Fibonacci

Did you know that pineapples have spirals going round them? Well they do, and the number of spirals is rather interesting...

We must first familiarise ourselves with the Fibonacci sequence which goes:

0,1,1,2,3,5,8,13,21,34,55,89…

Every term in this sequence is the sum of the previous two terms. For example, term 7 is 5+8, which is 13. (Note that in this series, the 0^{th} term is 0 and the 1^{st} term is 1.)

The Fibonacci sequence is rather interesting. If you divide a term into the term after it, then the answer averages out to a rather interesting number.

1÷1=1

2÷1=2

3÷2=1.5

5÷3=1.66667...

8÷5=1.6

13÷8=1.625

21÷13=1.61538...

34÷21=1.61905...

If we use much bigger numbers…

57314784401381708410÷35422484817926191507 5=
1.6180339887498948482045868343656381177203091798057628621354486227 05…

As we can see, this averages out to a number a little over 1.6, and if we did this infinitely, we would reach number called phi. Phi is represented by the Greek letter Φ, and it has a value around 1.618033. If we look at the last division we can see how accurate it is:

1.6180339887498948482045868343656381177203091798057628621354486227 05…

1.61803398874989484820740990001204904326284254042472288566070913139 4…

The exact value is $\frac{1+\sqrt{5}}{2}$. We will talk about phi later.

Another interesting property of the Fibonacci sequence is that it can convert miles and kilometres fairly accurately. For example:

55km = 34miles

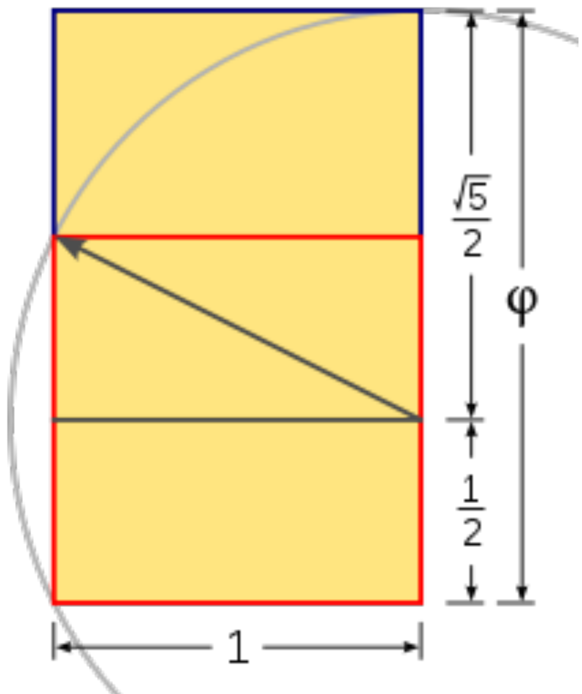
This is surprisingly accurate as the exact answer for the calculation above is 34.1754 miles. This is because phi is around 1.618 and the ratio of miles and kilometres is around 1.609, which are very close to each other. Sadly, it is not perfect.

4181km = 2584miles

The correct answer to this is 2597 miles, but this is accurate enough for most purposes.

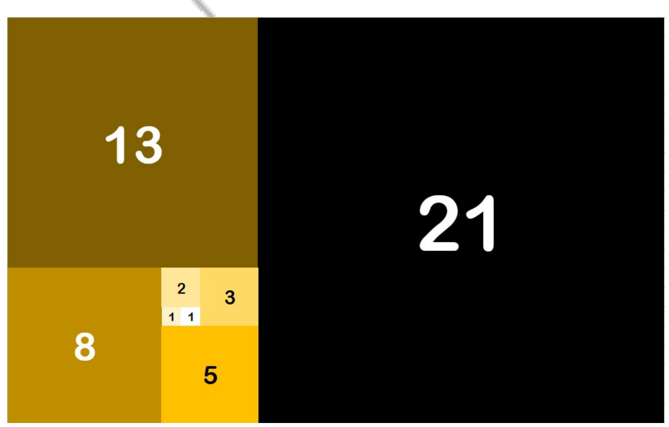
Numbers from the Fibonacci sequence come up as answers to a lot of problems. For example, if you only have 1p and 2p coins and you want to put them in your piggy bank, then the number of ways you can put in 1p is 1, the number of ways you can put in 2p is 2 (1p + 1p or just 2p), the number of ways you can put in 3p is 3 (1p + 1p + 1p, 1p + 2p or 2p + 1p), the number of ways you can put in 4p is 5, and so on.

Phi is also known as the golden ratio because it is meant to be the ratio which looks the best. Lots of attractive things are based on the golden ratio. Many buildings are based on the golden ratio and people who look attractive often have dimensions close to the golden ratio.

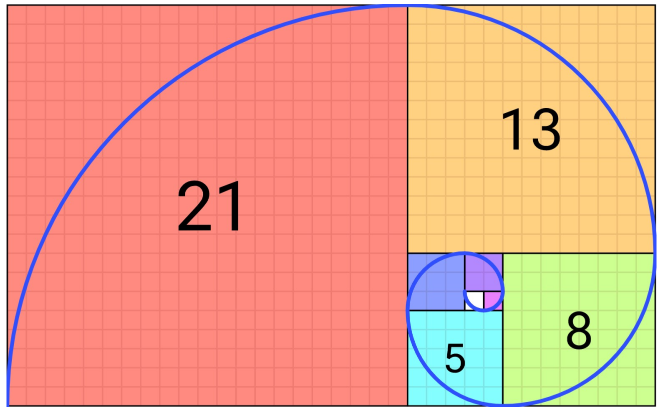


There is an interesting way of making the golden rectangle using a straight edge and a pair of compasses. You start off with a square and you the point of the compasses halfway across an edge. You then open up the compasses to the opposite corner. Then you make an arc and extend the edge up. To finish it, you complete the rectangle.

You can also get Fibonacci spirals. You start with two 1 by 1 squares next to each other. Then you put a square of side length of the rectangle you just made. So now we would put a 2x2 square, then add on a 3x3 square, and a 5x5 square and so on. This is another Fibonacci sequence.



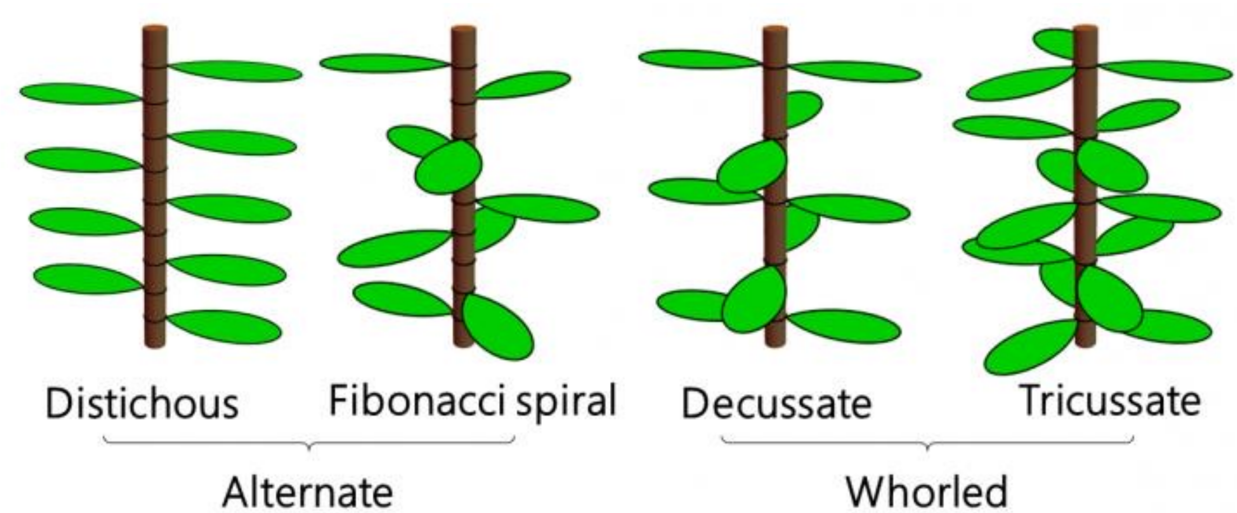
Then you put quarter circles in these squares so that they join up to make a spiral. Note that there is a difference between Fibonacci spirals and golden spirals. Fibonacci spirals are approximations of golden spirals, using numbers from the Fibonacci sequence. Golden spirals are a logarithmic spiral with a growth factor of φ.



What is very interesting is that plants often grow in patterns based on the Fibonacci sequence. For example, they may have a number of petals that is from the Fibonacci sequence. I had a look at some plants to see.

Pineapples have spirals on them. Most pineapples have 8 spirals going one way, 13 going the other way and 21 going almost straight down. This is because 8, 13, 21 are all in the Fibonacci sequence.

Pine cones also tend to have spirals on them. They are not as obvious on pine cones but they do exist. Pineapples even have spirals in their leaves, although these are harder to see. Most pine cones, like pineapples, have 8, 13, and 21 spirals, although some smaller ones have 5, 8 and 13. Plants such as daisies and sunflowers can have much bigger numbers, sometimes up to 144.



Even leaves tend to grow in a fixed pattern. I am not much of an expert at biology but I manages to find out that there are 4 main types of leaf arrangements. The Fibonacci spiral pattern is of most interest to us, and it might also be the most common. In the proper spiral, leaves grow at around 137.5° from the last. This comes from $\frac{360}{\phi}$ which is 222.5°, so $360 - \frac{360}{\phi}$ is 137.5°. This is also what causes these spirals mentioned above. Every circle around the stem the new leaves grow in between the old leaves, which is not only very helpful for the plant because every single leaf can reach sunlight and none are blocked from the sun by other leaves, but it also forms spirals in two directions.

When I went into my garden I immediately saw a plant that followed this pattern. It was a nice green bush which had branches with leaves growing off of them in this way. It wasn't completely perfect because plants will have some variation but it was clear this was meant to be a Fibonacci spiral.

I had a pineapple recently and it had 8, 13 and 21 spirals like above, but after eating said pineapple, I had a look at the leaves. This was also very clearly in spirals. I counted them and there were 3 spirals one way, 5 the other way and although a bit harder to see, there were 8 going almost straight down. Like all plants this one wasn't perfect either but all angles were around 120° to 150°.

Plants probably grow like this because in the part of the stem that grows new leaves, there is growth hormone. New leaves grow where there is most growth hormone. When a leaf grows it uses up growth hormone, but more growth hormone is constantly being made. The place where there is the most hormone is dependent on where the last leaves grew and when they grew. The place that has the most hormone is the place that is 137.5° from the last leaf. This is because 180° from the last leaf wouldn't have the most growth hormone as was is another leaf that grew before the last leaf.

Sometimes plants might not follow this pattern exactly. Sometimes this might be because it didn't grow properly for some reason, but other times there is a reason. There is another sequence of numbers called the Lucas numbers. Rather than starting with 1+2=3 like in the Fibonacci numbers, you start with 1+3=4 and continue adding to get the Lucas numbers. The Lucas numbers continue as follows:

1,3,4,7,11,18,29,47,76,123…

Sometimes you can get plants which feature numbers which are double the Lucas numbers. This would be because two leaves (or other plant parts) grow at once, following one of the two above patterns.

The way that plants use Fibonacci and Lucas patterns like is is fascinating and scientists are always trying to work out exactly how plants can do this. I hope that you enjoyed this article and will be fascinated by the way plants grow as well.