# ***ALL ABOUT TWEEN PRIME & PRIME NUMBER HISTORY***

***Abstract:*** In this article I talk about my own conjecture that Any integer n greater than 7 there exists at least one tween prime between n and 2n.and the history of primes and about tween prime conjecture.

## ***Topics***

1. ***A History of Prime number***
2. ***Fundamental Theorem of Arithmetic***
3. ***Infinitude of Primes***
4. ***Euler Polynomials***
5. ***Prime number theorem***
6. ***Some History about Twin Prime Conjecture***
7. ***Gaps Between tween Primes***
8. ***My Conjecture***
9. ***Conclusion***

## ***Introduction:***

***A History of Prime number:*** Number theory is the mathematical study of the natural numbers, the positive whole numbers such as 2, 1729,7,123576547566 and 5. A primary focus of number theory is the study of prime numbers, which can be viewed as the elementary building blocks of all numbers.

The natural numbers are those basic abstraction of quantity we first learn about as children; in particular they are the positive whole numbers:

$$\mathbb{N} = \{1,2,3,4,5,6,7,8,9, \dots\}$$

Prime numbers can be thought of as the building blocks of all natural numbers.

***Definition 1.*** natural number larger than 1 is called prime if it can be evenly divided only by 1 and itself; other natural numbers greater than 1 are called composite.

To give intuitive description, we might think of a set of objects and ask whether we can divide the set of objects into several small sets, each with an equal number of objects. Depending on the number of objects in the original set, we might be able to divide the set in several ways, or in no ways at all. If we have

20 chocolates in a box, we might distribute them into 4 sets of 5, or perhaps 2 sets of 10. If there are 25 chocolates in the box, then the only way we can evenly divide them is by separating them into five sets of 5 chocolates each. For some sets, though, there is no way at all divide them into separate sets. For example, if there are 13 chocolates in the box, there is no way to evenly divide them, except if want to divide them into 13 "sets" with one person each. Natural numbers which cannot be "broken up" into products of smaller natural numbers greater than 1 are called prime. In some sense, the prime numbers 2, 3, 5, etc. are the "atoms" that make up all natural numbers, in the way that hydrogen, helium, lithium, etc. make up the matter of our universe. Unlike the periodic table of the elements, however, the list of prime numbers goes on indefinitely.

***Fundamental Theorem of Arithmetic:*** Essential to everything discussed herein—in fact, essential to every aspect of number theory—is the notion of $a$ prime number. We know that any integer $a > 1$ is divisible by 1 and $a$; if these exhaust the divisors of a, then it is said to be a prime number. In Definition 2 we state this somewhat differently.

***Definition 2***. An integer $p$ is called a prime number, or simply a prime, if its only positive divisors are 1 and $p$. An integer greater than 1 that is not a prime is termed composite.

Among the first 100 positive integers, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97 are primes and others are composite numbers. Note that the integer 2 is the only even prime, and according to our definition the integer 1 plays a special role, being neither prime nor composite.

Proposition 14 of Book IX of Euclid's Elements embodies the result that later became known as the Fundamental Theorem of Arithmetic, namely, that every integer greater than 1 can, except for the order of the factors, be represented as a product of primes in one and only one way. To quote the proposition itself: "If a number be the least that is measured by prime numbers, it will not be measured by any other prime except those originally measuring it." Because every number $a > 1$is either a prime or, by the Fundamental Theorem, can be broken down into unique prime factors and no further, the primes serve as the building blocks from which all other integers can be made. Accordingly, the prime numbers have intrigued mathematicians through the ages, and although a number of remarkable theorems relating to their distribution in the sequence of positive integers have been proved, even more remarkable is what remains unproved. The open questions can be counted among the outstanding unsolved problems in all of mathematics.

***Infinitude of Primes***: Euclid of Alexandria was a Greek mathematician who lived several centuries before the common era. Aside from his many contributions to geometry, Euclid also made important contributions to our understanding of numbers. In particular, Euclid showed that for any finite number n, there are more than n prime numbers. This is equivalent to what we would say, "There are infinitely many primes". Euclid proved this by showing that if we take any finite set of prime numbers, we can always find another prime number that is not in that set.

***Theorem 2:*** There are infinitely many prime numbers

***Euler Polynomials:*** Euler himself, one of the most prolific mathematicians of all time, suggested another way of generating prime numbers using a polynomial. For our purposes, we can think of polynomials as expressions that look like $ax^2 + bx + c$; polynomials with higher powers, and with more than one variable, are also possible. Euler suggested the polynomial

$$f(n) = n^2 - n + 41$$

which gives prime numbers for natural numbers $n = 1$ through $n = 40$. The first few primes that result from this equation are 41, 43, 47, 53, 61, and 71. The fact that for the first 40 natural numbers we obtain a prime number might suggest that this pattern is true for all natural numbers. Indeed, if you were to conduct a scientific experiment to test a hypothesis, and the first 40 trials confirmed your hypothesis, you might conclude that indeed your hypothesis is correct, and that subsequent experiments should yield the same results. However, it is straightforward to see that if $n = 41$, then the given polynomial will result in a composite number, since $41^2 - 41 + 41 = 41^2 = 1681$, which of course is not a prime number. These examples again illustrate the care that must be taken when generalizing from particular examples, both in mathematics and more generally.

| $f(n)$ | $n$ | $f(n)$ | $n$ | $f(n)$ | $n$ |
|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| 41 | 0 | 61 | 4 | 113 | 8 |
| 43 | 1 | 71 | 5 | 131 | 9 |
| 47 | 2 | 83 | 6 | 151 | 10 |
| 53 | 3 | 97 | 7 | 173 | 11 |
| $f(n)$ | $n$ | $f(n)$ | $n$ | $f(n)$ | $n$ |
| 197 | 12 | 503 | 21 | 971 | 30 |
| 223 | 13 | 547 | 22 | 1033 | 31 |
| 251 | 14 | 593 | 23 | 1097 | 32 |
| 281 | 15 | 641 | 24 | 1163 | 33 |
| 313 | 16 | 691 | 25 | 1231 | 34 |
| 347 | 17 | 743 | 26 | 1301 | 35 |
| 383 | 18 | 797 | 27 | 1373 | 36 |
| 421 | 19 | 853 | 28 | 1447 | 37 |
| 461 | 20 | 911 | 29 | 1523 | 38 |

***Prime number theorem:*** The sequence of prime numbers, which begins 2, 3, 5, 7, 11, 13, 17, 19, 23,... has held untold fascination for mathematicians, both professionals and amateurs alike. The basic theorem which we shall discuss in this part of the article is known as the prime number theorem and allows one to predict, at least in gross terms, the way in which the primes are distributed. Let $x$ be a positive real number, and let $\pi(x)$ = the number of primes.

$$\lim_{n\to\infty} \frac{\pi(x)}{\frac{x}{logx}} = 1$$

The first published statement which came close to the prime number theorem was due to Legendre in 1798. He asserted that $\pi(x)$ is of the form $\frac{x}{B+Alogx}$ for constants A and B. On the basis of numerical work, Legendre refined his conjecture in 1808 by asserting that

$$\pi(x) = \frac{x}{A(x) + logx}$$

where A(x) is "approximately 1.08366". Presumably, by this latter statement, Legendre meant that

$$\lim_{n\to\infty} A(x) \approx 1.08366$$

Gauss had already done extensive work on the theory of primes in 1792-3. Evidently Gauss considered the tabulation of primes as some sort of pastime and amused himself by compiling extensive tables on how the primes distribute themselves in various intervals of length 1009. Each entry in the table represents an interval of length 1000. Thus, for example, there are 168 primes from 1 to 1000; 135 from 1001 to 2000; 127 from 3001 to 4000; and so forth. Gauss suspected that the density with which primes occured in the neighborhood of the integer n was 1 /log n, so that the number of primes in the interval [a, b) should be approximately equal to

$$\int_a^b \frac{dx}{logx}$$

In the second set of tables, samples from, Gauss investigates the distribution of primes up to 3,000,000 and compares the number of primes found with the above integral. The agreement is striking. For example, between 2,600,000 and 2,700,000, Gauss found 6762 primes, whereas

$$\int_{2,600,000}^{2,700,000} \frac{dx}{logx} \approx 6762$$

Gauss never published his investigations on the distribution of primes. Nevertheless, there is little reason to doubt Gauss' claim that he first undertook his work in 1792-93, well before the memoir of Legendre was written. Indeed, there are several other known examples of results of the first rank which Gauss proved, but never communicated to anyone until years after the original work had been done. This was the case, for example, with the elliptic functions, where Gauss preceded Jacobi, and with Riemannian geometry, where Gauss anticipated Riemann. The only information beyond Gauss' tables concerning Gauss' work in the distribution of primes is contained in an 1849 letter to the astronomer Encke. We have included a translation of Gauss' letter.

In his letter Gauss describes his numerical experiments and his conjecture concerning $\pi(x)$. There are a number of remarkable features of Gauss' letter. On the second page of the letter, Gauss compares his approximation to $\pi(x)$., namely $Li(x) = \int_2^x \frac{dx}{logx}$, with Legendre's formula. The results are tabulated at the top of the second page and Gauss' formula yields a much larger numerical error. In a very prescient statement, Gauss defends his formula by noting that although Legendre's formula yields a smaller error, the rate of increase of Legendre's error term is much greater than his own. We shall see below that Gauss anticipated what is known as the "Riemann hypothesis." Another feature of Gauss' letter is that he casts doubt on Legendre's assertion about A(x). He asserts that the numerical evidence does not support any conjecture about the limiting value of A(x).

Gauss' calculations are awesome to contemplate, since they were done long before the days of high-speed computers. Gauss' persistence is most impressive. However, Gauss' tables are not error-free.

After that Dirichlet's work contained two radically new ideas, which we should not discuss in here.

***Some History about Twin Prime Conjecture:*** There are many questions concerning primes which have resisted all assaults for two centuries and more. The set (3, 5), (5, 7), (11, 13), ... of twin prime pairs (p, p+ 2) has been studied by Brun (1919), Hardy and Littlewood (1922), Selmer (1942), Froberg (1961), Weintraub (1973), Bohman (1973), Shanks and Wrench (1974), and Brent (1975, 1976). Currently M. Kutrib and J. Richstein (1995) are completing a study similar to the present one.[16] Two of the oldest problems in the theory of numbers, and indeed in the whole of mathematics, are the so-called twin prime conjecture and binary Goldbach problem.

***Gaps Between tween Primes:*** A statement equivalent to the Prime Number Theorem is that an integer near x has a 1/ log x "probability" of being prime. If we wish to know the probability that p and p+2 are both prime, where p is near x, we might simply multiply the probability that each is prime individually to get a probability of $1/\log^2 x$. We could then "sum" over all primes up to x to conjecture that

$$\pi_2(n) \sim li_2(n) = \int_2^n \frac{dx}{\log^2 x}$$

There is one crucial flaw with this argument, however, namely that p and p + 2 being prime are not independent events. Consider, for example, that if p > 2 is prime, then p is odd, and therefore p + 2 is also odd, giving it an immediate leg up on being prime. Further, in order to be prime, p + 2 must not be divisible by any odd prime q. A random integer has a (1−1/q) chance of being not divisible by q, but if q ∤ p, then p+ 2 must fall into one of (q −2) out of (q − 1) remaining residue classes in order to be not divisible by q, and therefore we would expect that p + 2 has a (q − 2)/(q − 1) chance of being not divisible by q. If we assume that the primes are randomly distributed, we should then add a correction factor of $\frac{p/(p-1)}{(p-1)/(p-2)}$.

for each odd prime to the above approximation. Let us then amend our guess to

$$\pi_2(n) \sim 2 \prod_{\substack{p>2 \\ p\ prime}} \frac{p(p-2)}{(p-1)^2} li_2(n) = 2\alpha li_2(n)$$

where α is called the Twin Prime Constant, and $\alpha \approx 0.66016181 \ldots$

The estimate of $\pi_2(n)$has been refined by a determination of the constant and of the size of the error. This was done, among others, by Bombieri and Davenport, in 1966.

***My Conjecture:*** Any integer n greater than 7 there exists at least one tween prime between n and 2n.

***Conclusion***: There are lots of values that satisfy my conjecture! I thin my conjecture is true and it help us to proof tween prime conjecture but I am not sure at all because I'm not a professional mathematician. But I verified this statement for all integers less than 100,000.