Teddy Rocks Maths Essay Competition - Graph Theory and their use in Economics & Finance

Graph Theory is a fascinating and versatile field of mathematics that has been applied in numerous disciplines, including computer science, operations research, physics, biology, and social sciences. A graph is a mathematical model of a network, consisting of a set of vertices (or nodes) and a set of edges or links that connect them. It provides a powerful framework for analyzing and solving problems related to network structures and properties and has been used to represent a wide range of real-world problems, from social networks to transportation systems. In this essay, I will explore the foundational knowledge of graph theory, and explore its applications in network economics, as well as how it was used to analyse the 2008 financial crisis.

A graph may be defined as a collection of points, called vertices, and lines connecting these vertices, called edges, and they may be either directed or undirected. In an undirected graph, the edges do not have a direction, whereas, in a directed graph, the edges have a specific direction. For example, in a social network, we might use an undirected graph to represent the friendships between individuals, whereas, in a transportation system, we might use a directed graph to represent the flow of traffic between different locations. One important concept in graph theory is the degree centrality of a vertex, which is the number of edges incident to that vertex. In an undirected graph, the degree of a vertex is simply the number of edges connected to it. In a directed graph, we distinguish between the in-degree and out-degree of a vertex. The in-degree of a vertex is the number of edges that end at that vertex, whereas the out-degree is the number of edges that start at that vertex. We must also consider the paramount concept of a path, which is a sequence of vertices connected by edges. A path is called simple if it does not repeat any vertices or edges. Similarly, a cycle is a path that starts and ends at the same vertex. A graph is referred to as connected if there exists a path between every pair of vertices in the graph, and as such a graph that is not connected is said to be disconnected.

One of the most important applications of graph theory in network economics is in analyzing the structure of markets. Markets can be modelled as graphs, with firms or individuals as vertices and relationships between them as edges. Graph theory provides a way to analyze the structure of these markets, for example by identifying key players or clusters of firms with similar strategies. Graph theory is also used in network economics to optimize supply chains and transportation networks. By modelling supply chains and transportation networks as graphs, graph theory provides a way to optimize these networks, for example by finding the shortest path between two locations or by maximizing the flow of goods or information. Another important application of graph theory in economics is in the study of social networks. Social networks can be represented as graphs, with individuals as vertices and relationships between individuals as edges. Graph theory provides a way to model and analyze these social networks, and has been used to study topics such as the spread of information and the formation of social denominations. Within the field of graph theory, there are many complex methods for solving problems and analyzing graphs that are particularly useful in network economics. Some of these methods include spectral graph theory, algebraic graph theory, and random graph theory. Spectral graph theory uses the eigenvalues and eigenvectors of the adjacency matrix of a graph to study its properties, while algebraic graph theory uses algebraic techniques to study graph properties such as connectivity and graph colouring. Whereas, Random graph theory studies the properties of graphs that are generated randomly, which can be used to study the properties of networks that arise naturally in network economics.

One notable example of the application of graph theory is in the analysis of financial networks. Financial systems can be modelled as networks, with nodes representing financial institutions and edges representing transactions or relationships between them. By analyzing the structure of these networks, researchers can identify key players and vulnerabilities that may affect the stability of the financial system as a whole. The global financial crisis of 2008 was one of the most severe economic downturns in modern history. It was triggered by the collapse of the US housing market, which had been fueled by a surge in subprime lending and the securitization of risky loans. The crisis quickly spread to the financial system, as banks and other financial institutions that had invested heavily in mortgage-backed securities suffered massive losses and faced liquidity problems.

Graph theory has been used to analyze the structure of financial networks and understand the risks and vulnerabilities that can arise in these systems. In the aftermath of the 2008 financial crisis, researchers used graph theory to analyze the interbank lending network and identify key players and vulnerabilities.

One of the key insights from this analysis was that the interbank lending network was highly centralized, with a small number of large banks playing a critical role in the network. The diagram below shows a simplified representation of the interbank lending network, with nodes representing banks and edges representing lending relationships.

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       (25)/  |  \ /    |   \(30)
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    | /   \ | /   X/    X |/   \ |/   \
Bank E(20) Bank H(18)Bank I(12) Bank F(14)
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The total amount of money lent in this simplified interbank lending network is the sum of all the edge weights. In this network, the size of each node represents the size of the bank, and the thickness of each edge represents the size of the lending relationship. This structure makes the network vulnerable to the failure of a few large banks,i.e. the ones with the most centrality e.g. bank D, as their failure can trigger a domino effect of defaults and cascading failures throughout the network. In the context of the 2008 financial crisis, this meant that the failure of large banks such as Lehman Brothers and Bear Stearns had far-reaching consequences for the entire financial system. To quantify the risks and vulnerabilities in the interbank lending network, researchers used measures such as centrality, betweenness, and clustering coefficient. Centrality measures the degree to which a node is central to the network, while betweenness measures the extent to which a node lies on the shortest paths between other nodes. The clustering coefficient measures the degree to which nodes in a network tend to cluster together. By analyzing these measures for the interbank lending network, researchers found that the network was highly centralized and had low levels of diversity and redundancy. This made the network vulnerable to shocks and contagion, as the failure of a few large banks could have cascading effects throughout the network.

To address these vulnerabilities, policymakers have proposed various reforms, such as increasing capital requirements for banks, improving risk management practices, and promoting diversity and competition in the financial system. Graph theory has been an essential tool in identifying these risks and developing policies to address them, highlighting the importance of mathematical tools in understanding and managing complex systems.

Another important application of graph theory in finance is the analysis of stock market networks. In recent years, researchers have used graph theory to study the structure and dynamics of stock market networks, to understand how information and risk are transmitted through these networks. In a stock market network, nodes represent stocks or financial assets, and edges represent relationships between these assets, such as correlations or co-movements in prices. By analyzing the structure of these networks, researchers can identify clusters of stocks that tend to move together, as well as individual stocks that play a critical role in the network. One example of this is the analysis of the network of hedge fund holdings. Researchers have used graph theory to analyze the portfolio holdings of hedge funds and identify the key players and relationships in the network. By doing so, they have been able to identify the sources of systemic risk in the hedge fund industry and develop policies to mitigate these risks.

Graph Theory has also been used in finance for analysing Credit risk. Credit risk refers to the risk of default or non-payment by borrowers, and it is a critical concern for banks and other financial institutions that lend money. Graph theory can be used to model the network of credit relationships between borrowers and lenders and identify the key players and vulnerabilities in the network. For example, researchers have used graph theory to analyze the network of credit relationships between European banks and identify the banks that are most vulnerable to default or insolvency. By analyzing the structure of the network, they were able to identify clusters of banks that were highly interconnected and vulnerable to contagion, as well as individual banks that were critical to the functioning of the network.

Overall, graph theory has proven to be a valuable tool for analyzing and understanding complex financial networks. Its applications in network economics and social networks have allowed for the development of innovative solutions to complex problems in finance and other fields. As the world becomes increasingly interconnected, the importance of graph theory in the modelling and analyzing complex systems will only continue to grow. By representing these systems as graphs, we can gain insights into their structure and dynamics, and identify potential risks and vulnerabilities. The case of the 2008 financial crisis highlights the importance of graph theory in understanding the interbank lending network, and the role that it played in the crisis. The highly interconnected nature of the network, along with the lack of transparency and regulatory oversight, contributed to the rapid spread of contagion throughout the financial system. Graph theory provides a framework for modelling and analyzing such networks, allowing us to identify key players and potential sources of instability.
Graph Theory has proven to be an indispensable tool in various fields, including computer science, social sciences, economics, and engineering. Its ability to model and analyze complex systems has enabled us to understand, predict, and optimize a wide range of phenomena, and going forward, with access to advanced computational power and data analytics allowing us to create more sophisticated models and analyses, graph theory can be used to transform our understanding of the world around us.