

Basketball Kinematics: An insight into what makes shooters great

Intro:

Basketball shots require such skill, precision, and (surprisingly) physics. What separates the great shooters from the rest? Although it may be instinctive to them, every amazing shot is controlled by key factors. What makes the perfect swish? Is it the launch angle? Velocity? Spin? We will be using mathematics and physics to see if these factors have an influence and to what extent.

Base Units used

m Represents meters – unit for distance or displacement

s Represents seconds- unit for time

kg Represents kilograms – unit for mass

$m.s^{-1}$ Represents meters per second – unit for velocity

$m.s^{-2}$ Represents meters per seconds² – unit for acceleration

N Represents newtons – Unit for force

radians and degrees ($^{\circ}$) are the units for angles

The Arc - The Shot



First of all, what is happening in a shot?

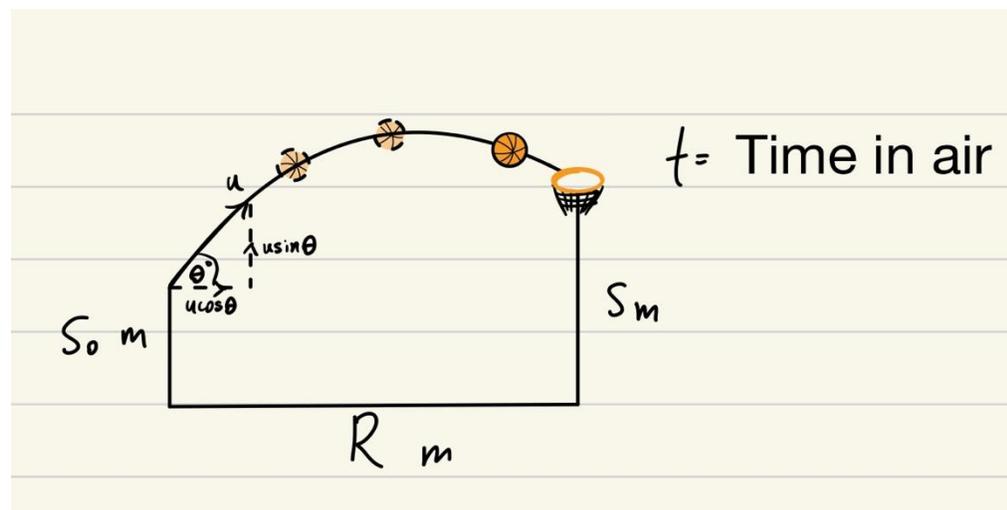
Let's analyze the energy transfers in Steph Curry's shot (the greatest shot of all time). In the first frame, Steph Curry has his legs loaded and bent like a spring. The chemical energy in his body converts to kinetic energy in his muscles as he extends his legs. His arms and legs rise together to maximize energy efficiency. As he releases the ball, the wrist flick creates backspin, which stabilizes the flight and softens its bounce on the rim. As the ball rises, kinetic energy (the only energy put into the system due to the conservation of energy) converts to gravitational potential energy ($GPE = mg\Delta h$), with minor losses to heat and sound. The velocity is 0 at the peak, and it then begins accelerating downward due to gravity, regaining kinetic energy ($KE = \frac{1}{2}mv^2$). By looking at this equation, we see the main variable that affects the energy needed to shoot is the speed of the ball's projection. Obviously as a player you do not want to waste unnecessary energy. Using this knowledge, we will investigate and identify the initial velocities and angles to shoot the ball.

General equation/model

To start with, let us look at the physics of a thrown ball in general. The ball is a projectile moving through the air. This means, that the classic SUVAT (displacement, velocity, acceleration and time) equations, can be used as tools to determine the speed at which the ball needs to be projected at a certain angle.

Projectile motion tells us that the horizontal and vertical components of an object's speed are independent. We use the basic kinematic equations to find the time taken to reach the hoop and the launch speed.

Firstly, let's produce a general equation to find the optimum angle that gives us the minimum speed. A general diagram is shown below:



The initial velocity ' u ' is split up into its horizontal and vertical velocities $u \cos \theta$ and $u \sin \theta$

Some assumptions –

- The player shoots the ball in a perfectly straight line towards the basket.

- Negligible air resistance.
- Negligible spin.
- The ball will pass perfectly through the center of the hoop.
- The player's shot timing is consistent.

Now for some equations!

Horizontal component:

$$R = u_x t \quad \therefore \quad t = \frac{R}{u_x}, \quad u_x = u \cos \theta \quad \therefore \quad t = \frac{R}{u \cos \theta}$$

\therefore Represents therefore, showing a logical step/process

In this equation, ' u_x ' is the initial horizontal velocity of the ball (the subscript 'x' here shows that it acts in the horizontal direction or along x-axis), 'R' is the horizontal displacement (range) and 't' is the time taken for the ball to hit the hoop. Due to Newton's first law, we know the horizontal velocity experiences no acceleration and will be constant throughout the journey. The ball only comes to a stop due to the vertical acceleration.

Vertical component:

$$s - s_0 = u_y t + \frac{1}{2} a t^2, \quad a = -g, \quad t = t, \quad u_y = u \sin \theta, \quad s - s_0 = \Delta s \quad \therefore$$

$$\Delta s = u t \sin \theta - \frac{1}{2} g t^2$$

In this formula, 's' represents the final displacement, so the height of the hoop in this scenario, while 's₀' is the initial displacement (the shot release height). This means 's-s₀' or ' Δs ' is telling us about the change in vertical displacement of the ball. For these equations, 'u' represents the initial velocity of the ball (subscript 'y' shows it's the vertical velocity) and 'a' is the acceleration. By observing the diagram, we know.

$$\Delta s = u t \sin \theta - \frac{1}{2} g t^2$$

Substitute 't' from the horizontal component into this equation to get

$$\Delta s = u \sin \theta \times \frac{R}{u \cos \theta} - \frac{1}{2} g \left(\frac{R}{u \cos \theta} \right)^2$$

$$\Delta s = R \tan \theta - \frac{R^2 g}{2u^2(\cos \theta)^2}$$

$$\frac{R^2 g}{2u^2(\cos \theta)^2} = R \tan \theta - \Delta s \quad \therefore \quad \frac{R^2 g}{u^2(\cos \theta)^2} = 2(R \tan \theta - \Delta s)$$

$$R^2 g = 2(u^2(\cos \theta)^2)(R \tan \theta - \Delta s)$$

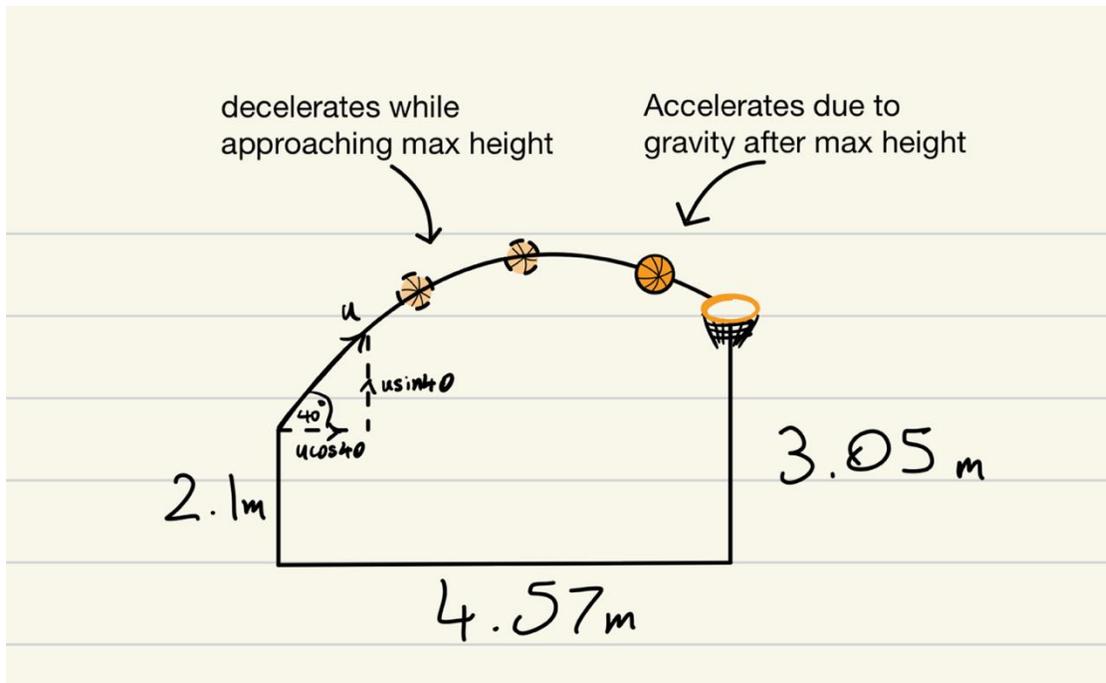
$$u^2 = \frac{R^2 g}{2(\cos \theta)^2 (R \tan \theta - \Delta s)} \quad \therefore \quad u = \sqrt{\frac{R^2 g}{2(\cos \theta)^2 (R \tan \theta - \Delta s)}} \quad \{0 < x < 90^\circ\}$$

I have restricted the domain between $\{0 \text{ and } 90^\circ\}$. We restrict the domain as the ball will only be shot between 0 and 90 degrees. For the players shooting at a 140-degree angle, say goodbye to your NBA dreams! Firstly, the horizontal displacement/range is represented as 'R'. Secondly, the ' Δs ' in our equation would just be the height of the hoop, s (3.048 m) minus the height of the shot release s_0 (the vertical difference in height). By putting all the numbers in the model, you can create a graph that plots the required speed to project the ball at different angles from different distances.

The Speed of a Free Throw:

The free throw is a free shot taken at the line when a player is fouled during a shot attempt – It is just you and the basket. Steph Curry has an insane 91.0% free throw percentage

By exploring different angles, the perfect speed for the perfect shot can be identified. When shooting a free throw, the player stands **4.57 m** (or 15 feet) away from the hoop. The hoop is **3.05 m** (or 10 feet) tall. Let's assume the average non-6-foot player releases the ball **2.1 m** (including the jump) above the ground.



For **vertical motion**, we can use the SUVAT equation $s - s_0 = u_y t + \frac{1}{2} a t^2$

$$u_y = u \sin 40 \text{ ms}^{-1}, \quad s = 3.05 \text{ m}, \quad s_0 = 2.1 \text{ m}, \quad a = -g \text{ ms}^{-2}$$

Putting these numbers in shows:

$$3.05 - 2.1 = u \sin 40 \times t + \frac{1}{2} \times -g \times t^2$$

$0.95 = u \sin 40 \times t - \frac{1}{2}gt^2$: We have an equation linking the initial velocity and time

Now the **horizontal motion** will be found by using a simple speed=distance/time

$u_x = \frac{R}{t}$ Again, from the diagram we know:

$u_x = u \cos 40 \text{ ms}^{-1}$, $R = 4.57 \text{ m}$, By substituting these values, we get

$$u \cos 40 = \frac{4.57}{t} \quad \therefore \quad u = \frac{4.57}{t \cos 40}$$

'u' Can be substituted into the vertical component as it is the same in both equations. Doing this and solving simultaneously provides us with:

$$0.95 = t \sin 40 \times \frac{4.57}{t \cos 40} - \frac{1}{2}gt^2$$

$$\frac{1}{2}gt^2 = \frac{4.57 \sin 40}{\cos 40} - 0.95$$

$$t^2 = \left(\frac{4.57 \sin 40}{\cos 40} - 0.95 \right) \div \frac{1}{2}g \quad \therefore \quad t^2 = 0.588 \text{ s} \quad \therefore \quad t = 0.77 \text{ s}$$

Substitute this back into our equation ' $u = \frac{4.57}{t \cos 40}$ ' to find that

$u = 7.78 \text{ ms}^{-1}$ for a **40°** projection

A **40-degree** projection is calculated first as shots in games with an angle less than **40** are considered too flat. The range of **40 to 60** degrees has been selected as the majority of shots are shot between 40 and 60 degrees.

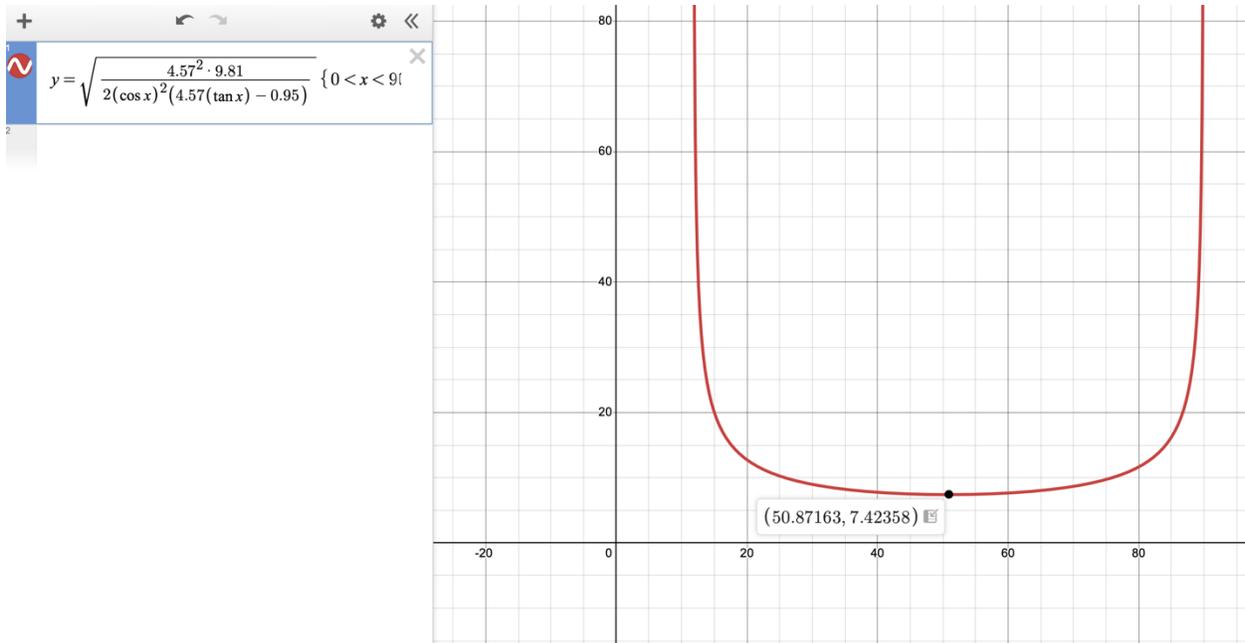
$$\text{When } \theta = 45, \quad t = 0.86 \text{ s}, \quad u = 7.52 \text{ ms}^{-1}$$

$$\text{When } \theta = 50, \quad t = 0.96 \text{ s}, \quad u = 7.43 \text{ ms}^{-1}$$

$$\text{When } \theta = 55, \quad t = 1.07 \text{ s}, \quad u = 7.47 \text{ ms}^{-1}$$

$$\text{When } \theta = 60, \quad t = 1.19 \text{ s}, \quad u = 7.67 \text{ ms}^{-1}$$

interesting, right? The data shows it requires less speed to project the ball at an angle of around **50** degrees. Now we have found the speeds required to shoot the ball at angles between 40 and 60 degrees. However, this isn't as mathematically accurate as we would like. We mathematicians love to be as precise and exact as possible. Using the model made earlier, we can find the exact angle that demands the lowest speed of projection. Our range in this situation is **4.57 m** while 's' is **3.05 m** and 's0' is **2.1 m** Subbing these into the model and graphing, it gives us:



After using the formula, we derived at the start, we can see, the angle of release that demands the least release velocity of **7.42 m/s** is an angle of **50.87 degrees!!** We have now concluded the angle of **50.87** degrees is the optimum for a free throw. Thus, showing the launch angle does have a significant role in shots. It decides how easy it is to shoot the ball.

But wait, why exactly is the lower speed optimal. Let's use our optimum angle's speed to do some calculations.

$$KE = \frac{1}{2}mv^2, \quad m = 0.6 \text{ kg}, \quad v = 7.42 \text{ ms}^{-1}$$

$$KE = \frac{1}{2} \times 0.6 \times 7.42^2, \quad KE = 16.52 \text{ J}$$

This doesn't require too much energy, displaying why shooting is not the most tiring part of the game. I am sure you all would rather shoot 100 shots than run 10 laps.

Let us also show that a lower speed results in less force needing to be exerted

$$p = mv$$

'p' is momentum, 'm' is mass and 'v' is velocity. Momentum is how difficult it is to stop or move a moving object. An object with greater mass and velocity requires more momentum and is more difficult to move or stop.

A lower velocity results in less momentum needing to be transferred (since the mass of the ball is constant) for the shot. We can now use Newton's second law to find the force needed to launch the ball, $F = ma$, Force is the mass multiplied by the acceleration of an object,

We know the acceleration of an object is the change in velocity over time $a = \frac{v-u}{\Delta t}$, so we can substitute this into Newton's second law to get

$$F = \frac{mv - mu}{\Delta t}, \text{ we know } p = mv \text{ therefore this equation shows } F = \frac{\Delta p}{\Delta t}$$

This equation we have found is the impulse momentum theorem which shows that an impulse is the change in momentum over a period. In our scenario it is the change in momentum over the contact time with the ball. A lower velocity causes a lower change in momentum which then causes less force to be exerted (if we assume the contact time of the player and the ball is the same in every shot angle).

Using our initial example shown in the first diagram, we can then find the force required to shoot the ball at a **40 degree** launch angle, **2.10 m** from the hoop. We are assuming the ball starts at rest.

$u = 7.42 \text{ ms}^{-1}$ from earlier which will be 'v' in our momentum equation

$$'v' = 7.42 \text{ ms}^{-1}, \quad m = 0.6 \text{ kg} \quad \therefore \quad p = 4.45 \text{ kgms}^{-1}$$

The average contact time with the ball is **0.5 s** during the shot

So, by substituting in our values, we find that:

$$F = \frac{\Delta p}{\Delta t}, \quad F = \frac{4.45}{0.5} \quad \therefore \quad F = 8.90 \text{ N}$$

Which is roughly **0.91 kg** of force. Not much at all for the 6'7, 100 kg NBA Players. This shows that for a player whose shot timing is consistent, a lower shooting speed requires less force to be exerted for the perfect shot. The force needing to be exerted is also affected by many other factors such as the motion of the player. In some shots the player jumps backwards (known as a fadeaway) which would require a greater force to be exerted.

The 3 pointer:

The three-point line is the line around the court where any shot from behind it counts as 3 points if scored.

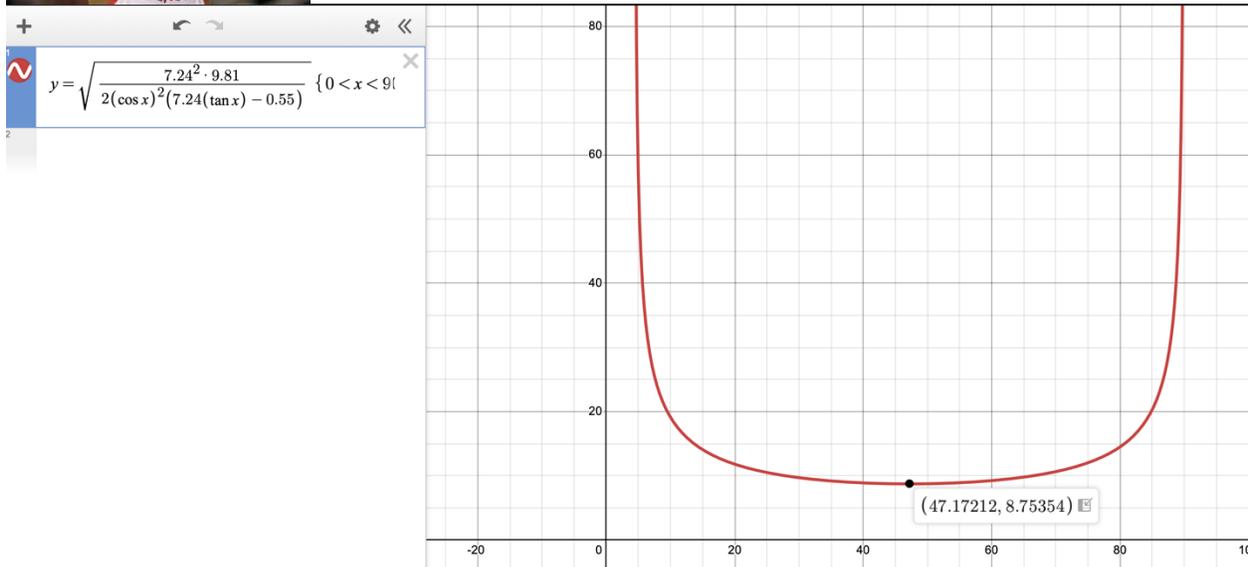
The range of a 3-point shot varies around the court. Directly in front of the hoop, the distance is the greatest. Whereas parallel to the hoop in the corner is the shortest distance. The reason for this is that there is no aid from the backboard, hence the shorter distance.

The largest range from the hoop to the NBA 3-point line is **7.24 m** from the basket. Extremely far! Despite this, players make around **40%** of these shots. How impressive!

Using our model, let's find the optimal angle that another great shooter James Harden should shoot at. He releases the ball **2.5 m** above the ground. This makes our 's-s0' **3.05-**



$2.5 = 0.55 \text{ m}$ Our range will be **7.24 meters**. Let's sub these values into the model and find our value.



As expected, the speed required is greater than the free throw we calculated earlier, now being 8.76 ms^{-1} , while the angle this time is **47.17 degrees**. This angle is not too far off from James Hardens' actual launch angle of **49.6 degrees**. For him to shoot even better he should shoot at a slightly lower angle (I'm sure he wouldn't want a random kid saying that to him though).

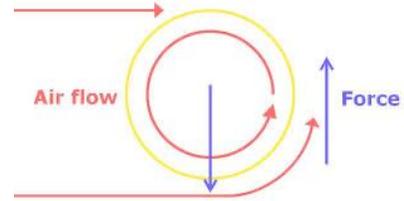
A lower degree of launch compared to the free throw is expected as you want the ball to travel further than it did before. A speed increase is also needed as more kinetic energy and force is needed to push the ball.

To conclude this section, we have discovered that the launch angles and shot velocities influence shot success immensely. It determines how close the shot is and how easily it can be projected. Arguably, this is the most important component to a jump-shot, because without good accuracy there can be no lucky bounces or backboard assistance. In the future, to improve such a model, the 3D plane could be considered. In this scenario it is assumed the ball is shot perfectly straight. However, we know that there is a possibility that the ball is too far to the right or left of the hoop despite having perfect launch angles. Now that we have discovered the influence of the balls' speed and angle, the final part is to see the extent and influence of its spin.

Spin

The final factor we haven't accounted for. Spin in the game of basketball is something all coaches encourage. It enhances control and feel over shots. Why does spin help?

A small factor affecting the flight of the ball is the magnus effect, which is where a pressure difference is caused on each side of the ball due to its backspin. This creates a slight lifting force.



However, the main benefit to be investigated is the friendly bounce. Our initial model focused on perfect swishes, but didn't account for rim bounces that keep crowds on the edge of their seats. Spin softens these bounces, increasing the probability of scoring.

Let's look at one of the tensest shots in NBA history, a game 7 buzzer beater which bounces on the rim multiple times. This shot was during the playoffs where its outcome decided who advanced to the NBA Eastern conference finals. You can watch the shot here using this video https://www.youtube.com/watch?v=ojM9nVvigyA&pp=0gcJCfcAhR29_xXO

Before starting, it is key to know that linear values have rotational version/rotational counterparts. For example, velocity is how many meters are travelled per second. While angular velocity is how many angles are travelled per second (the spin speed). Acceleration is meters per second squared while angular acceleration is angles per second squared (the rate of the spin speed). Linear motion uses mass while angular motion uses moments of inertia, momentum is linear while angular momentum is rotational, force is linear while torque/moments are rotational. So on and so forth. All these values are applied in rotational dynamics which is what the following section investigates. This means classic kinematic equations can be used but applied with rotational values. This section will also use radians rather than degrees as it is the preferred value for calculations involving circles.



Firstly, the trajectory of the ball will be analyzed so we can find the velocity the ball has as it hits the rim (it's final velocity which is represented as 'v'). The ball is in the air for 43 frames. Since I know the video used was in 30 frames per second, the ball is in the air for **43/30 seconds**. We also know that the acceleration due to gravity is -9.81 ms^{-2} . The standing reach of Kawhi Leonard is **2.7 m**. He jumps and bends his arm in this shot so it will be assumed this is the shot release height (s_0). Lastly, the height of the hoop is **3.05 m**. Using these and the following equation gives us:

$$s - s_0 = vt - \frac{1}{2}at^2, s = 3.05 \text{ m}, s_0 = 2.7 \text{ m}, v = ? \text{ ms}^{-1}, a = -g \text{ ms}^{-2}, t = \frac{43}{30} \text{ s}$$

$$3.05 - 2.7 = \frac{43}{30}v - \frac{1}{2} \times -g \times \left(\frac{43}{30}\right)^2$$

$$0.35 = \frac{43}{30}v + 10.07705$$

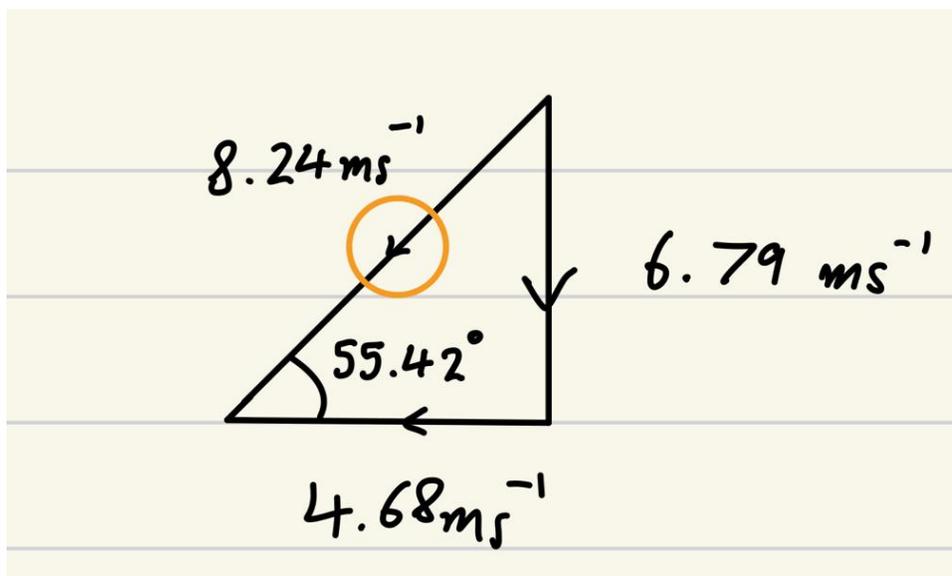
$$v = -6.79 \text{ ms}^{-1}, \text{ The ball comes in at this speed downwards}$$

Horizontally, we use $v_x = \frac{R}{t}$

$$R = 6.71 \text{ m}, t = \frac{43}{30} \text{ s} \quad \therefore \quad v_x = \frac{6.71}{\frac{43}{30}} \quad \therefore \quad v_x = 4.68 \text{ ms}^{-1}$$

Now that we have horizontal and vertical speed, we can find the magnitude of the speed that it has hitting the hoop obliquely. By using Pythagoreans theorem, the speed is found using $\sqrt{4.68^2 + (6.79)^2} = 8.24 \text{ ms}^{-1}$

$$\text{The ball also enters at an angle of } \tan^{-1}\left(\frac{6.79}{4.68}\right) = 55.42^\circ$$



The rim is being modelled as a flat plane to make matters easier (if modelled as round calculations get much more complex). This means if the ball comes in at this angle, it will also be reflected as this angle. The friction will slow down the speed of the ball by decelerating it. Depending on the angle of impact, the ball can still go in the basket. However, in most cases, the bounce might cause the ball to miss and not enter the basket.

In our scenario, an angle like this without spin might have caused the shot to miss. Understanding this, the frictional force can be calculated to justify why the ball slows down.

Firstly, the normal reaction force can be calculated by using the vertical change in velocity, since the normal reaction force acts vertically. We can do this as: $F = \frac{m(v_y - u_y)}{t}$, $m = 0.6 \text{ kg}$, $t = 0.01 \text{ s}$, $F = N$ (normal reaction)

We can figure out the vertical velocity after the ball hits the rim by utilizing the vertical coefficient of restitution. This tells us what the velocity of the ball after the impact is. Wikipedia quotes that "The International Basketball Federation (FIBA) rules require that the ball rebound to a height of between 1035 and 1085 mm when dropped from a height of 1800 mm". Using this, let's assume the coefficient of restitution is 0.75. So, the vertical velocity after is $0.75 \times 6.79 = 5.0925 \text{ ms}^{-1}$.

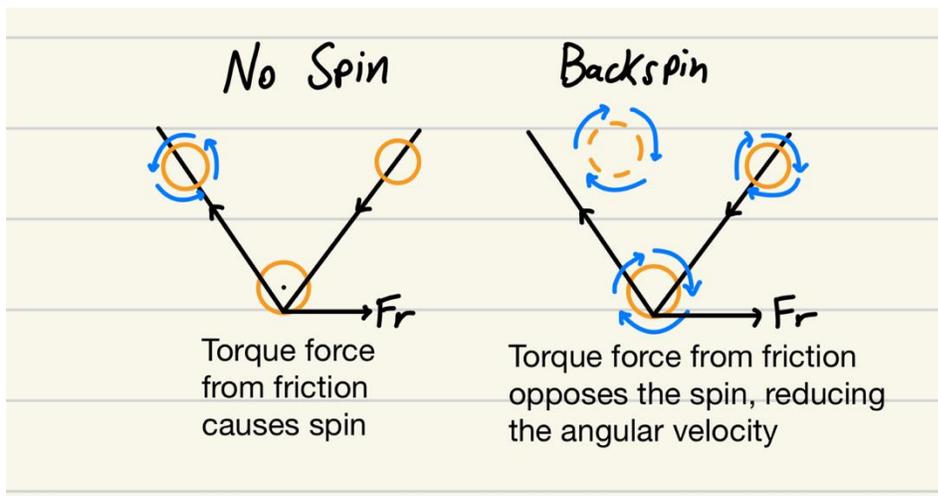
Using this we can find that:

$$N = \frac{0.6(6.79 - 5.0925)}{0.01} \quad \therefore N = 101.85 \text{ N}$$

Now we can find the friction by using $Fr = \mu_k \times N$, Friction is equal to the kinetic coefficient of friction multiplied by the normal reaction force. A quick google search tells us that the kinetic coefficient of friction between the rim and the ball is 0.5. Using this we get:

$$Fr = 0.5 \times 101.85 \quad \therefore Fr = 50.925 \text{ N or } 50.93 \text{ N To 2 decimal places}$$

We have now found the frictional force experienced by the ball as it contacts the rim, spin or no spin. This large value justifies our observation. Showing that the ball's motion is opposed greatly. This force will act horizontally reducing the speed of the ball and opposing the slip from the ball and the rim. Now the scenarios can be compared.



$F_{r_{backspin}} > F_{r_{no\ spin}}$, both diagrams experience a friction of at least 50.93 N

In the first diagram we see the friction causes the ball to spin due to torque. Torque is a turning force that creates an anti-clockwise moment about the center of the ball. The equation to find the torque is $\tau = r \times Fr$, where ' τ ' is torque, ' r ' is the radius and ' Fr ' the friction. Torque is a twisting force, and in a circle, it is always the radius multiplied by the perpendicular force acting on the surface of the ball. This means:

$$r = 0.12\ m, Fr = 50.925\ N$$

$\tau = 0.12 \times 50.925 \therefore \tau = 6.11\ Nm$, to 2 decimal places. This tells us the torque creates a small turning force in the anti-clockwise direction that spins the ball slightly forward.

Now let us look at this same scenario but introduce backspin. Now we will involve more rotational values during calculations. Firstly, the backspin causes more friction to be applied since it is trying to oppose the ball's motion and spin into the ground. Since it is a tangential force, the friction creates torque and therefore creates angular deceleration of the ball. By working backwards, we will first find the angular deceleration, then discover what torque caused it and therefore what friction caused the torque. But before that, let us use the kinematic equation $v = u + at$ but using rotational values to find the angular acceleration. This means that:

$$\omega_{final} = \omega_{initial} + \alpha t$$

In this equation, ' ω ' represents the angular velocity which is the change in angles per second, ' α ' is the angular acceleration and ' t ' the time the ball decelerates in (the contact time between the ball and the rim which is 0.01 seconds). Now let us find what the angular velocities of the ball are in Kawhi Leonard's shot.

Firstly, by analyzing the video frame by frame, the ball spins around 4.5 full rotations in its ascent journey to the rim.

$$\omega = \frac{\theta}{t}, \quad \omega = \frac{4.5 \times 2\pi}{\frac{43}{30}} \quad \therefore \quad \omega = \frac{270\pi}{43} \text{ or } 19.73 \dots \text{ radians/s}$$

After the bounce on the rim, it does around 1.5 full rotations in 26 frames before coming back down to the rim again. This means:

$$\omega = \frac{1.5 \times 2\pi}{\frac{26}{30}} \quad \therefore \quad \omega = \frac{45}{13}\pi \text{ or } 10.87 \dots \text{ radians/s}$$

Since we know the ball contacts the rim for roughly 0.01 seconds, we can substitute our values into the formula to find that:

$$\frac{45}{13}\pi = \frac{270}{43}\pi + \alpha \times 0.01$$

$$0.01\alpha = \frac{-1575}{559}\pi$$

$$\alpha = -281.75 \text{ radians/s}$$

This insanely large number is because the collision happens over an extremely short period of time. Our values are also negative because the ball is angularly **decelerating**, reducing spin/ angular velocity during the impact.

Before our final step, we must calculate the moment of inertia for a basketball. A moment of inertia is simply the distribution of the mass around any point. It is also the rotational version of mass as it is how the mass is distributed around the axis. Since a ball is a sphere, we take this point at the center of the ball. It is a hollow sphere meaning we can calculate the moment of inertia with the formula:

$$I = \frac{2}{3}mr^2$$

(the $\frac{2}{3}$ is just a constant that changes depending on the dimensions of the object, a solid sphere would be $\frac{2}{5}mr^2$)

$$m = 0.6 \text{ kg}, \quad r = 0.12 \text{ m}, \text{ which are mass and radius}$$

$$\text{Substituting these gives us } I = \frac{2}{3} \times 0.6 \times 0.12^2$$

$$I = 0.00576 \text{ kgm}^2$$

Now, with all our values required, we can finally use the equation $\tau = I\alpha$, which is just Newtons second law with rotational values.

We can derive this equation by multiplying both sides of newtons first law by r (radius) as the friction force 'F' acts tangentially to the surface of the ball (creating torque):

$$F = ma$$

$$F \times r = ma \times r, F \times r = \tau \quad \therefore \quad \tau = ma \times r$$

Tangential acceleration (a_t) is equal to angular acceleration multiplied by the radius

$$a_t = \alpha \times r \quad \therefore \quad \tau = m(\alpha r) \times r, \quad \tau = mr^2\alpha, \quad I = mr^2 \quad \therefore \quad \tau = I\alpha$$

$$\tau = I\alpha, \quad I = 0.00576 \text{ kgm}^2, \quad \alpha = -281.75 \text{ radians/s}$$

$$\tau = 0.00576 \times -281.75 \quad \therefore \quad \tau = -1.62 \text{ Nm}, \text{ to 2 decimal places.}$$

It is a negative value as it shows that the torque acts in the anti-clockwise direction which is opposite to the clockwise backspin of the ball.

The spin of the ball in Kawhi Leonard's shot had angular deceleration to create a torque force that opposes the direction of the back spin, explaining why the value calculated is negative and. It also shows us why the ball's angular velocity reduces, giving a number to back up this observation. This torque had to have come from a tangential force however, and in our impact, the only force that could cause this is friction.

This means we can find the frictional force that the spin caused by rearranging our previous equation $\tau = r \times Fr$ to get:

$$Fr = \frac{\tau}{r}, \quad Fr = \frac{1.62}{0.12} \quad \therefore \quad Fr = 13.5 \text{ N}, \text{ (negative sign is ignored as we only observe the magnitude of the friction force since we already know its direction)}$$

Now we also know an extra friction force of 13.50 N is created which a spinless ball doesn't have. This along with the 50.925 N frictional force calculated earlier from the projectile motion of the ball answers our final question. To what extent does spin affect shots? As we have discovered, it has a significant impact (over an extra 25% of friction is added) which causes the motion of the ball to be altered in such a way that players are able to have another shot at scoring if they hit the rim. This **13.5 N** force was a small but contributive reason as to why the ball was able to stay on the rim in the video, and eventually dropped right in. This extra spin saved the season and gave the NBA team a ticket to the next round.

Conclusion

Basketball shots are a beautiful display of physics and mathematics. As we have evaluated, the shots success is more interlinked with math than one would expect. As investigated, 3 key factors affect the shot, the launch speed, the launch angle and finally the backspin. Through many calculations we discovered the average free throw should be shot at 50.87° since it requires the least speed of projection at 7.42 ms^{-1} . Also finding what angles the giants of the NBA, like the 6'5 James Harden, should shoot at. Finding that for his towering 2.5 m shot release, 47.17° is the best angle. We also discovered the hidden benefit of spinning the ball, showing that the spin adds over **25% more** friction to oppose motion and angular velocity. Using all this knowledge, it is now possible to mathematically

make your shot better. Next time you watch your favorite NBA player, you will know what separates those great shooters from the rest.

References

- All diagrams drawn personally.
- Nba.com provided insight into players' heights and standing reaches so that the numbers for shot release heights could be estimated. It also gave dimensions of ball such as radius and mass.
- The coefficient of friction was found from the following report (<https://core.ac.uk/download/pdf/81927888.pdf>)