917, The Report of the Committee on the Teaching of Arithmetic in Public Schools recommends the use of brackets (not any other grouping methods) to avoid ambiguity in case an equation contains both both a division and multiplication, or both an addition and subtraction (*Mathematical Gazette 8*, page 238).
- It is suspected that around 1917, the abbreviation "*PEMDAS*" was invented in the United States to teach students about the Order of Operations and its importance. This could be classed as the starting point for BIDMAS!
- However, in the 1928 book *A History of Mathematical Notations - Volume 1*, mathematician Florian Cajori outlines that "*If an arithmetical or algebraical term contains ÷ and ×, there is at present no agreement as to which sign shall be used first.*" This surprising statement highlights how the Order of Operations (taught as PEMDAS at the time) was often ignored by mathematicians yet still taught to students.
- In the 1960s, it was conventionally agreed that indices come before multiplications and divisions, which come before additions and subtractions. It was also quite clear that one should always use parentheses to make the expression easier to comprehend.

As you can see, the Order of Operations initially originated out of pure convention and agreement between mathematicians. The harmony that the Order brings relies on this ongoing agreement (even though it has been engraved into the minds of mathematicians since the late 1920s) and that nobody dares challenge it. In case you needed a reminder, yes, that is vaguely what I'm doing - however, let's highlight some of the features I believe benefit the Order, in terms of how easy it is to teach them to students and their clarity. I believe that, in the first place, the Order of Operations should be shaped for the mathematicians of tomorrow, therefore I have taken into consideration that the Order is initially taught in late Key Stage 2, when the students are roughly aged between 8 and 10.

- First of all, let's talk about GROUPING METHODS. They are an essential part of the Order, perhaps its most useful feature, as they clean an expression up by eradicating any confusion that could potentially still lurk around. Furthermore, I appreciate that multiple grouping methods exist, not just the parentheses - often miscalled "brackets" - represented by the symbols ( ), but also brackets [ ] and braces { }. Although it is conventional to calculate expressions in parentheses first, then brackets, followed by braces, this can be easily maneuvered around by placing parentheses in the centre, surrounded by brackets, which are then surrounded by braces. Here is an example: **2**{**3**+[**6** (**2**+**1**)+**4**] } = **50**. This wouldn't be too difficult to teach to students once they reach Year 6 or 7. To summarise, grouping methods are excellent in clarifying an expression.
- Secondly, I am of the opinion that it is a good move that, out of the four operators, multiplication and division come first. As you know, the most common symbol for the multiplication operator is "×", a symbol which very closely resembles the letter "x", which is famously present in the word "extreme". This adjective's definition (according to Oxford Languages) is "reaching a high or the highest degree". Now, this is where the education method comes into play: students could be taught that multiplication (× or x) is calculated before addition and subtraction because it is an extremum - it has a higher degree of importance in the Order.
- Finally, I agree that subtraction and addition are last.

However, as most things in life, the current "*correct*" Order of Operations has its shortcomings. Namely, I believe that exponents (often called powers) and roots should NOT be calculated before the basic arithmetic operators. A young student is not expected to comprehend how exponents, let alone roots, work. In a primary school classroom, students are not even allowed to have a calculator in their possession, which is often the only reasonable way to calculate these operations. Therefore, teachers shouldn't mention them if they are not understood by anyone. Speaking from my own experience, this causes a lot of confusion, which I believe is the most common reason for prejudice against mathematics among students. After all, as educators, it is your secondary role to ensure that your students enjoy learning your subject. (The only more important role is actually teaching the subject in the first place - I am ignoring a child's health or wellbeing, safeguarding, etc.) That put aside, I think that exponents and roots (as two separate operations) should be put in the last position in the Order of Operations, since they are only taught in KS3 and above.

THE NEW ORDER

Here is my proposal for a new Order of Operations, a new BIDMAS. This one would seem less complex to students.

1. Of course, the grouping methods would clarify if an expression was meant to be calculated first. This is to avoid confusion and to have clarity.
2. Secondly, multiplications and divisions are to be calculated, from left to right.
3. After that, additions and subtractions are to be calculated, from left to right.
4. Finally, exponents and roots should be calculated last, also from left to right.

As to how the New Order should be taught and remembered by young students, I propose the following mnemonic:

Graham - grouped expressions

Ma**D**e - multiplications and divisions

Sals**A** - subtractions and additions

Eage**R**ly - exponents and roots

As you can see in the latter 3 words, there are two operations in each one. This is to avoid confusion as to which one of the two comes first - it clearly shows that they are equal. Unfortunately, the New Order no longer has a fun abbreviation, like BIDMAS. I have made the decision to change the B in BIDMAS to G, standing for grouped expressions, in order to include all the other grouping symbols, not just parentheses ( ) - the ACTUAL brackets [ ] and braces { }.

Because I myself am ultimately correct in every thing that I say, I expect the New Order of Operations to be henceforth used in all English-speaking classrooms, and those who don't speak English are to invent a suitable mnemonic device for the same Order. Failure to comply will not result in anything.