Can Formal Logic (FL) act as a viable substitute for natural, informal reasoning?

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Introduction

People frequently find their lives influenced by the actions of their government and/or society, either positively such as the slaves who found themselves freed by Lincoln (Manning, 1998) or negatively, such as the casualties of the Vietnam war (United States Senate, 1972). Present in our everyday life, these decisions are influenced by many things, ranging from informed logical conclusions produced by an epistemic use of rationality (Termini, 2019), to factors more akin to personal prejudice, feelings, and bias (Matlock, et al., 2017; Sharp, et al., 2012). In the theatre of modern discourse, accusations of irrationality and logical fallacies abound; such as the conjunction fallacy (Tversky & Kahneman, 1983) or that of Post hoc ergo propter hoc (Damer, 2013) to name a few. Additionally, as noted by Wittgenstein, natural language as a mode of communication misleads those engaged in discourse to a misfitting picture of the matter at hand (Wittgenstein, 1974). The aim of this report is to investigate the possibility of a formalised system, in the mathematical sense; concerned solely with form, (Tiles, 1989) as a complementary alternative to human decision making, with the intent of rigorizing thought processes while seeking to minimise ambiguity and irrationality.

Methodology

The research method utilised was literature review of important texts on the subject matter including:

1) Essays, articles, and books which presented the problem of the shortfalls of natural reasoning to provide justification for the study of formalisation.

2) Review of textbooks, books, and papers regarding the mathematical subject matter to gain an understanding of the mathematical theory at hand.
3) Review of proofs and articles to determine the viability of formal systems (FS), and examine prior usage of FS to demonstrate their strengths and weaknesses.

A literature review was chosen as the research method for this report as it was the only viable option, mathematics, especially that contained within this paper is not a practical subject. At best, one may be able to observe the results of mathematics in the real world, one apple add one apple makes two apples, however, with regards to the concept of proof, practical realisation is impossible (Hoftstadter, 1979). As Mathematical proofs rely on a generalisation over infinites, no finite amount of empirical evidence can constitute a proof (Tiles, 1989). Expert opinion is also not a valid method of proof in Mathematics, a mathematical proof, if true exists regardless of one's opinion on it, no matter how learned (Pantsar, 2021). In addition, Mathematics, as a discipline, necessitates a review of proofs and theory to be spoken of sensibly, hence a literature review of existing material on the subject was deemed the only viable research methodology.

While theoretically, the mathematical theory could have been constructed and proved from scratch, a literature review in mathematics allows for the building upon of past bodies of work without going through the exercise of proving everything from scratch. It also allows for the use of universal nomenclature for ease of communication.

With regards to the location of sources, the name of many important papers on the subject are well known, these were obtainable by name search. Further information was obtainable from a library search and location of physical mathematics textbooks/books on the subject matter.

Physical sources were located by locating a particular book in a library with a Dewey decimal system and browsing around that general area. Electronic sources were mainly obtained from miscellaneous websites as most papers on this subject matter are not easily locatable on University corpuses.

All sources of a mathematical nature used for this research paper can generally be regarded with high confidence, the objective nature of mathematics and the fact they mostly concur with each other lends credibility to them. Most, if not all sources concerning the theoretical framework of the mathematics can be considered highly reliable. They are also published by reputable publishers, some of which are attached to institutions well known for their mathematical proficiency, such as Cambridge or the London Mathematical Society. A lot of sources also offer first-principle derivations of many results which increases reliability.

Regarding ethical considerations for the report, these are non-existent bar proper citation of sources and accurate representation. All research has been desk research and there are no test subjects or confidentiality issues. There are hardly any ethical considerations arising from reading a Mathematical paper.

Though some of the sources used are not academic in nature, such as certain books they are mostly to do with the mathematical theory, which is concurred with by scholarly sources and is a point of consensus between the sources themselves. Moreover, the theory is fundamentally objective bar notation.

Conceptual Framework: FL and theory thereof.

Through use of syllogisms in natural language valid and true outcomes can be reached.

E.g.

Premises:

1) All humans are mortal
2) Socrates is a human

   Conclusion: Socrates must be mortal.

In addition, a somewhat rigorous theory regarding syllogisms was developed by Aristotle (n.d.) in *Prior Analytics*. This prompts the question as to why FL should be studied. However, informal reasoning allows for issues of the following nature to arise (Bostock, 1997; Termini, 2019):

1) False premises: if these are present, the argument can be false. E.g. I am blue. All things blue are Donald Trump. I am Donald Trump. This applies to FS as well.
2) Incorrect reasoning (IR): When a conclusion is reached that does not flow from the premises E.g. I have legs. Chickens have legs. Therefore, I am a chicken
3) Ambiguous definitions (AD) : E.g. This soup is hot, hot people are attractive, therefore this soup is attractive

In addition to this, reliance on intuition can result in antimonial paradoxes such as the question of whether the set of sets which do not contain themselves contains itself (Hoftstadter, 1979; Hrbacek & Jech, 1978). If it does not, then it fulfils the criteria to be an element of itself and vice versa. In linguistic terms, is the word heterological heterological (sic).

When scaled up to the level of complex real-world dilemmas, Informal logic can become highly prone to IR and AD, for example in 1843, doctors at the time demonstrated the "correlation implies causation" fallacy, with regards to mothers who died of childbirth fever after inflammation was noted. Doctors presumed the cause of the fever was inflammation, and many lives were lost as a result, as the inflammation was actually a symptom of a common cause (Holmes, 1809-1894). This was prevailed against by Oliver Holmes. IR and AD can be found in the myriad of logical fallacies seen in today's political discourse (Edward, 1995; Matlock, 2017; Sharp, 2012; Tversky, 1983). Meanwhile, FS, by definition, cannot act outside of their own rules, and so seem like they may be less vulnerable (Bostock, 1997; Hofstadter, 1999; Tiles, 1989). For these reasons, research into the formalisation of logic is undertaken, with an intent to investigate the possibility of FL as a viable alternative to intuition.

Elucidating the difference between 'Formal' and 'Informal' logic; While Informal reasoning focuses on the substance and merit of arguments, FL is solely concerned with the form of statements and typographical manipulation of statements, following prescribed rules (Aristotle, n.d.; Bostock, 1997; Hofstadter, 1979; Tiles, 1989). Hence, while in our intuitive assessment of arguments, we may examine the plausibility of the conclusion, no such reasoning is allowed in FL. If one accepts the axioms, one accepts the theorems, no matter how absurd. The means justify the ends.

A note on terminology: Within the language of FL, quantifiers, $\forall$;for all and $\exists$;there exists, truth functors $\neg$;not, $\Rightarrow$;implies, $\Leftrightarrow$;if and only if, $\wedge$;and, and $\vee$;or, and the element relation, $\in$;is an element of, e.g. Joe Biden$\in$M, the set of all men, are used. A truth table setting out the usage of the truth functors is provided, where P and Q are propositions. These will be used freely.

P	Q	$P \wedge Q$	$P \vee Q$	$P \Leftrightarrow Q$	$P \Rightarrow Q$	$\neg P$
True	True	True	True	True	True	False
True	False	False	True	False	False	False
False	True	False	True	False	True	True
False	False	False	False	True	True	True

(Bostock, 1997; Fraenkel, 1958; Hrbacek & Jech, 1978; Roitman, 1990; Smullyan, 1968).

Within this context, the following terms are defined as follows (Hoftstadter, 1979; Tiles, 1989):

Axioms- Fundamental assumptions held to be true. The 'premises' of an argument in FS.

Theorems- Statements which proceed from Axioms by rules of a FS.

Valid- If there is no scenario in which all premises a conclusion is premised off are true and the conclusion itself is false, then it is valid.

True- A case of validity in which the premises are true.

Complete- A FS is complete if every true statement within its language can be reached from the Axioms.

Consistent- A FS is consistent if there is **at least one** possible world in which all theorems are true.

Decidable- A FS is decidable if every statement can be checked to see if it is a theorem in a finite amount of time.

In a FS, only explicitly typographical operations may be performed. These consist of:

1) reading and recognising any of a finite set of symbols;
2) writing down any symbol belonging to that set;
3) copying any one of those symbols from one place to another;
4) erasing any of those symbols;
5) checking to see whether one symbol is the same as another;
6) keeping and using a list of previously generated theorems. (Hofstadter, 1979, p. 64)

When operating within FS the 'Requirement of Formality' (ROF) must be followed; put simply, "You must not do anything outside the rules" (Hofstadter, 1979, p. 33).

This comprises, a restriction to work within and only within the framework provided by the axioms and rules. One can only go from statement to statement by the rules of the FS, using purely typographical operations (Hoftstadter, 1979). As an example, consider the following formal system:

Axiom: I+I=II

Rule of production (ROP):

Suppose one has I+x=Ix

Then, I+Ix=IIx is a theorem where x stands for any arbitrarily long string of I's.

Hence from I+I=II we have I+II=III etc. This may be reminiscent of addition; however, it is wrong to presume that II+I=III is a theorem as no rule within the system has allowed 'adding' an additional 'I' to the first 'I'. Though this may seem trivial, it is important. When systems grow more complicated, so does following the ROF, more below.

A useful toy system to demonstrate the "architecture of logical thought" (Hoftstadter, 1979, p. 195) and demonstrate the mechanics of FS, the 'propositional calculus' consists as follows (Bostock, 1997):

Atoms: Sentence letters which stand for particular statements; P,Q,R etc. Atoms are well formed.

Formulation rules: If 'a' and 'b' are well formed, the following are well formed:

$\neg x$

$x \wedge y$

$x \vee y$

$x \Rightarrow y$

Let z be the set of all theorems and 'x' and 'y' be well formed strings, the ROP are as follows:

(1) $(x \epsilon z) \wedge (y \epsilon z) \Leftrightarrow (x \wedge y) \epsilon z$
(2) $x \epsilon z \Leftrightarrow \neg\neg x \epsilon z$
(3) $(x \Rightarrow y) \Rightarrow (x \Rightarrow y) \epsilon z$ The assumption of the premise x as a theorem is denoted with [and an indent
(4) If another conditional after x is present, x can be presumed a theorem within that supposition.
(5) $((x \Rightarrow y) \epsilon z) \wedge x \epsilon z \Rightarrow y \epsilon z$
(6) $x \Rightarrow y \Leftrightarrow \neg y \Rightarrow \neg x$
(7) $\neg x \wedge \neg y \Leftrightarrow \neg(x \vee y)$
(8) $x \vee y \Leftrightarrow \neg x \Rightarrow y$

Essentially, within each [, the premise is assumed to be a theorem for the formulation of statements, then when 'translated' out, it becomes a conditional statement.

As an example derivation of a theorem:

[		(3)
	x	Premise
	[	(3)
	$\quad y$	Premise
	$\quad x$	(4)
	$\quad x \wedge y$	(1)
	]	
	$y \Rightarrow x \wedge y$	
	$\neg(x \wedge y) \Rightarrow \neg y$	(6)
	$\neg\neg(\neg x \vee \neg y) \Rightarrow \neg y$	(7)
	$\neg x \vee \neg y \Rightarrow \neg y$	(2)
]		

$x \Rightarrow (\neg x \vee \neg y \Rightarrow \neg y)$ is a theorem.

Hence it can be seen that increasingly complex theorems can be constructed. The propositional calculus is limited, but it is consistent, complete and decidable, and all its theorems are tautologies, that is they are true by virtue of their form (Bostock, 1997; Hoftstadter, 1979). The propositional calculus also proves a useful tool in that it can be embedded in other FS and variations can approximate the human thought process to varying degrees. Though it is a very limited example of the codification of logic, it serves as a starting point from which more complex study may be undertaken.

Applications of FS

The above description of the propositional calculus may make FS seem a severely limited tool for decision making, however while FS seem, on the surface simple, they can reach profound depths. To illustrate this point, two examples will now be outlined of the use of FS in mathematics to great success, namely, Zermelo-Fraenkel Set Theory with Choice (ZFC) and Euclidian/Non-Euclidian Geometry.

FS have also enjoyed much success in the field of AI, especially in recent years with them being able to outperform humans at chess, problem solving, etc. however this will not be covered here due to lack of specialty expertise (Hoftstadter, 1979).

ZFC consists in 10 axioms, from which the vast majority of mathematics can be derived. Exact formulation may vary by text as there are multiple modes of expression (Ciesielski , 1997; Fraenkel, 1958; Hrbacek & Jech, 1978; Roitman, 1990; Tiles, 1989). The 10 are:

Extensionality

$\forall_X \forall_Y \forall_z [(z \epsilon X \Leftrightarrow z \epsilon Y) \Rightarrow X = Y]$

Regularity

$\forall_X [\exists_a (a \epsilon X) \Rightarrow \exists_y (y \epsilon X \wedge \neg \exists_z (z \epsilon y \wedge z \epsilon X))]$

Schema of Specification

$\forall_X \exists_Y \forall_z \ [z \epsilon Y \Leftrightarrow z \epsilon X \wedge P(z)]$

P(z) is a well-defined property in the language of FL.

Pairing

$\forall_x \forall_y \exists_Z \forall_w (w \epsilon Z \ \Leftrightarrow w = x \ \vee w = y)$

Union

$\forall_F \exists_A \forall_Y \forall_x [(x \epsilon Y \wedge Y \epsilon F) \Rightarrow x \epsilon A]$

The operation defined is abbreviated $\cup A$. $x \cup y$ is defined as $\cup z$ where z = {x,y} - existence justified by pairing.

Empty set

$\exists_x \forall_y \neg(y \epsilon x)$

This set is denoted $\varnothing$

Schema of Replacement

Let F(y,z) be a well-defined function of y which maps an input y to unique output z, such that $F(y, z) \Rightarrow \forall_w (F(y,w) \Rightarrow w = z)$. Then:

$$\forall_x [\forall_y [y \epsilon x \Rightarrow \exists_y (F(y,z))] \Rightarrow \exists_v \forall_u [u \epsilon v \Leftrightarrow \exists_t (t \epsilon x \wedge F(t,u))]]$$

Infinity

$$\exists_X [\varnothing \epsilon X \wedge \forall_y (y \epsilon X \Rightarrow (y \cup \{y\}) \epsilon x)]$$

Power Set

$$\forall_X \exists_Y \forall_Z [Z \epsilon Y \Rightarrow \forall_w (w \epsilon Z \Rightarrow w \epsilon X)]$$

Well-ordering theorem

$$\forall_x [\forall_y (y \epsilon x \Rightarrow \neg(y = \varnothing)) \wedge \forall_y \forall_z (y \epsilon x \wedge x \epsilon x \ (y = z) \Rightarrow \neg(\exists_W (w \epsilon y \wedge w \epsilon z))) \Rightarrow \exists_U \forall_y (y \epsilon x \Rightarrow \exists_z (z \epsilon u \wedge z \epsilon y \wedge \forall_w (w \epsilon u \wedge w \epsilon y \Rightarrow w = z)))]$$

Though the axioms may look complicated on first blush, after deciphering using the meaning of the symbols, one realises they are quite self-evident and simple. The reader is encouraged to 'interpret' the axioms with the definitions given above.

ZFC shows the power of FS, because, even with a limited set of rules such as those of ZFC, most of modern mathematics can be derived, working purely within ZFC itself (Fraenkel, 1958; Hoftstadter, 1979; Hrbacek & Jech, 1978; Roitman, 1990; Smullyan, 1968; Tiles, 1989).

In addition, Whitehead and Russell (1968), penned their monumental text, Principia Mathematica, which serves as a prime example of the power of FS, using the language of set theory/FL to construct arithmetic in a rigorous fashion.

Seemingly, the above paradox can arise in a formally formulated fashion, consider set R, $\forall_y \{y \neg\epsilon y \Rightarrow y \epsilon R\}$, a formal version of the definition as above. However, at this point one must distinguish between active and passive meanings of isomorphisms in FS. An isomorphism is a one to one correspondence, it can be thought of as an 'interpretation'. (Hoftstadter, 1979). E.g. the word apple is isomorphic with the fruit. Here, it is important to note, that the meanings isomorphisms give symbols in FS is passive, not active. This means, while strings of symbols in a particular FS are allowed to be 'interpreted' into natural language, the converse is not. This is important, because though it seems that the above set can be specified, as it makes sense upon 'interpretation', it is not a valid set (Hrbacek & Jech, 1978; Tiles, 1989). This is because it is not, by the above axioms, guaranteed to exist. Hence the above definition is invalid. Such a paradoxical definition, for the most part, does not appear in FL. Later, exceptions to this idea will be visited. This demonstrates another benefit of FS, that they avoid confusion arising from 'active' meanings of isomorphisms.

This prompts a stressing of the ROF.

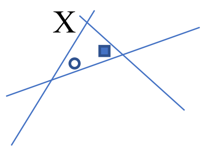
Another system in which FS have been employed to great success is in the field of Geometry (Hoftstadter, 1979). For this, an examination of basic Euclidian geometry and variant offshoot branches is warranted.

Euclidian Geometry is essentially the branch of geometry characterised by the five postulates laid down by Euclid in his “elements” (Hoftstadter, 1979). It is a geometry that deals with shapes and objects that exist within a world which has no inherent curvature of space. (Euclid, n.d.).

The Five postulates which characterise Euclidean Geometry respectively are:

1) A straight line segment can be drawn joining any two points.
2) Any straight line segment can be extended indefinitely in a straight line.
3) Given any straight line segment, a circle can be drawn having the segment as radius and one end point as centre.
4) All right angles are congruent.
5) If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough (Euclid, n.d.; Hoftstadter, 1979, p. 90).

E.g. observe that the two angles, ∘ and ■ sum to less than two right angles, and observe they intersect at the point X.



‘5)’ is equivalent to the parallel postulate; for a line p and point x not on p, there is one and only one line through x which never intersects p (Hoftstadter, 1979).

Euclid’s work in the volumes of the elements is a monumental demonstration of proofs by construction and contains proof of many important results in geometry, for example that of

Pythagoras's theorem (Euclid, n.d.). This theorem has many applications in everyday life. Maths fellow Matt Parker at Queen's University is on record, speaking of its usage in data transmission (Khaleeli, 2016). Many important results can be found in Euclid's elements, all derived from the five postulates above, some definitions, and common-sense notions (Hoftstadter, 1979). However, these will not be discussed. The text of Euclid's elements is assumed to be straightforward enough reader. The reader is encouraged to conduct their own examination of the text to gain an understanding and appreciation of the significance and formality of the of the results, flowing from the simple premises above.

The reader may note that the fifth postulate is not as intuitive as the first four, indeed it seems that Euclid had a dislike for the fifth postulate, avoiding its usage in his first 28 propositions (Euclid, n.d.; Hoftstadter, 1979). Many mathematicians have felt that the fifth postulate was not obvious enough to be held as an axiom, yet proving its truth/falsity was, for the longest time, fruitless (Ramsay & Richtmyer, 1995; Sommerville, 1910). Eventually, in Euclides Vindicatus, Saccheri, via assuming the falsity of the fifth postulate, managed to arrive at a conclusion which he deemed "repugnant to the nature of the straight line" (Saccheri, 1733, p. 233).

The fifth postulate seemed proven, except it was not, as would be shown by Lobachevsky, Bolyai and Gauss around 90 years later (Hoftstadter, 1979; Lobachevsky & Papadopoulos, 2010; Sommerville, 1910; Ramsay & Richtmyer, 1995). Saccheri's error was a violation of the ROF. His result was only "repugnant to the nature of the straight line" as defined in the informal conception of a line (Encyclopedia of Mathematics, 2001; Saccheri, 1733).

This then led to the development of a theory of two forms of non-Euclidian geometry. In one, space has constant positive curvature, and all lines through x intersect p (Encyclopedia of Mathematics, 2001; Hoftstadter, 1979). In elliptical geometry a line becomes defined as a great circle on a sphere. If, however, there exist multiple possible lines through x which do not intersect p, then we reach a space with constant negative curvature, hyperbolic geometry, where a line takes on a stranger definition still (Ramsay & Richtmyer, 1995). In both cases, though the nature of a line in these geometries may directly clash with our conception of a line in everyday thought, it is important to note that these definitions of "line" are consistent with the first four postulates. Hence, thinking that any of these definitions constitutes a contradiction breaches the ROF.

Hence, a failure, in this case, to follow the ROF could have impeded research into elliptical and hyperbolic geometry, both of which in modern times have become increasingly relevant, especially with Einstein's discoveries of the curvature of spacetime (Einstein, 1905; Lenárt, 2010). This is an important substantive supporting the importance of the ROF and showcases the power of formality in terms of avoiding AD.

Hence, FS allow for a certain circumvention of AD and IR, at least to a greater degree than natural intuition, while still managing to produce monumental results.

Limitations of FS

Cantor's notions of actual infinites and the existence of transfinite numbers caused fracturing in the mathematical community (Tiles, 1989). The debate was split between intuitionists who believed that mathematics was a pure human construct, and formalists who believed that mathematics could be built on absolute, secure, and logical foundations. In the midst of this, German mathematician David Hilbert, posed the questions of whether mathematics was complete, consistent and/or decidable (Hilbert, 1900). Hilbert believed the answers to these questions was yes, saying “we must know, – we will know!” (Hilbert, 1930).

Shortly after Hilbert’s declaration in 1930, Kurt Gödel would show that Hilbert was misled, proving that any sufficiently powerful, consistent, FS is incomplete (Gödel, 1931). Essentially, if a FS is complete, then it is either sufficiently limited, or it is inconsistent. For an elucidation of the concept of ‘sufficient power’, the reader is referred to Chapters 2 & 4 of Hofstadter’s “Gödel, Escher, Bach”. Essentially, Gödel managed this by introducing his Gödel numbering system, which assigns a unique natural number to each symbol and well-formed statement in FL. Gödel used a system reliant on prime factorisation to achieve this. Exact methods vary, the essential idea is that by the fundamental theorem of arithmetic, proved by Euclid, any number formed in this manner has a unique prime factorisation which can be determined to find the original statement (Euclid, n.d.). Gödel then constructs a statement with Gödel number ‘G’, which states that the formula with Gödel number ‘G’ cannot be proven. Either a proof for this statement exists, in which case the statement is false and hence a contradiction as a false statement has been proven, or there is no proof for it and it is true and hence the system is incomplete. Therefore the system cannot be both consistent and complete. This is a gross oversimplification. Gödel also proved that a consistent FS cannot prove its own consistency, however the proof is too complex to be summarised here. The reader is referred to Gödel’s original paper, referenced here, if deeper understanding is desired.

Similarly, in 1936, Turing’s research complimentarily demonstrated that FS are not decidable as well by solving an equivalent problem, whether a given Turing machine (TM) would halt (Turing, 1936). If one could solve the problem, one could program a TM to generate theorems from the axioms using the ROP and stop when it generates a desired theorem, then check to see if that machine will halt and hence determine if the theorem is a part of the FS. Turing reasoned, if a TM; H could determine if any program would halt, it could be connected as part of a TM; I. If H halts, I goes into a loop and vice versa. If I is fed its own code, logically it can do neither, hence the assumption that H exists must be false. Since Turing had also showed that a TM is, in theory, capable of any operation FS can perform, logically, FS are undecidable.

Hence, considering Gödel and Turing's work, a key weakness of FS is that they are incomplete and undecidable, with an inability to prove their consistency.

Note well, by the Church-Turing Thesis, all real-world computations are performable by TM (Church, 1936). If one accepts that the brain functions by a complicated pattern of electrical activation of neurons, then it should be the case that a TM, and by extension, a FS, can perform any feat the human mind is capable of (Church, 1936). Proponents of this thinking would argue that the human mind itself is simply a complex FS, as the chemical and biological processes are determined by deterministic laws (Hoftstadter, 1979). For further reading, the philosophy of Determinism proves informative (Berofsky, 1971). Unfortunately, there has been no proof for the Church-Turing Thesis, instead with supporting evidence coming from the fact that all discovered, realistic models of computation have been proved equivalent (Rowland, n.d.).

Strengths and Weaknesses

Regarding the strengths of the project, one is that it is primarily grounded in the field of mathematics which is highly objective, opinion and/or bias has no effect on the results. No matter how biased, one cannot prove 1+1=3, unless one fundamentally changes the meaning of the symbols, and even then, the essence remains. Hence, research proved to be relatively easy with regards to the theory of mathematics, there are disputes over notation and conventions between sources, but most sources, except the flawed ones such as Sachheri's, agreed with each other in essence and idea. This is because of the inherently objective nature of Mathematics (Pantsar, 2021).

Much of the material reviewed has tried to derive as much of its own content from first principles as possible, with Principia Mathematica being a prime example, however, the more philosophical works, such as those of Hofstadter and Wittgenstein prove to have a philosophical element that could have potentially been prey to bias. This proves to be a rather frustrating limitation of the research project. Nonetheless, the authors are respected authorities in their respective fields. The truth of Hofstadter's work as a source is supported by its winning of the Pulitzer Prize. To its credit, a range of scholarly and non-scholarly sources were used in writing this report.

It has been attempted to improve the reliability of the non-mathematical sources by ensuring that most major works cited are peer reviewed and of an acceptable academic level, as well as ensuring they have reputable publishers.

This report does contain significant weaknesses, chief among these being the fact that not much work has been done or is easily available on this topic due to its niche and complex nature. Unlike calculus or algebra, the discipline of formal mathematics is not regularly applied in everyday life, and I struggled to find sources even with access to two university corpuses and frequently had to resort to hunting down sources by other means. Also, one could argue that logic itself is subjective and an object being in and out of a set simultaneously is not a contradiction. Responses to this are sparse for obvious reasons. There is also the issue that a lot of questions within this field are open, such as the truth of the Church-Turing Thesis. Evidence, and all practical experience suggests that all the above theory holds true, yet this does not constitute proof.

Additionally, Gödel and Turing have shown that mathematics is fundamentally undecidable, incapable of proving its consistency, and incomplete. Yet it remains an open question as to whether FL could prove to be better than informal logic.

Another weakness of this report is that a lot of sources were quite old, however the effect of this is minimised by the fact that mathematical truths do not usually change overnight.

Conclusion and Future Research

Though some basic conceptual framework has been established with regards to the construction of a formal codification of logical reasoning, much is lacking, and many simplifications have been made due to the complexity of the subject matter. The predicate calculus provides a starting block from which further research must be conducted to attempt the creation of a viable formal system of logical thought.

The power of FS is showcased in both the fields of geometry and set theory. In both cases, incredibly simple axioms allowed for the construction of highly complex results, in the case of Euclid his postulates and the findings of non-Euclidian geometry, and in ZFC, the derivation, in principle, of most of modern mathematics.

In either case, both the power of FS as well as their ability to supersede Informal reasoning in cases has been showcased.

Perhaps even more important, the limitations of FS have been discussed, both in Gödel and Turing's respective work with respect to completeness, consistency and decidability.

There is also the question of if an 'informal' system actually exists or if our brains are simply complex FS.

In conclusion, there is simply too little known about FL at the moment for it to viably replace Informal Logic. The current limitations outweigh the potential advantages

In terms of future research, the truth of the Church-Turing thesis must be ascertained. Further development of FS and FL to extend their capabilities should be made. Research should be performed into the usage of FS in AI, which essentially are complex FS, where they have seen much success to examine the viability of using FS in decision making.

Reflection

When I began this report, I was highly confident that a formalised system of logic could clear up all ambiguities and fallacies within our modern discourse, however, as I read deeper into the results of Gödel and Turing, I am faced with a similar sentiment to mathematicians in the wake of Cantor's discoveries. Where they realised that the concept of a limit in calculus was poorly defined, I now realise that for all its dreams of clearing up ambiguities, the field of FS is still very much unexplored territory, perhaps it is the domain of Mathematics to explore it, or maybe that of Philosophy. I can only hope that, like how mathematicians defined the limit formally with the epsilon-delta definition, that one day the field of FS will see a similar reviewal and, whilst not perfection, improvement. While my eyes are opened to the nature of the situation, that FS are not a silver bullet for irrationality, as shown by Gödel and Turing, I still have informed hope that FS may provide humanity with better solutions to its problems, as it does have tremendous power, as shown in ZFC, AI and Geometry.

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