The Search for the USS Scorpion:
A Modern Application of Bayesian Statistics

How many times have you lost an object, having no viable and systematic way to locate it? I'm talking about *any* object- from as trivial as your phone charger to as significant as your social security number (awkward indeed). All of those events, frustrating as they may be, could have been easily solved with the Bayesian Search Theory.

Due to its effective nature, this theory has been used by scientists and even governments over the years to solve the biggest mysteries known to man. One of the most prevalent events of pop culture, the 1968 disappearance of the USS Scorpion, was quickly solved using this search method.

Throughout her deployment, the Scorpion participated in training in the Mediterranean east and monitored Soviet vessels in her course back west. As the submarine's mission was finally accomplished, families congregated on the docks of Norfolk, Virginia to welcome back the crew. However, as time passed, collective horror dawned on the families as the submarine failed to resurface.

After eight days passed, the Navy issued a statement declaring the crew lost at sea and presumed dead. Though the prospect of saving the crew members was gone, there was another high-stakes predicament- strapped to the Scorpion were two nuclear-tipped torpedoes, now lost somewhere at the bottom of the ocean.

In a search led by Dr. John Craven, chief scientist in the Navy's Special Projects Office, a team of experts employed a strategy called Bayesian Search to find these dangerous warheads. This search method heavily relies on Bayesian statistics to outline a systematic search procedure.

Bayesian search theory is heavily inspired by Bayes' Theorem. This mathematical formula is rooted in conditional probability and determines the likelihood of an event occurring based on previous outcomes in similar circumstances.

$$P(A|B) = \frac{P(B|A)\, P(A)}{P(B)}$$

The formula determines the probability of event A occurring given that a previous condition is met; Event B is also true. It may also be referred to as the posterior probability of A given B.

With this theorem in mind, it is clear how helpful the Bayesian theorem would be in locating lost objects such as the warheads of the USS Scorpion. The application of this theorem allows for the location of lost objects in the most efficient way, rather than arbitrarily searching all the possibilities. The procedure is relatively straightforward, following a methodical set of rules.

1. Brainstorm a set of all possibilities for the missing object. This includes which locations are most probable, the last known location, and the approximate time it was lost.

Prior to the search, Craven gathered and evaluated the opinions of various specialists regarding each possible scenario. These specialists were able to provide predictions and voices of expertise over several fields- oceanography, nuclear bombs, and more. These opinions were analyzed and used to form a prior possibility for each scenario, and the definition of the term is exactly how it looks- these possibilities are determined before any data is collected in the search. For example, a highly valuable piece of information collected prior to the search is the occurrence of unusual activity approximately 400 miles off the coast of the Azores, which matched the anticipated course of the USS Scorpion. This information was helpful as it served to narrow the searchable region from a 3000-mile-long area to a few square miles.

Craven's team then took a map of the narrowed-down region and divided it into small rectangles, each one representing a searchable area and assigned a label. Then, the team collected many possibilities of the submarine's sinking- with theories ranging from natural disasters to an unprecedented Soviet attack. Each of these scenarios was examined and the likely probabilities were marked.

2. Compose a probability density function to model each of these scenarios. In probability theory, this is a function that represents the relationship between a distinct variable and its probability while being restricted between a specific range of values.

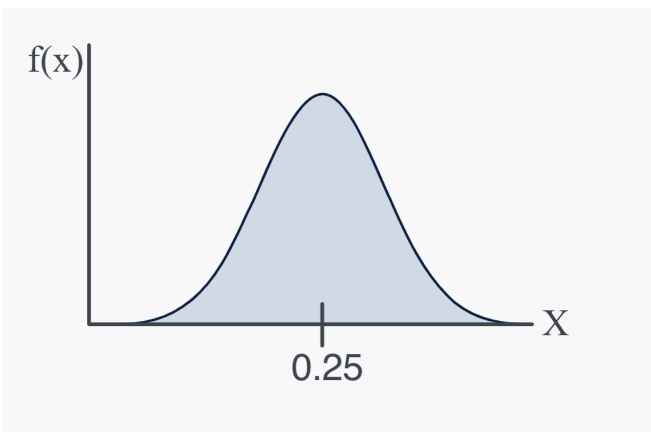

Image taken from Penn State University, Eberly College of Science

For the USS Scorpion, this function pointed out many high-probability areas- starting the search off with rectangle F6. The team treated the searchable region as a Euclidean space, with coordinates represented by ordered pairs (x_1 , x_2), where x_1 and x_2 are independent, normally distributed variables. Mathematicians estimate that the function looked similar to this:

$$p(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2} exp\left[-\frac{1}{2}\left(\frac{x_1^2}{\sigma_1^2} + \frac{x_2^2}{\sigma_2^2}\right)\right] \; for \; (x_1, x_2) \in X$$

The significance of this step is not to be overlooked- it simplified a complicated search process to a more simple probability density function.

3. Compose another function predicting the probability of finding an object in a certain location given that it is actually there.

In the search for the USS Scorpion, this function usually models water depth- if an object is located in more shallow water, there is a higher probability of finding an object given that it is located there. This probability is reduced in deeper water. Another important factor is the effectiveness of the instruments used in the search, provided by the US Navy. Given a searchable area, instruments were evaluated for how likely they were to find a bomb in that area. This postulate was also heavily influenced by the prior possibility of each scenario and its limitations. For example, although an area could have a high prior possibility, other factors including how dark the area is or the limitations of an instrument in that area would affect how likely the bomb is to be found there.

4. Compile all of the data collected thus far. With the data collected, you can coherently put together an overall probability density map. This map can be visualized as a contour map, effectively giving the probability of finding an object according to its location and the various factors surrounding that region.

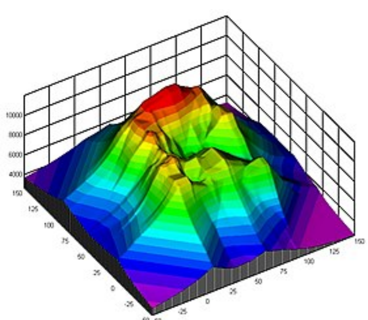

Image taken from Wikipedia.

In the search for the USS Scorpion, Craven put together a map that compiled all collected information throughout the search process;

interviews, prior possibility regions, calculations, etc. The map most likely resembled this one:

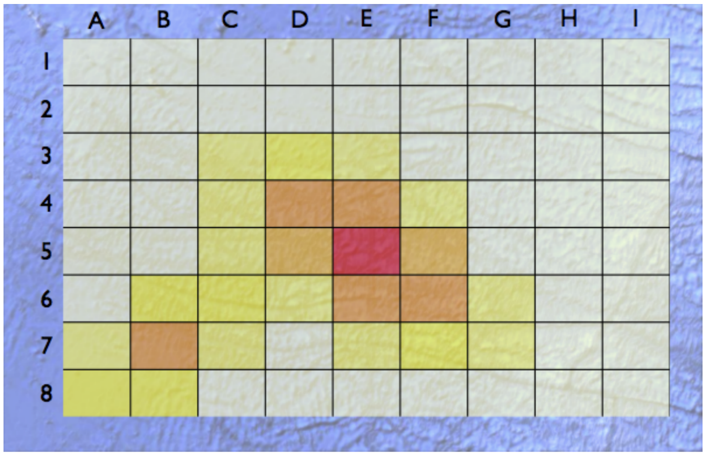

Image constructed by Jared Murray.

5. Next, construct a search path. This path starts with the locations with higher probabilities, but often not in exact order.

When searching these areas, a linear approach is often used. This is because it is simply not effective to search high-probability regions first- it saves more time to follow a linear path, as more locations are searched.

It is also important to note that although a region may have been previously searched, that does not mean other factors couldn't have prevented the object from being found. These factors include water depth and effectiveness of equipment.

6. Finally, throughout your search, apply Bayes' Theorem!

This theorem helps update the probability of finding an object in an area. A distinct quality of this search method, Bayesian search is highly iterative as the data collected would be updated throughout the search. As more areas are searched, the probability of all areas is adjusted so that each prior probability location is then turned into a posterior probability location. These areas are continuously revised, while the probabilities of other areas increase or decrease accordingly. For example, if you search through an area and don't find the bomb, the next day you search a different area, and the previous area's probability is reduced and your assessments of the regions change. This process occurs daily, with the next high-probability area then being searched.

And thus, with the application of this search process, the USS Scorpion had finally been found. With the discovery of this lost warhead, a definitive victory had been made with the concept of Bayesian search. The submarine's location was actually only a few hundred yards away from one of the initial high-probability regions marked on Craven's map at the first step of this process.

It was a brilliant accomplishment for both the team and the two hundred fifty year old Bayes' Theorem. The theorem's applications do not end here- it is heavily used in other fields today, with applications ranging from financial probability to machine learning. To conclude this essay, I hope you realize the importance of this seemingly-simple probability theorem and its significance in finding lost objects.

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