The Science of Swimming:
Decoding the Mathematics that underlies the swimmer's motions.

An Essay Presented by
Swagato Ray
for
The Tom Rocks Maths Essay Competition

March 2025

A couple of days ago, on our family vacation, my little brother challenged me to a race in the hotel pool. I felt confident; after all, I had trained as a kid, so I thought beating him would be easy. But to my surprise, he beat me. Initially, I blamed it on my lack of practice, but later, on further introspection, I wondered—was it just a lack of practice or was there something else—something mathematical—that determined the winner?

Introduction:

Mathematics forms the backbone of everything we see in the world around us, and swimming is no exception. Indeed, understanding the Mathematics behind any sport provides one with a hefty advantage over their opponent. But let's start at the basics. A lot of concepts come to mind when I think of swimming—Newton's Laws of Motion, Archimedes Principle, Multivariable Calculus, and so on. At the physical level, though, be it the right posture or stroke length, the right swimsuit or diving angle—everything that a swimmer does stems back to mathematics. But the question persists—how much of swimming is skill, and how much is science?

Is Swimming Novel?

Swimming as an activity has been around for thousands of years and can be dated back to 2500 BCE in Egypt, Greece and Romania. But back then, swimming was just a means of survival and recreation—not a scientific sport. The global spotlight turned towards competitive swimming when it was featured in the first modern Olympic Games in 1896, marking the beginning of its evolution as an actual sport. In the early days of competitive swimming, technique lacked efficiency and optimization. The breaststroke was one of the first strokes to be used, mainly because it resembled natural human movement in water. With time, however, swimmers started to experiment—an effort which saw the evolution of freestyle, butterfly stroke and other techniques of swimming. These strokes did not develop overnight—they were the product of years of experimentation, trial and error, and the intuitive pursuit of speed and efficiency. While formal mathematics wasn't yet part of the picture, every adjustment—from arm angles to body positioning—was slowly laying the groundwork for a more scientific, math-based approach to swimming.

When Math Entered The Pool:

With the growth of sports culture, competition became tougher—coaches wanted to understand what made one swim faster. A mathematical intervention took place. It all started with splitting time and stroke analysis in the 1950s.

Split Time:

A split time is an intermediate time recorded during a race, which is generally the time taken to swim 50m or 100m. It allows swimmers to monitor their pace in different segments of the race and identify their areas of improvement. Standard deviation in their split times throughout the race helped them quantify consistency in their pace using the formula,

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (x_i - \mu)^2}$$

– σ: shows how much the split times vary

– n: number of splits in the race

–x_i : time of the i-th split

– μ: average split time

A standard deviation of 0.56 seconds would mean the swimmer's splits are varying by over half a second on average — leaving room for improvement. A smaller σ (e.g., <0.2s) indicates better pacing and control.

Stroke Analysis:

Stroke analysis, by definition, is studying how a swimmer moves their body in the water, how

fast they do so, and how efficiently they do it. These are mainly judged based on Stroke Rate(SR)—number of strokes taken per minute and Stroke Length(SL)—the distance covered in one stroke, where:

$$Speed = Stroke\ Rate \times Stroke\ Length$$

Now, to get the ideal output, a swimmer should not only increase their stroke rate by moving their hands faster, but also focus on simultaneously increasing Stroke Length, which reflects better propulsion and technique. This balance can be best found using partial derivatives, where:

$$\frac{\partial v}{\partial SR} = SL,\ \frac{\partial v}{\partial SL} = SR$$

This helps find the balance using the conservation of energy, using constraint based optimization.

Another metric for judgement is the Stroke Index, which is calculated as:

$$Stroke\ Index = Speed \times Stroke\ Length$$

The higher the stroke index, the more efficiently the swimmer is moving.

Stroke Force Efficiency:

Let's say when a swimmer makes a stroke, he applies a force F, making an angle θ with the horizontal. This applied force can be broken down into two components where the components are:

$$\text{Horizontal Component: } F_{horizontal} = F \cdot cos\theta$$

$$\text{Vertical Component: } F_{vertical} = F \cdot sin\theta$$

Here, the vertical component is mainly responsible for maintaining buoyancy whereas the horizontal component of the applied force is responsible for the forward motion of the swimmer. Although the vertical component does play a role, forward motion is the primary objective in swimming, and maximizing efficiency requires focusing on the horizontal component of the force.

Therefore, to maximize the forward force, we must find the maxima of the horizontal force component ($F_{horizontal}$) which is given by:

$$\frac{d}{d\theta}(Fcos\theta) = 0 \Rightarrow \theta = 0^{\circ}$$

Thus, in theory the most efficient stroke angle would be as close to 0° as possible but in reality, due to human biomechanics and the restrictions of the human body, it usually lies between 15°—25° during the pull motion(underwater arm movement).

Acceleration and timing analysis

As years passed by, small devices like accelerometers and other sensors were attached to the swimmers body to analyze their performance, mainly their acceleration and velocities. This can be used to analyze change in acceleration of a swimmer during each stroke cycle and the effect of fatigue on their performance, also helping to understand how the swimmer can conserve energy, by reducing stroke rate and increasing stroke length(as discussed earlier).

$$a(t) = \frac{dv}{dt}$$

Acceleration peaks during the pull phase. Sudden drops or irregularities in acceleration are signs of inefficient energy use and wrong technique. This data helps swimmers understand their faults and form the basis for optimization.

Although accelerometers give data of the velocity of the swimmers, it isn't as accurate as the data found using video camera analysis, where the video is split frame by frame, and the swimmers motion is then analyzed. Velocity is calculated as:

$$v_i = \frac{x_{i+1} - x_i}{t_{i+1} - t_i}$$

where

– v_i: velocity at the i-th frame

– x_i: position at the i-th frame

– t_i: time at the i-th frame

Achieving the Highest Velocity:

When a swimmer dives, they do so at an angle with the horizontal, acting like a projectile. The path followed by a projectile is given by:

$$y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$$

where,

– x: horizontal position

– y: vertical position

– θ: angle of entry

– v: launch velocity

– g: acceleration due to gravity

Usually the optimized angle of entry is somewhere between 30°—40°, as they provide the maximum horizontal distance with the dive, but also minimum splash and thus, minimum drag. Sometimes in races, swimmers have to push off of walls, here, the Impulse—Momentum Theory comes into play.

$$F \cdot \Delta t = m \cdot \Delta v$$

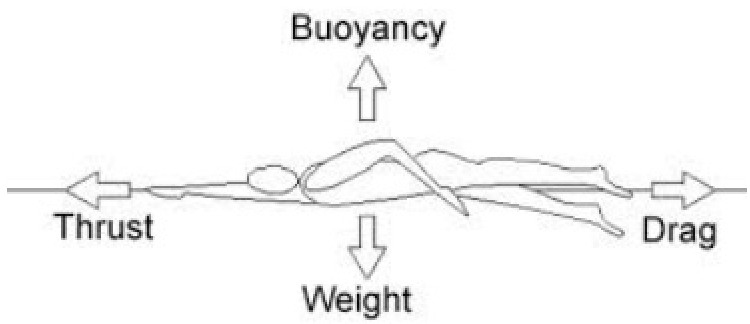
Thus, the lesser time one takes while pushing off with great force, the greater velocity they achieve.

Drag and Resistance

Now, on to my personal favourite part—essentially the most mathematically deep part.

Drag in water refers to the physical resistance encountered by swimmers as they move through it. There are mainly three types of drags:

1. Form Drag: It depends on the position of the body of the swimmer.
2. Wave Drag: It arises due to turbulence created by the swimmer near the surface of the water.
3. Frictional Drag: It arises due to friction between water and the swimsuit and skin.



The general formula for drag is:

$$F_d = \frac{1}{2}\rho A C_d v^2$$

where,

– F_d: drag force

– ρ: density of fluid

– A: cross sectional area

– C_d: drag coefficient

– v^2: relative velocity of the swimmer with respect to water.

From this formula, we get to know a lot of things. Swimmers streamline their body (arms extended, head tucked) which reduces frontal area A. The streamlined shape with overlapping hands and a tight posture reduces turbulence and flow separation, indirectly reducing drag coefficient C_d, thus, reducing drag.

Computational Fluid Dynamics (CFD) is used in modern day training to simulate using the Navier Stokes Equation, areas where drag force can be reduced. The Navier Stokes Equation is a set of partial differential equations that helps us understand how fluids, in this case water, move. Its basic form is:

$$\rho(\frac{\partial v}{\partial t} + v \cdot \nabla v) = -\nabla p + \mu\nabla^2 v + f$$

where,

— ρ: fluid density
— v: velocity vector of the fluid
— t: time
— ∇v: velocity gradient
— ∇p: pressure gradient
— μ: dynamic viscosity of the fluid
— $\nabla^2 v$: Laplacian of velocity (accounts for internal fluid friction)
— f: external forces (e.g., gravity, swimmer thrust)

The Navier Stokes Equation is mainly used in CFD (Computational Fluid Dynamics), where it helps provide simulations to understand which swimsuits are most efficient, optimize dive angles, and study stroke cycles in great detail.

Getting faster?

Modern day swimsuits are no longer just pieces of cloth stitched together, they are engineering marvels, carefully crafted with meticulous calculations in mind. CFD is of great use here, as designers use it to simulate water flow over the body, and find areas to minimize drag. Swimsuits are now made using textured surfaces which reduce turbulence and hinder flow separation.Smooth seam lines reduce flow interruptions. Tight swimsuits produce compression which reduces fatigue and keeps the body aligned. All of this combined, reduced drag coefficient C_d and frontal area A, thus, reducing drag.

The **LZR Racer suit** by Speedo, developed with NASA using CFD and wind tunneling, was a technological marvel. It reduced drag by up to a marvelous 24% more than the last Speedo model. It highly improved efficiency with its ultrasonically welded seams and graduated compressions. 93 world records were broken wearing the suit. This shows the sheer power of Mathematics and Science in swimming. Its unfair advantage made FINA (now World Aquatics) ban the LZR racer suit and similar suits in 2009.

Conclusion

From the angle of entry to the fabric of a swimsuit, everything ultimately roots back to the vast depths of Mathematics and Science. Calculus, Fluid Mechanics and Optimization are some of the most powerful tools in the field of swimming. The right technique and correct implementation of modern maths and technology can have a huge influence on an athlete's career. For example, the swim team of the University of Virginia, under the guidance of Mathematics Professor Ken Ono, trained using high speed cameras and mathematical modelling, analyzing their movements and stroke cycles. This led to them winning 11 Olympic Medals at Paris Olympics 2024. This shows the real life importance of maths and how success can be achieved with the right training, guidance and mathematical understanding. As science continues to advance, the sport of swimming keeps transforming into a data driven sport.

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