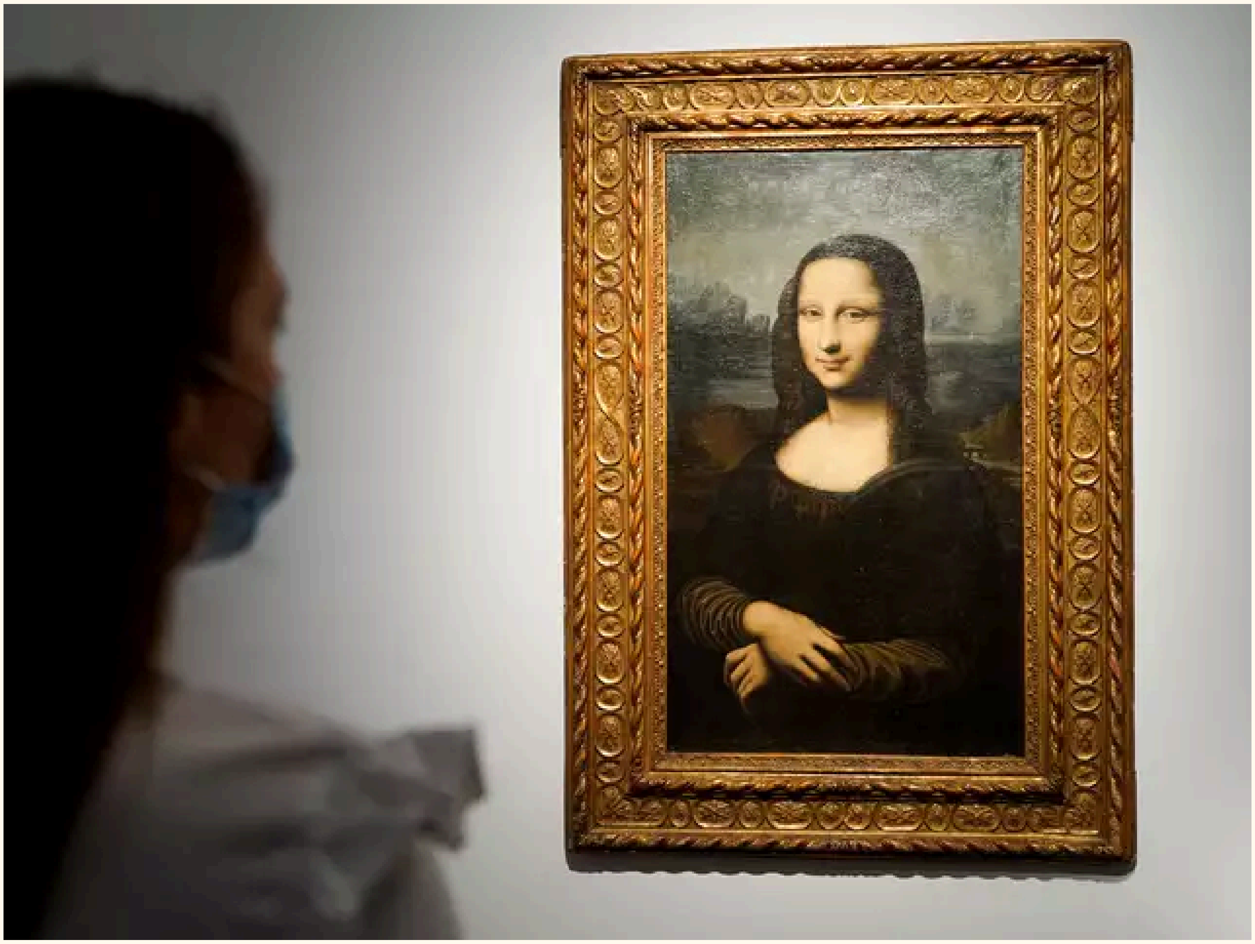
TOM ROCKS MATHS ESSAY COMPETITION

SPIRALS OF SYMMETRY

Mathematics and the Mona Lisa's timeless charm

By Vidushi Arora



INTRODUCTION

Mathematics and art, two seemingly disparate fields, have an unexpected yet profound intersection that has intrigued scholars for centuries. Mathematics has itself been described as an art motivated by beauty and has inspired renowned artists such as Leonardo da Vinci , Max Ernst , Man Ray .

From the intricate patterns of Islamic geometric art to the harmonious proportions of Greek sculptures, mathematics has served as a silent architect, shaping the aesthetics of human creativity .

One such iconic artwork that embodies this symbiotic relationship is Leonardo da Vinci's enigmatic masterpiece, the Mona Lisa. Within this timeless portrait lies a fascinating connection to the mathematical concept of the golden spiral, a phenomenon that underscores the inherent beauty and harmony of both art and mathematics.

In this essay, we will delve into the intriguing relationship between mathematics and art, exploring how the golden spiral manifests within the Mona Lisa.

THE SECRET TECHNIQUE USED BY DA VINCI FOR MONA LISA

Leonardo considered himself more a scientist than an artist . He saw everything through the eyes of mathematics including his paintings.The Mona Lisa, arguably the most iconic painting in the world, has long been a subject of scholarly inquiry and artistic fascination.

The Mona Lisa has been described as "the best known, the most visited, the most written about, the most sung about, and the most parodied work of art in the world". Scholars believe that The Mona Lisa shows evidence that da Vinci may have used a golden spiral in many of his paintings.

The Golden Spiral is simple mathematics beginning with the Golden Ratio, expanding to Golden Rectangles, and then by drawing curves through the corners of the Golden Rectangles to reflect the Golden Spiral.

But what exactly is the golden spiral ? Well to answer this question , We first need to be familiar with the golden ratio ,the golden rectangle and the Fibonacci Sequence.

THE GOLDEN RATIO

What is The Golden Ratio ?

The golden ratio – also called the divine proportion – is a true magic formula that has been seducing the eyes for centuries.The Golden Ratio is a design concept based on using the Fibonacci sequence (I will explain this in greater detail shortly in the essay) to create visually appealing proportions in art, architecture, and graphic design.
The proportion, size and placement of one element compared to another creates a sense of harmony that our subconscious mind is attracted to.

In simple words , It is the ratio of a line segment cut into two pieces of different lengths such that the ratio of the whole segment to that of the longer segment is equal to the ratio of the longer segment to the shorter segment.

The origin of this number can be traced back to Euclid, who mentions it as the “extreme and mean ratio” in the *Elements* (his treatise on geometry and mathematics)

How is the golden ratio expressed in mathematics ?

In mathematics two quantities for example a and b (where $a > b > 0$) are in the golden ratio if their ratio (a : b) is the same as the ratio of their sum to the larger of the two quantities [(a+b) : a] . The golden ratio is denoted by the letter φ (phi)

$$\frac{a+b}{a} = \frac{a}{b} = \varphi$$

One method for finding a closed form for φ starts with the left fraction. Simplifying the fraction and substituting the reciprocal $b/a = 1/\varphi$,

$$\frac{a+b}{a} = \frac{a}{a} + \frac{b}{a} = 1 + \frac{b}{a} = 1 + \frac{1}{\varphi}.$$

Therefore ,

$$1 + \frac{1}{\varphi} = \varphi.$$

Multiplying by φ gives,

$$\varphi + 1 = \varphi^2$$

Which can be rearranged to

$$\varphi^2 - \varphi - 1 = 0.$$

The constant φ satisfies the quadratic equation $\varphi^2 = \varphi + 1$

And yields two solutions

$$\frac{1+\sqrt{5}}{2} = 1.618033\ldots \text{ and } \frac{1-\sqrt{5}}{2} = -0.618033\ldots.$$

Because φ is a ratio between positive quantities , it has to be the positive root which is the irrational number with a value of

$$\varphi = \frac{1+\sqrt{5}}{2} = 1.618033988749\ldots.$$

This mathematical formula is considered by some as a universal rule of beauty. The golden ratio in art creates a balanced relationship that the mind's eye loves. More precisely, it is about obtaining a precise ratio between the different parts of a work, an image, or an object.

But we still need to understand how Leonardo could have used this formula in his painting .The Greeks had observed that the golden ratio provided the most aesthetically pleasing proportion of sides of a rectangle. Leonardo did in fact use the golden rectangle to paint the Mona Lisa.

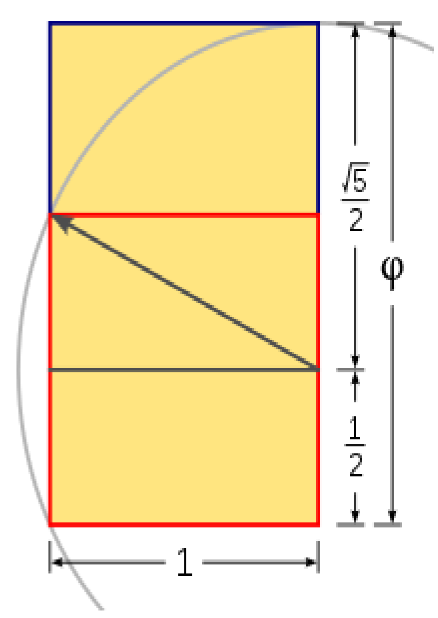
THE GOLDEN RECTANGLE

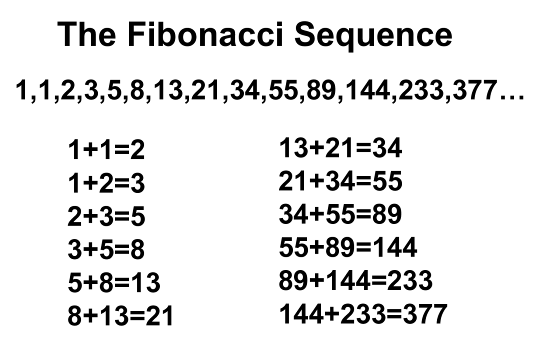
In geometry, a golden rectangle is a rectangle whose side lengths are in the golden ratio $1 : \frac{1+\sqrt{5}}{2}$, which is 1 : φ where φ is approximately 1.618 .

How can we construct The Golden Rectangle ?

A golden rectangle can be constructed with only a straightedge (a ruler) and compass in four steps:

1. Draw a square
2. Draw a line from the midpoint of one side of the square to an opposite corner
3. Use that line as the radius to draw an arc that defines the height of the rectangle
4. Complete the golden rectangle



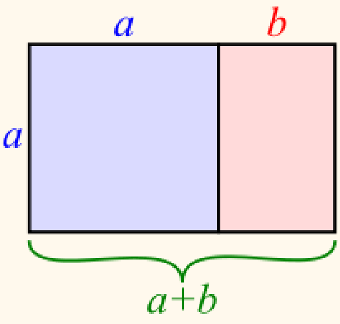


The Fibonacci Sequence

1,1,2,3,5,8,13,21,34,55,89,144,233,377…

1+1=2
1+2=3
2+3=5
3+5=8
5+8=13
8+13=21

13+21=34
21+34=55
34+55=89
55+89=144
89+144=233
144+233=377



The diagonal dividing one half of a square equals the radius of a circle whose outermost point is also the corner of a golden rectangle added to the square.A golden rectangle with sides ***a* , *b* *placed*** adjacent to a square with sides of length ***a*** produces a similar golden rectangle.

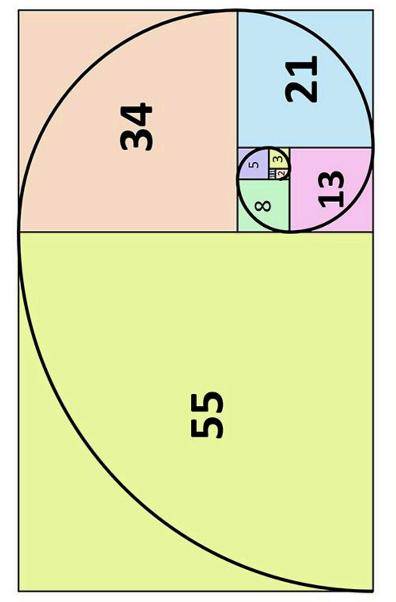
A distinctive feature of this shape is that when a square section is added—or removed—the product is another golden rectangle, having the same aspect ratio as the first. Square addition or removal can be repeated infinitely, in which case corresponding corners of the squares form an infinite sequence of points .

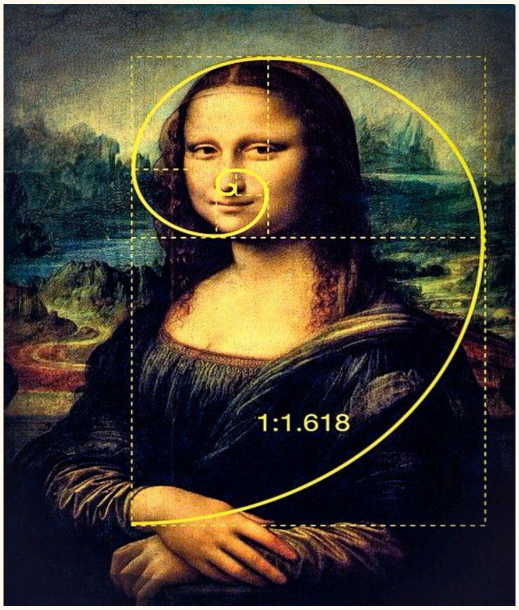
THE FIBONACCI SEQUENCE and THE GOLDEN SPIRAL

Now that we have understood what the golden ratio and the golden rectangle is , we need to understand The fibonacci sequence in order to be able to recognise the golden rectangle that borders the spiral .

The Fibonacci Sequence is defined as: a sequence of numbers in which each number equals the sum of the two preceding numbers.

If you add the two shortest lines of a work, they must be the same size as the third line. This third line will then show the proportions of the golden ratio.



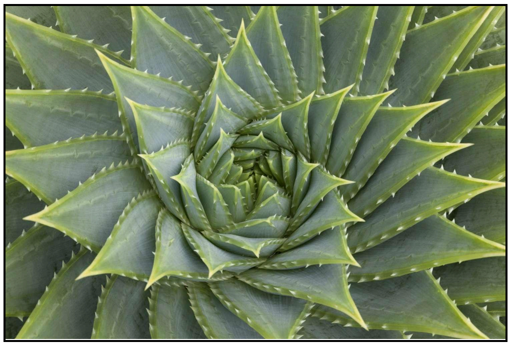


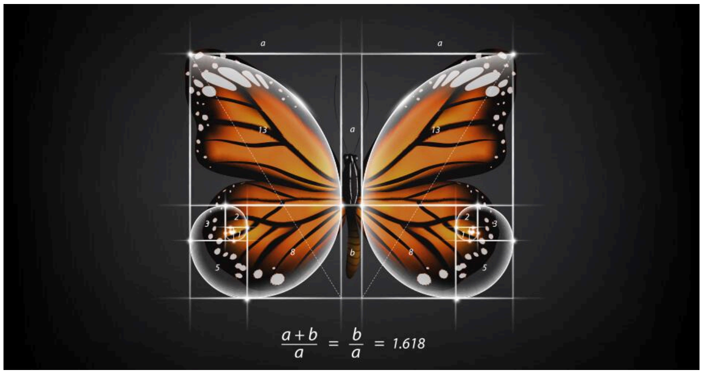
The Golden Rectangle is formed by creating rectangles within the corresponding dimensions of 1.618, from each descending Fibonacci Number (8, 5, 3, 2, 1, etc.) .The spiral comes from touching each side in the Perfect Rectangle.

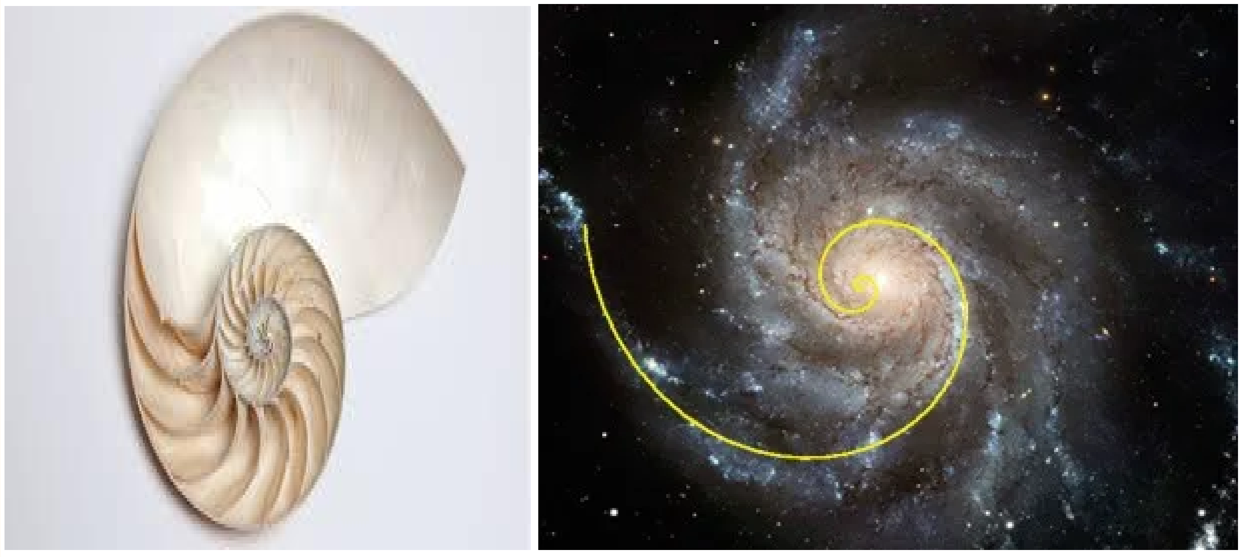
First, Leonardo uses the rectangle to frame Mona Lisa in the painting. The spiral begins at her left wrist then travels to the background of the image, which contrasts the beauty of her face. It then skims over her forehead and continues turning until it kisses her chin. It rises, going past the slight of her dimple. Lastly, it completes one rotation which ends at the tip of her nose.

HISTORY OF THE GOLDEN SPIRAL

The Golden Spiral was first discovered by Pythagoras in the 5th century B.C. as a mathematical expression but it has been recognised by man in nature for 4000 years and has been around since the beginning of time. It can be seen in galaxies, hurricanes, shells, sunflowers and all aspects of nature including DNA.







We don’t know for sure if Leonardo da Vinci did this on purpose, but we do know that Leonardo was close friends with Luca Pacioli, the author of *On Divine Proportion* (a treatise on the golden ratio). We also know that Leonardo believed that mathematics was a foundational aspect of all life and work, including works of art.

WRAPPING - UP

In conclusion, the exploration of mathematics in art, as exemplified by the Mona Lisa and the Golden Spiral, unveils a profound synthesis of rationality and creativity, precision and beauty .

In Mona Lisa, the subtle alignment of her features with the proportions of the Golden Spiral underscores the meticulous attention to detail and the intuitive grasp of mathematical principles that characterize da Vinci's genius.

Moreover, the study of mathematics in art challenges us to reconsider our perceptions of beauty and to appreciate the underlying order that governs the universe, from the smallest spiral of a seashell to the grandeur of a galaxy.

It invites us to see the world with new eyes and to appreciate the beauty that surrounds us, not only in galleries and museums but in the patterns of nature and the rhythms of existence.

As we contemplate the enigmatic smile of the Mona Lisa and the graceful curves of the Golden Spiral, we are reminded of the boundless potential of human imagination and the constant presence of rationality and math in everything around us.