The Mathematics Behind Game Shows

Game shows provide entertainment to millions of households across the UK. The tense atmosphere, flashing lights and life-changing prizes keep viewers constantly engaged as they anticipate whether the next contestant will make the most effective decisions in order to win this monumental prize. However, beneath these dazzling lights and consciously crafted theatrics lies a range of mathematical concepts such as probability, expected values, variance, risk aversion and game theory, which ultimately underpin the success of the contestants on these shows. This essay will explore how these mathematical concepts form the foundations to these shows and how contestants can use them to break down uncertain situations into logical and quantifiable decisions that can move them one step closer to achieving the astonishing game show prize.

One popular game show in the UK is Deal or No Deal. This game show begins with a contestant selecting a box containing an unknown prize out of 22 sealed boxes. Throughout the game, the contestant chooses a set number of boxes each round and once a prize is revealed in an open box, it is removed from the prize pool. The Banker will then make an offer to the contestant after each round, resulting in the contestant having to make a decision whether to accept the offer from the Banker or to continue playing the game, risking a lower prize or potentially winning a higher one. The contestant's prize can range from 1p to £100,000, depending on their final decision.

One key mathematical concept used in this show is expected values, which helps contestants to make rational decisions about whether to continue playing the game. Expected values are defined as the theoretical mean value of a numerical experiment over many repetitions of the experiment. In the context of Deal or No Deal, expected values can help a contestant decide whether to keep playing. For example, the contestant may be left with 8 boxes after Round 4 which contain the following prizes: 50p, £1, £100, £750, £1,000, £7,500, £25,000 and £50,000.

Deal or No Deal Show
12th February 2025

The probability of the contestant choosing each remaining value is equal, with each box having a 12.5% chance of being selected. This means that the expected value can be calculated by finding the sum of the values and dividing by the number of possible outcomes, as shown below.

$$EV = \frac{0.50 + 1.00 + 100 + 750 + 1000 + 7500 + 25000 + 50000}{8} = £10{,}543.94$$

This calculation suggests that, on average, the contestant could win a prize of £10,543.94 if they choose to continue the game under the same circumstances. In theory, if the game were to be repeated many times under these same conditions, the average winnings would converge to this expected value. However, the Banker offers the contestant £3,990, which is significantly below the predicted expected value; this mathematically suggests that the contestant should reject this offer.

However, risk aversion - defined as the tendency to avoid situations that involve uncertainty or the potential for negative outcomes - may contribute to a contestant's decision-making, as the contestant may fear walking away with a very low prize, such as 50p. Therefore, we are able to quantify this risk by calculating the variance between the remaining outcomes, which will highlight the spread of the possible outcomes.

$$\text{Variance} = \frac{\sum (x_i - \mu)^2}{n} = £286{,}678{,}194.70$$

In this situation, the variance is extremely large due to the wide spread of values from the expected value (£10,543.94). This greater uncertainty makes it difficult for the contestant to decide whether they should continue playing, as they could leave with any prize ranging between 50p and £50,000. As a result, contestants may become more risk-averse and may favour lower Banker offers, such as £3,990, rather than continuing the game. This is due to players opting for lower safety values (despite them being lower than the expected value) rather than gambling for a higher payout. This links to the concept of game theory as continuing the game would mathematically maximise the expected value; however, human psychology often prefers to minimise risk, especially where there are large disparities between potential outcomes. This deviation from the rational decision-making model is a key aspect of game theory, where both mathematical calculations and psychological factors influence player strategies.

Mathematics also plays a role in Deal or No Deal, particularly in how the Banker calculates their offers. The Banker often uses the Root Mean Square Value (RMS) of the remaining prize amounts to determine an offer. This is due to the RMS value providing an average of the remaining prize amounts, while also accounting for both the high and low amounts. The Banker then typically applies an additional multiplier to this RMS value to provide a balance between the potential winnings and the psychological factors influencing the contestant's decision.

Root Mean Square Formula

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$$X_{rms} = \sqrt{\frac{x_1^2 + x_2^2 + x_3^2 \ldots + x_n^2}{n}}$$

where, x_1 , x_2 , x_3 ,.... x_n are given "n" observations.

$$RMS = \sqrt{\frac{0.5^2 + 1^2 + 100^2 + 750^2 + 1000^2 + 7500^2 + 25000^2 + 50000^2}{8}} = £19{,}946.25$$

Banker Offer = £19,946.25 x *20% = £3,990 (3SF)*

The calculation above confirms the use of the Root Mean Square Value by the Banker, along with the application of a multiplier (20% in this case), as it matches the actual offer made by the Banker in this episode of Deal or No Deal (£3,990). The Banker applies this multiplier because their primary goal of being to profit from the game. By offering appealing offers to the contestant, which is below the expected value, the Banker encourages contestants to accept the offer and leave the game, reducing the risk of high pay-outs and subsequently increasing the Banker's chances of making a profit.

A number of factors can influence the multiplier applied to the RMS by the Banker including the stage of the game, contestant's behaviour and entertainment value of the game show. For example, early in the game the Banker may apply a lower multiplier to the RMS to make the offer more tempting for the contestant to take the deal due to the higher risks they face. Alternatively, the contestant's behaviour also plays a crucial role, as the Banker is aware that emotions, such as the fear of leaving the show with a low prize, often affect contestant's decision-making. As a result, the Banker may adjust the multiplier accordingly depending on the confidence of the contestant and how reluctant they are to take their deal. Additionally, entertainment value can influence the multiplier value as the Banker may adjust the multiplier depending on how engaging or entertaining this segment of the episode is. For example, during a dramatic moment of the show, such as reaching the climax of the episode, the multiplier may be tweaked to create a more dramatic offer that leads to the contestant having to make a high-risk decision, this further builds suspense and adds entertainment value to the episode as there are higher stakes at risk.

In conclusion, game shows are delicately designed around mathematical principles that guide every contestant's decision. As demonstrated through Deal or No Deal, mathematical concepts such as probability, expected value, risk aversion and game theory work in unison to shape outcomes, which then determine the success of both the contestant and Banker throughout the show. Through exploring game shows through this perspective, I have been able to gain a valuable insight into how mathematical concepts can be applied to a variety of scenarios as well as other disciplines such as economics, finance and behavioural science. Ultimately, this essay has highlighted how mathematical thinking allows us to navigate uncertainty and make informed decisions – even in the high-pressure world of game shows.

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