The philosophical problems of foundations of logic: By Jiya Dave:

Humans are generally inclined to label and categorise skills . Of these skills, mathematics prevails as a key one and is more often than not hailed as a science. However, is this always the case? At first glance it always appears to us that mathematics has to be the study of abstract entities, but in reality, when one changes their perspective they quickly realise mathematics can be used to answer such complex yet simple questions about maths, more specifically the philosophy and logic behind the genius we call maths, which are often pondered upon by many philosophers.

Now in order to understand what these philosophers ponder upon, we must first understand what logic (more specifically mathematical logic) is, and what better way to do that than look to history? Logic derives from the Greek word 'Logos' which has various different translations but the closest one being reasoning. Despite this, logic has no universally accepted definition, and this makes it all the more complicated. Whilst we may not have a universal definition, we most certainly do have a traditional definition and that is that logic is the study of the laws of thought or correct reasoning, and is usually understood in terms of inferences or arguments. In an attempt to understand and examine the black hole we call logic, a mathematician, named Douglas Walton, split logic into 2 main types: Formal logic and Informal logic. Formal logic looks into formulae and formally expresses a situation using syntax and semantics. Whereas, informal logic focuses more on the use of argumentation in the context of language.

Now that I've given a brief overview of what logic is, I now need to introduce a certain area of logic solely dedicated to mathematics: mathematical logic. Mathematical logic is a discipline that seeks to understand the principles of reasoning and argumentation. It is based on the notion that mathematical concepts can be used to represent and analyse logical relationships. In simpler words, it is the study of formal logic within the endless sea of mathematics. Mathematical logic is also split into different theories and sectors. The main ones (also namely the most famous) include: Model theory, Set theory, Graph theory, and Recursion theory. Research in mathematics plays a vital role in the sense that the results of these areas of research can be used to characterise and theorise correct mathematical reasoning. However,

the foundations of mathematical logic have been the subject of much debate in the philosophical community. This is where the problem of the philosophical flaws comes into play.

The first flaw of the foundations of mathematical logic is the assumption that logic can be reduced to a formal system. This assumption is based on the idea that logic is a purely mechanical process that can be reduced to a set of rules. However, this view neglects the fact that logic is a human activity that involves creativity and intuition.While formal systems can be useful tools for analysing logical relationships, they cannot fully capture the complexity of human reasoning. Human reasoning involves a variety of factors such as context, emotion, and experience, which cannot be fully captured by formal systems. In fact, the very act of creating a formal system requires human judgement and creativity.Furthermore, the assumption that logic can be reduced to a formal system ignores the fact that different individuals may have different interpretations of the same formal system. This is because formal systems are inherently abstract and require interpretation to be applied to real-world situations. Therefore, the idea that logic can be reduced to a formal system oversimplifies the nature of reasoning and neglects the complexity of human cognition.

The second flaw of the foundations of mathematical logic is the assumption that logic is independent of language and culture. This assumption is based on the idea that logical principles are universal and apply to all languages and cultures. However, this view neglects the fact that language and culture shape the way we think and reason. Different languages and cultures have different concepts and ways of expressing ideas, which can affect the way we reason about them. For example, the Chinese language has a different grammar structure than English, which can affect the way Chinese speakers reason about logical relationships. Additionally, different cultures may have different values and beliefs, which can influence the way individuals reason about ethical and moral issues. Therefore, the assumption that logic is independent of language and culture oversimplifies the nature of reasoning and neglects the impact of language and culture on human cognition.

The third flaw of the foundations of mathematical logic is the assumption that logic can provide a complete and consistent account of reality. This

assumption is based on the idea that logic is a tool for discovering objective truths about the world. However, this view neglects the fact that reality is complex and multifaceted. Logic can only provide a limited perspective on reality, and it cannot capture the full richness of human experience. For example, logic may be able to provide a mathematical proof for the existence of black holes, but it cannot capture the subjective experience of seeing a black hole or the emotional impact of such an experience .Additionally, the very act of observation can affect the reality being observed, which complicates the notion of objective truth. As a result, the assumption that logic can provide a complete and consistent account of reality oversimplifies the nature of knowledge and neglects the complexity of human experience.

The fourth flaw of the foundations of mathematical logic is the assumption that logic is a value-neutral discipline. This assumption is based on the idea that logical principles are neutral with respect to ethical and political considerations. However, this view neglects the fact that logic is often used to justify and reinforce existing power structures. Logical arguments can be used to justify oppression and discrimination, and they can serve as a tool for maintaining the status quo. For example, the use of logic in the legal system can result in the perpetuation of unjust laws and practices. Additionally, the very act of choosing which logical principles to apply in a given situation involves ethical and political considerations .And so, the assumption that logic is a value-neutral discipline oversimplifies the nature of reasoning and neglects the impact of power dynamics on human cognition.

The fifth and final flaw I will touch upon is, in my opinion, one of the biggest flaws, and that is the problem of induction. The problem of induction is a philosophical challenge to the justification of inductive reasoning, which is a type of reasoning that draws general conclusions based on specific observations or instances. In other words, it is the problem of how we can justify using inductive reasoning to make predictions about the future based on past experiences. It was famously articulated by the Scottish philosopher David Hume in the 18th century. He argued that we cannot logically justify the assumption that the future will be like the past, based solely on past experiences. In other words, we cannot prove that the laws of nature will remain constant over time, because we have no direct experience of the future. Hume's argument points to the fact that inductive reasoning is based on a leap of faith, rather than on logical deduction. No matter how many observations we make, we cannot logically infer from them that a given event

will always occur in the future. For example, we might observe that the sun has risen every day for millions of years, but we cannot prove that it will continue to rise tomorrow. The problem of induction has important implications for science and everyday life. Scientific theories are based on inductive reasoning, and we rely on inductive reasoning to make decisions about the future. However, the problem of induction suggests that these forms of reasoning are ultimately based on unprovable assumptions. There have been many attempts to address the problem of induction, such as the idea of falsifiability, proposed by Karl Popper; the notion of Bayesian probability and the concept of inductive logic. However, the problem of induction remains a contentious issue in philosophy, and it continues to inspire ongoing debates and discussions.

In conclusion, the foundations of mathematical logic are based on several philosophical assumptions that are problematic. Logic cannot be reduced to a formal system, it is not independent of language and culture, it cannot provide a complete and consistent account of reality, and it is not a value-neutral discipline. While mathematical logic can be a useful tool for analysing logical relationships, it cannot fully capture the complexity of human reasoning and the richness of human experience.

As such, it is important to approach mathematical logic with a critical eye and recognize its limitations. Additionally, we must acknowledge the impact of language, culture, power dynamics, and subjective experience on our reasoning and seek to incorporate these factors into our understanding of logic. By doing so, we can develop a more nuanced and comprehensive understanding of reasoning and argumentation. It is important to note that while the foundations of mathematical logic have their flaws, they are still valuable tools for analysing logical relationships. However, we must be aware of their limitations and approach them with a critical eye. By recognizing the impact of language, culture, power dynamics, and subjective experience on our reasoning, we can develop a more nuanced understanding of logic and argumentation. And so, while these flaws do not necessarily undermine the usefulness of mathematical logic as a tool for analysing mathematical concepts and reasoning, they do raise important questions about the foundations of mathematics and the limits of our knowledge. Philosophers of mathematics continue to grapple with these questions, and ongoing debates in the field reflect the richness and complexity of the philosophical issues that arise in the study of mathematical logic.