Chaos Theory in Formula 1: Modelling the Unpredictability of Wet Weather Racing

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March 2025

Introduction

Chaos theory is a branch of mathematics that focuses on dynamic systems that are highly sensitive to initial conditions. It states that within the apparent randomness of certain systems, such as the climate, there are patterns, feedback loops, fractals and interconnections. Known as the butterfly effect, chaos theory is effectively the study of systems where small differences in starting conditions can lead to extremely different outcomes. The butterfly effect's name is derived from the question famously posed by Edward Lorenz: Does the flap of a butterfly's wings in Brazil set off a tornado in Texas? [1] Minuscule changes in initial conditions such as rounding errors can result in widely varied outcomes. For example, Figure 1 below shows three double pendulums with massless rods and equally weighted ends. They all start positioned horizontally to the right with deviations of ±0.5°, however as we see below, they all have varied different paths and outcomes.

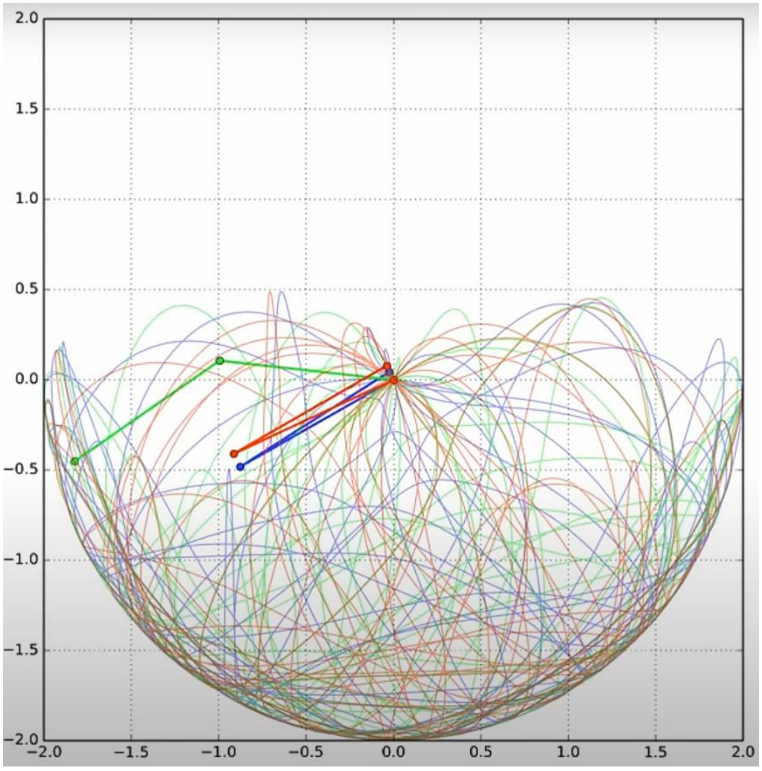

Figure 1: Double Pendulum Chaos [2]

You might now be wondering what chaos theory has to do with wet weather racing in Formula 1. Wet weather in Formula 1 can create varying conditions where grip levels constantly change due to factors like standing water, drying track and marbles (rubber debris from tyres). Variations in these conditions, including tyre temperature and type of wet tyre (intermediate or full wet) can lead to large changes in grip. For example, small changes in temperature can majorly alter performance and handling which are sensitive to initial conditions – a key feature of chaotic systems. Furthermore, feedback loops can develop where a small slide or correction can escalate into a spin depending on the driver's inputs.

Changing track conditions is also related to chaos theory as the drying rate of the track depends on temperature, wind and water displacement from the tyres. This can cause uneven drying across different parts of the circuit and creating an unpredictable surface where grip varies at different corners. Wet weather in Formula 1 often leads to chaotic race strategies since teams must decide what tyres to use and when to change compound as the track dries or if there's a sudden rain shower. Miscalculations therefore can lead to critical losses in position and performance and can lead to crashes or spins. Therefore, in this essay I am going to try and design a model for wet weather racing in Formula 1 using chaos theory.

Mathematical Basis of Chaos Theory

Chaotic systems are modelled with the general formula below:

$$\frac{dx}{dt} = f(x, t)$$

Where:

- $x = x_1, x_2, \dots, x_{\mathrm{n}}$ which are vectors of state variables
- $t =$ time

This general formula helps to describe how a complex system evolves over time and helps to identify how sensitivity to initial conditions and feedback can lead to unpredictable behaviour. These small differences in initial conditions are known as chaotic divergence. The state variables represent the key quantities that define the system's state at any given moment. For instance, in weather, the state variables could include temperature, wind velocity, pressure, humidity and others. There also exists coupling within the system between variables where changes in one variable affects other variables which can cause knock on effects and added complexity.

The general formula is used to describe any chaotic system in abstract terms, this makes it less specific for modelling Formula 1 wet weather racing using chaos theory. The general formula is also harder to analyse and model as the functional form isn't defined making it more sensitive to rounding errors and numerical instability.

Therefore, I am going to use a more specific model instead of the general formula for chaotic systems to model wet weather racing in Formula 1. The Lorenz equations, developed by Edward Lorenz in 1963 is a set of three differential equations, seen below, that relate the properties of a two-dimensional fluid layer [1]. The equations describe the rate of change of the three quantities with respect to time.

$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = x(\rho - z) - y$$
$$\frac{dz}{dt} = xy - \beta z$$

$x =$ rate of convection
$y =$ horizontal temperature variation
$z =$ vertical temperature variation
$\sigma, \rho, \beta =$ system parameters (proportional to the Prandtl number, Rayleigh number and physical dimensions of the layer itself)

Modelling Wet Weather Racing

I am going to use the Lorenz equations to initially model wet weather racing in Formula 1 because they reflect the chaotic nature of the sport and its sensitivity to changes in initial conditions like rain intensity and track temperature. Since the equations describe fluid convection (as they were developed to model atmospheric convection) I will need to modify them. I will start by changing the constants, where:

$x =$ lateral position (or trajectory stability) of the car
$y =$ grip level of the car
$z =$ track surface wetness
$\sigma =$ responsiveness of car to changing grip conditions
$\rho =$ critical grip threshold for stability
$\beta =$ rate of grip loss due to water build up

Changing these constants and approximating them whilst using them to expand upon the Lorenz equation will help provide a more accurate and representative model.
Firstly, I am going to find an approximation for sigma, which represents the responsiveness of the car to changing grip conditions and how quickly the cars trajectory adjusts to grip variations. In wet weather, the response is slower due to reduced tyre contact due the accumulation water forming between the tyres and track which causes a reduction in friction and hence a loss of grip. The equation on the next page will allow me to estimate the value of sigma as a higher wet grip allows the car to respond more effectively to changes since the tyres maintain better contact with the track. Hence, if μ_{wet} increases the car becomes more stable. Despite higher tyre temperatures (T_{tyre}) generally improving grip, they can reduce responsiveness if they overheat as it leads to degradation. Additionally, the equation below makes sense since a high grip with a low temperature means that the tyres are maintaining responsiveness and are working efficiently.

$$\sigma \approx \frac{\mu_{wet}}{T_{tyre}} \times 100$$

μ_{wet} = wet friction coefficient
T_{tyre} = tyre temperature

If we let μ_{wet} and T_{tyre} be typical values for wet weather racing in Formula 1 we get:

$$\sigma \approx \frac{0.4}{60} \times 100 \approx 0.667$$

$\mu_{wet} \approx 0.4$
$T_{tyre} \approx 60°\mathrm{C}$

The lower value for sigma reflects a slower adaptation to grip changes in wet conditions whereas in dry conditions the value of sigma would be ≈ 1.7 which reflects a higher adaptation to grip changes and more predictable driving patterns.
Initially I was going to use the hydroplaning threshold equation to model ρ, however I have derived an equation shown below which is a modified critical grip threshold equation as a function of pressure which now includes a constant *c* which reflects how pressure influences grip retention. A higher pressure raises the threshold for instability, so the system resits chaos for longer.

$$\rho = y_{crit}(p) = \frac{\mu_0 mg}{F_{downforce}} e^{-kz} + cP$$

y_{crit} = critical grip level needed to maintain stability (dependent on tyre pressure)
μ_0 = wet friction coefficient
m = mass of the Formula 1 car (kg, mass in 2025 season)
g = gravitational constant ($9.81\mathrm{ms}^{-2}$)
$F_{downforce}$ = aerodynamic downforce generated by Formula 1 car (N)
k = grip decay constant
z = wetness of track (0 = dry, 1 = full wet)
c = coefficient determining how tyre pressure influences grip
P = tyre pressure (bar)

The first half of the equation represents the available grip from friction which I have modelled to decrease exponentially as track wetness increases (z) so as track wetness increases grip decreases, and the system becomes more unstable. The second term cP represents a stabilising effect due to tyre pressure. Since higher pressure increases the stiffness of the tyre it reduces contact area. This improves water clearance delaying hydroplaning and also increases the hydroplaning threshold velocity increasing the system's ability to resist chaos as:

$$v_{hydro} \approx 10.35\sqrt{P}$$

As wetness increases (z increases) the grip is reduced due to the exponential decay term making the system more chaotic and unstable as ρ also drops. I am going to estimate a value of ρ to use in the Lorenz equations using the values below where:

$$\rho \approx y_{crit}(p) \approx \frac{0.4 \times 800 \times 9.81}{20000} e^{-0.15\times0.5} + 0.05 \times 1.6$$
$$\rho \approx y_{crit}(p) \approx 0.15696 \times 0.92774 + 0.08$$
$$\rho \approx y_{crit}(p) \approx 0.226$$

$\mu_0 \approx 0.4$
$m \approx$ 800 kg
$g \approx$ 9.81 ms^{-2}
$F_{downforce} \approx$ 20000 N
$k \approx 0.15$
$z \approx 0.5$
$c \approx 0.05$
$P \approx$ 1.6 bar

I decided to model the value of ρ in moderate wetness ($z \approx 0.5$) to reflect racing conditions as if there were full wet conditions ($z \approx 1$) then there would most likely be a red flag in Formula 1 so they wouldn't race until the conditions improved. In dry conditions F1 cars can generate up to approximately 31392N of downforce (4 times the cars weight), however I let $F_{downforce} \approx$ 20000N since in wet conditions set ups would be adjusted and due to lower speeds less downforce will be generated. The calculated value $\rho \approx 0.226$ is below the Lorenz system's threshold ($\rho = 1$). This means that the system shows unstable behaviours but hasn't entered full chaos yet. This is accurate as the car is still drivable, despite being in a reduced grip zone, until more water accumulates. However, this does mean that minor inputs will have amplified knock on effects.
I am going to model β using the equation below:

$$\beta = \frac{\dot{z}}{C + kv}$$

$\dot{z}$ = rainfall rate (mms^{-1})
C = track drainage efficiency (mms^{-1})
k = tyre effectiveness
v = speed of car (kmh^{-1})

The value kv represents the water displaced and cleared by the tyres. The model is good because it shows that higher speeds increase water displacement but also shows how grip deteriorates if $\dot{z} > C + kv$ ($\beta > 1$) then water builds up on track increasing hydroplaning risk. I am going to estimate a value for β using the values below:

$$\beta \approx \frac{0.00083}{0.0015 + 0.000015 \times 150}$$
$$\beta \approx 0.221$$

$\dot{z} = 0.00083$ mms^{-1}
$C = 0.0015$ mms^{-1}
$k = 0.000015$ mmkm^{-1}
$v = 150$ kmh^{-1}

My value of $\dot{z}$ is assuming moderate rain at 3 mmh^{-1}. The track efficiency is set for tracks like Silverstone with good drainage (tracks like Monaco with poor drainage would have a value of 0.0008 mms^{-1}). The value of k is set for intermediate tyres and v is for a medium speed section of the track. The value of β shows that grip is reduced by $\approx 22.1\%$ due to water buildup. Currently β is in an area that shows the car is still drivable but if beta increases to $\beta > 0.4$ hydroplaning is likely. Therefore, if we were to model this with heavy rain and on a track with poor drainage at lower speeds, we would get a much larger value of beta that would reflect a greater rate of grip loss due to water build up and an increased risk of hydroplaning.

After finding estimations for the constants σ, ρ, β I am now going to make more detailed models for x, y, z to expand upon the Lorenz equations. I am going to start by finding a model for x. Since x is the lateral position (or trajectory stability) of the car I am going to create a perturbation model for it using the Lyapunov exponent. A perturbation model is a method of analysing small disturbances in a system and how they evolve like how car position can lead to large consequences like a loss of control. The Lyapunov exponent, named after Russian mathematician Aleksandr Mikhailovich Lyapunov, characterises the rate of separation of infinitely close trajectories. It shows that two trajectories with initial separation δ_0 diverge at a rate given by $|\delta(t)| = e^{\lambda t}|\delta_0|$ where λ is the Lyapunov exponent [3]. By modifying this equation, we can create a model for x shown below.

$$x(t) = x_0 + \in e^{\lambda t}$$

$x(t) =$ cars lateral position (or trajectory) over time
$x_0 =$ initial trajectory position
$\in =$ small initial deviation (caused by wet track, grip variation, steering input…)
$\lambda =$ Lyapunov exponent

This model is good at showing amplitude and intensity of errors as if $\lambda > 0$ then the perturbations increase over time so the system is unstable, this could be when rain is increasing or continuing to fall. But, if $\lambda < 0$ then the perturbations are decaying over time, this could be

as the track dries making the car follow a more predictable trajectory. So, the Lyapunov exponent determines whether small perturbations grow and amplify over time.
To find a model for y, the grip level of the car, I am going to begin by modelling tyre grip based on friction using the equation below.

$$F_{grip} = \mu N$$

$\mu =$ coefficient of friction
$N =$ normal load

This equation provides an accurate estimation of grip based on friction for dry conditions but not as much in wet weather racing. Therefore, I am going to adjust it to make it the coefficient of friction dependent on water film thickness as seen below.

$$\mu(h) = \mu_0 e^{-kh}$$

$\mu(h) =$ coefficient of friction dependent on water film thickness
$\mu_0 =$ maximum coefficient of friction when $h = 0$ (the track is completely dry)
$h =$ water film thickness
$k =$ decay constant

This equation shows how the coefficient of friction decays as water film thickness (wetness of the track) increases. Substituting this into the equation from earlier gives us a model for y shown below.

$$y = F_{grip} = \mu_0 e^{-kh} N$$
$$y = \mu_0 e^{-kh} N$$

Finally, we can simply estimate z, the track surface wetness, using the equation below.

$$z = \frac{R}{C}$$

$R =$ rainfall rate
$C =$ water clearance rate (water drainage rate and water displacement rate)

Before we substitute the constants σ, ρ, β and the new models for x, y, z, I am going to add one more term $R(t)$ which will be a stochastic term that will reflect random bursts of rainfall during the race. I am going to use sinusoidal pulses to model intermittent rain bursts shown by the equation on the next page.

$$R(t) = Asin(\omega t) + Bsin(2\omega t) + C\varepsilon(t)$$

$A, B =$ amplitude of rainfall bursts

ω = frequency of rainfall bursts
C = intensity of white Gaussian noise
$\varepsilon(t)$ = white Gaussian noise

Since real life rainfall isn't periodic like a sinusoidal wave, I have used white Gaussian noise to add a randomness to generate random fluctuations which mimic unexpected rain bursts [4]. This equation for $R(t)$ ensures that track wetness (z) won't evolve evenly but will have small unpredictable variations due to the stochastic term used.
After adapting the Lorenz equations, we now have the general equation shown below for wet weather racing in Formula 1 with the added term $R(t)$.

$$\frac{dx}{dt} = \sigma(y - x) + R(t)$$
$$\frac{dy}{dt} = x(\rho - z) - y$$
$$\frac{dz}{dt} = xy - \beta z + R(t)$$

x = lateral position (or trajectory stability) of the car
y = grip level of the car
z = track surface wetness
σ = responsiveness of car to changing grip conditions
ρ = critical grip threshold for stability
β = rate of grip loss due to water build up
$R(t)$ = random rainfall burst

However, by substituting in all the equations we've derived and estimations for constants found earlier, we get an equation shown below:

$$\frac{dx}{dt} = 0.667\big((\mu_0 e^{-kh}N) - (x_0 + \in e^{\lambda t})\big) + Asin(\omega t) + Bsin(2\omega t) + C\varepsilon(t)$$
$$\frac{dy}{dt} = (x_0 + \in e^{\lambda t})\left(0.226 - \frac{R}{C}\right) - (\mu_0 e^{-kh}N)$$
$$\frac{dz}{dt} = (x_0 + \in e^{\lambda t})(\mu_0 e^{-kh}N) - 0.221\frac{R}{C} + Asin(\omega t) + Bsin(2\omega t) + C\varepsilon(t)$$

$x = x_0 + \in e^{\lambda t}$
$y = \mu_0 e^{-kh}N$
$z = \frac{R}{C}$
$\sigma = 0.667$
$\rho = 0.226$
$\beta = 0.221$
$R(t) = Asin(\omega t) + Bsin(2\omega t) + C\varepsilon(t)$

This may look very complicated, however when you break it down into each component it actually makes sense. Throughout this essay I have tried to find an accurate and representative model of wet weather racing in Formula 1 using chaos theory. There will always be things you can add like the effect of graining (which occurs when the tyres are lower than optimal temperature and bits of the tyre rips off and sticks to the tyre itself) which reduces grip. However, I believe that I have created a good model for chaos theory within Formula 1 whilst learning lots about the Lorenz equations.
Thanks for reading, Sachin Vohra.

References

[1] https://en.wikipedia.org/wiki/Edward_Norton_Lorenz

[2] https://www.youtube.com/watch?v=pEjZd-AvPco

[3] https://en.wikipedia.org/wiki/Lyapunov_exponent

[4] https://en.wikipedia.org/wiki/Additive_white_Gaussian_noise