Love letter to leftovers

My dear friend,

is right always right?
Is the right side right?
Is looking to the right side of life always the bright side of life - and automatically the right sight?
I look to the left side: Are the leftovers not on the right side because they are left over? We should not leave them lying around, they are loveable, important, useful and valuable.
I see all the leftovers, that have ever played a role in my life. How cute. I would like to hug you all. But you are so many, the mount grows every minute, every second. It grows and grows. I lie buried under the mountain…

Dingdong - that's my alarm clock. Oh, I had a dream.
What remains of my dream?
It's 6:30. Fortunately, the same procedure as every day. Imagine if all the people counted the hours individually. Then there would be around 528768 hours on my watch, the number of hours since I was born. How could I recognize when I have to get up? I'm glad that we count the hours anew every day. And in the same way for everyone (in the same time zone).
Observing and recognizing that each day has a recurring pattern of brightness is not so far-fetched, but inventing a time system from it - formerly in the form of a sundial - is very useful. Sundials exist since the old Greek. Who actually came up with this ingenious idea? The fact that the units are now 24 hours and 60 minutes is for my taste not so practical. 6:30 - Why not 6.5? Imagine if we also calculated time using the tens system. 6:30 would be 25.5 o´clock. We'd have to make a big adjustment.

Remainder and modulo

6 o´clock is strictly speaking a remainder. A remainder if we divide the time by 12 (or 24 on my alarm clock), beginning at any midnight you want.
If we divide by 12 we say modulo 12 and write mod 12.
Equality of remainders is written with three stripes (looks very sporty, especially in Germany) or with two dashes and a hat: a wonderful notation to express how strongly they are connected.
We know for example that

$$3 \equiv 15 \bmod 12.$$

And 30 hours later instead of 3 + 30 = 33 o´clock we would use:

$$33 \equiv 9 \bmod 12.$$

Modulo, you are the most frequent application of mathematics in my everyday life!

How chaotic the world would be if modulo had not been invented. This is true for hours, days…
Days?

Calendar

Today is February 28th.
Tomorrow March starts? No, tomorrow we have February 29, because the year is divisible by four. If it were divisible by 100, we wouldn't have a leap year, but if it were divisible by 400, we would again. It can be useful to be good at dividing. Phew, I prefer to check the calendar and trust the mathematicians.

Easter formula

And I took the opportunity of the calendar check to find out when Easter is this year. On the first Sunday after the first full moon after the beginning of spring. I can't remember that. But the clever Gauss developed a formula for it:
First, divide the year Y by nineteen and call the remainder a.

$a \equiv Y \bmod 19$

2024 divided by 19 gives 106 and the remainder 10.
Then divide the year by 4 and call the remainder b.

$b \equiv Y \bmod 4$

2024 divided by 4 gives 506 and the remainder 0.
Next, divide the year by 7 and call the remainder c.

$c \equiv Y \bmod 7$

In this year, this results in 289 and remainder 1.
Now multiply a by 19 and add 24. Divide the result (here 214) by 30. The remainder is called d.

$d \equiv 19a + 24 \bmod 30$

In our case, the result is 4.
Then multiply b by 2, c by 4 and d by 6, add everything together and then add 5. Divide the result by 7 and call the remainder e.

$e \equiv 2b + 4c + 6d + 5 \bmod 7$

In this year, the remainder is 5.
Finally: If you now add the values d and e to 22, you get the March date of Easter Sunday.
Easter Sunday = (22 +d + e) March

This year: 22 plus 4 plus 5 equals March 31.
Of course, if you get for example March 35 your Easter Sunday is April 4.
(And there are a few exceptions for this formula, which we skip here.)

My age

If we couldn't think in circles, there would be no Easter, no Christmas and no birthday celebrations! I would say, I am 22032 days old, counted as in the young adult novel "Every Day", in which the protagonist - a person without gender - counts his life in days. Every day is both new and the same, because the person has to borrow a body for one day and remains the same at its core. The German title of the novel "Letztendlich sind wir dem Universum egal" (Ultimately, the universe doesn't care about us) shows even better that we are so insignificant, no matter what influence we imagine we have.

Place value system

When I write a number, I use the place value system modulo 10. My computer prefers the binary system and thinks in modulo 2, 2^2, 2^3 and so on. Place value systems are so much better to handle than the romian notation of number (I am luckily born in the right time).

Music

Where's the music coming from now? Ah, my radio is still playing. Listen to the music. The motif repeats itself an octave higher, now lower again. Thinking, singing and listening in circles (of fifths) is so beautiful.

Banking

Today I wanted to have a quick look to my bank account. Thanks to RSA, my transfer is securely encrypted. Guess what we need for a good encryption: Yes: again modulo!
And thanks to Fermat's little theorem.

$a \equiv a^{p-1} \bmod p$ for any prime and number a.

And to Fermat-Euler, which is a bit more general

$a \equiv a^{\varphi(n-1)} \bmod n$,

where ϕ(n) is the Euler phi-function, which counts the ….
Instead of explaining the RSA encryption here I show you some entertaining applications of modulo which are useful in invitations and parties.

Pull the fifth root in your head

If you want to impress the guests, you claim, you can pull the fifth root in your head within seconds.
A person in the audience starts with a two-digit number n (you do not see it).
For example the person chooses 68. n to the power of 5 is $n^5 = 68^5 = 1,453,933,568$.
Then you get to see only the result 1,453,933,568.

First you cut off the last 5 digits - in your head - and look to the remainder: 14539.
Sorry I forgot, in advance you had to learn by heart these powers of 5:

$1^5 = 1$
$2^5 = 32$
$3^5 = 243$
$4^5 = 1024$
$5^5 = 3125$
$6^5 = 7776$
$7^5 = 16807$
$8^5 = 32768$
$9^5 = 59049$

Now you look for the biggest number which is smaller than you remainder 14539:
This is $6^5 = 7776$.
Therefore your first digit of the root is 6.
Then you have to know:

$a \equiv a^5 \bmod 10$ for every digit a = 0, 1, …9.

That makes it easy: The last digit of n and the last digit of n^5 are the same.

So your result is 68. Very impressive!

Find your equal-number-friends

Another impressive gag is the following: you choose a group of p persons. Each of them gets a number k on the back (not seeing, not knowing the own number). The individual number k was randomly chosen by the audience between 0 and n (thus repetitions are allowed). There are n+1 tables (they are not numbered). The challenge: The p persons have the task to group together so that in every group at one table all members of them have the same number and the groups are complete. The persons are allowed to make up a strategy in advance, later (as soon as they have their numbers) they can see the other numbers, but are not allowed to speak.
For example: 5 persons A, B, C, D and E get numbers between 0 and 3: A3, B3, C0, D1, E0. There are four tables.
Is there a strategy better than guessing?

You wont´t believe it: yes. You can even bet, that they will succeed each time.

First in advance they give every table a number from 0 to n. (This number will not necessarily the number on the back of the group members.)
Every person looks at the numbers of the others. And add these numbers. The sum is s.
In our example they calculate:
A 4, B 4, C 7, D 6, E 7
Then calculate s mod n+1 and take the remainder which is between 0 and n. Then go to that table to which they had given the number in advance. There you will find only persons who have the same number on the back like you.
Thus A and B go to table number 0, C and E go to 3 and D goes to 2.

Why does this work?
All Persons with the same number on the back will calculate the same sum s, because they see the total sum of all numbers S minus their own number:
s = S – own number
is equal for all persons with the same number on the back.

Can two persons with a different number end up with the same sum (modulo n+1)?
The sums of two persons P and Q differ only by one summand (in the sum of P there is as a summand the number of Q and vice versa, the difference of the two sums is the difference of the numbers of P and Q.). This difference can be at most n. So P and Q have a different result modulo n+1 as soon as their numbers on the back are different.

Acknowledgements

Dear remainders, you save my daily life, my money, my fun.

Without leftovers, the world would remain poor. I would be lost. Not only me…

In eternal love and gratitude,
your friend of numbers

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Acknowledgement: The last example can be read in “Raven´s Hat” by Jonas Peters and Nicolai Meinshausen.