A monkey wrote this essay

Part 1: The literary abilities of primates

Suppose you were to give a monkey a typewriter, but why would anyone do that you might understandably be asking? Well in 2002 that's exactly what a group from the University of Plymouth did. They brought six Celebes crested macaques into an enclosure with a computer and left them to become the next Booker prize winners. However, after one and a half months with the keyboard, the monkeys had only produced 5 pages and had mostly spammed the 's' key. They had also spent a considerable amount of time hitting it with a rock and later began to urinate on it. A successful result if you ask me!

The experiment was actually a piece of performance art designed to critique the 'Infinite monkey theorem' which was first proposed by French mathematician Émile Borel in 1913, stating that if a monkey were to be given a typewriter and began to hit keys at random, after an infinite amount of time it will have written the entire works of William Shakespeare. And since this immortal monkey has been given an infinite amount of time and no hard deadline, it will have surely written every possible piece of finite text ever, an infinite number of times.

Of course this could not be done with a real monkey as Plymouth university have shown us. Instead, Borel's monkey is purely figurative, an example of something that can produce long sequences of random characters. But what is the proof that our metaphorical monkey will end up replicating the Bard?

Well, let's start with a short word first – 'maths'. A standard keyboard has 105 keys, but for simplicity I'm going to make the assumption that our monkey doesn't know any alt key codes so can only use 40 of the keys. This gives it access to the alphabet, all ten digits, the space key and some basic punctuation. Furthermore, assume that the chance of pressing down one key is the same as any other key and that each key press is independent of any other, for example if an 'r' is pressed down, it won't affect the probability of a 'p' being pressed.

The chance that the monkey will type an 'm' first is $\frac{1}{40}$, and then an 'a' is $\frac{1}{40}$ and then 't' is also $\frac{1}{40}$ etc. So the probability of the primate writing the word 'maths' is $\left(\frac{1}{40}\right)^5$. This is a one in 102,400,000 chance. The odds are certainly not in its favour as the probability that the 5 characters that it typed out do not spell out 'maths' is $1-\left(\frac{1}{40}\right)^5$ or 0.99999…

If we call each set of 5 characters typed by the monkey a 'block' Then the chance of not typing the word 'maths' in 'n' number of blocks is $\left(1-\left(\frac{1}{40}\right)^5\right)$. As 'n' increases, the probability that the word maths is not typed decreases. Therefore, as n approaches infinity, the probability that the monkey won't type the word 'maths', tends to zero. Inevitably the word will feature somewhere in the text produced, and the same applies to much larger chunks of text as well.

Despite the probability of a finite piece of text appearing in the infinitely long chain of characters tending towards one, as the length of the desired text increases, the chance of it occurring decreases exponentially. A common example used to demonstrate the infinite monkey theorem is the likelihood of the monkey writing Shakespeare's *Hamlet*, which has around 130,000 letters in it, making the probability of recreating it (ignoring punctuation and spaces) $\left(\frac{1}{26}\right)^{130000}$ which is a probability of one in three point four multiplied by ten to the power of 183,946. To put that into

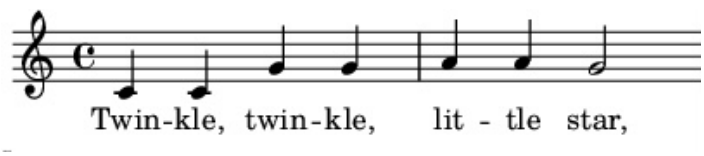
perspective, it is estimated that there are between only 10^{22} and 10^{24} stars in the observable universe and even more mind-blowingly, only between $10^{78} - 10^{82}$ atoms! It feels completely wrong to put the word '*only*' next to these staggeringly large numbers, but it demonstrates just how unlikely it is for specific pieces of writing to be generated completely randomly, let alone a play written by one of the most celebrated authors in our history. Furthermore, there is still a very small, non-zero probability that *Hamlet* could never appear if the monkey types an infinitely long string of characters instead, such as an irrational number or even a completely random repeating sequence of characters. There is no reason to say that it wouldn't just type the same character over and over again. However, this too becomes statistically improbable because $\frac{1}{40}$ to the power of infinity tends to zero. All in all, although the theorem shows that any text will eventually appear when an infinite string of characters is created, it is perhaps a little misleading as the probabilities are so inconceivably small. That being said, there is still the chance that this essay could have been written by our monkey friend, a one in 40^{7218} to be exact.

Part 2: The monkey switches to a different type of keyboard

The infinite monkey theorem refers only to typing, but could it be applied to any other situations. For example, could we turn our monkey into the next Mozart? Applying the theorem to a musical situation becomes more complicated as music has far more variables to consider. Instead of just the probability of the monkey pressing down a particular character on the keyboard, it must now press down a particular note for a particular amount of time, and we could also factor in timbre, dynamics, tempo and harmonies, all key features of a piece of music. But for the sake of simplicity let's just keep it to note duration and pitch. There are 88 notes on a standard piano and we'll limit this scenario to have only eight different note durations, from a breve (with a relative value of 2) to a hemidemisemiquaver (relative value of one sixty-fourth).

The eight different note durations

This means that the chance of the monkey playing a middle c on a piano, $\frac{1}{88}$, with a crotchet note value, $\frac{1}{8}$, is $\frac{1}{704}$ assuming that pitch and length are independent of one another. This is around a 0.14 percent chance! As we add more notes to the sequence, the probability of it being played randomly will decrease even further. For example, the first part of Twinkle, Twinkle, Little Star has seven notes, hence the probability of the monkey playing this tune is $\left(\frac{1}{704}\right)^7$ or a $1.17 * 10^{18}\%$ chance.



The first 7 notes of Twinkle Twinkle Little Star

However, like before if the monkey plays the piano for an infinite amount of time, then of course it will eventually get round to twinkle twinkle little star as $\left(1-\left(\frac{1}{704}\right)^{7}\right)^{n}$ will tend to zero as n, the number of sets of seven notes played, increases. The probability that we will hear Twinkle Twinkle Little Star will therefore approach one.

Ultimately the infinite monkey theorem doesn't have any practical uses, but it is interesting to look at these improbable probabilities and to see how it can be applied to other scenarios. It also can hopefully give some reassurance to musicians and authors that their creative output has meaning and it's unlikely that anyone else could create something so individual and unique – especially not an immortal monkey with a typewriter.

Bibliography:

http://news.bbc.co.uk/1/hi/3013959.stm
http://www.apstatsmonkey.com/StatsMonkey/TPS3e_files/TypingMonkeys.pdf
https://www.arttimesjournal.com/music/mathematical.htm
https://en.wikipedia.org/wiki/Twinkle,_Twinkle,_Little_Star