Specific Impulse and The Starship Enterprise

Anybody who calls themself a sci-fi fan has a fictional ship that they are hoping will become a reality. How could you not? In all good fantasy and science fiction, the writer's dedication to meticulous world building and explaining the physics behind all inventions, makes fiction seem closer and closer to real life. For me, Star Trek’s attention to detail does this. And in particular, “Star Trek: The Next Generation”, lulled by Geordi’s specialist vocabulary and Picard’s optimistic speeches about how humans can do anything, I, like many others, hope that the Galaxy-class USS Enterprise (NCC-1701-D) is not just fictional and could become a reality.

In this essay, I will be trying to determine whether this is completely foolish or if it’s still valid to hold out hope.

To do this, I will compare the Enterprise’s engine efficiency with that of our current rockets. The closer it is, the more likely the Enterprise can be made (preferably in my lifetime!). Although I would love there to be a ship travelling at Warp 9, I would be content with anything close to the Enterprise, so we will look less ambitiously at the sub-warp speeds (below the speed of light) that are controlled by the Impulse Engine, rather than the Warp drive.

A typical way of comparing rocket (or spacecraft) efficiency's is through their "specific impulse," which measures how well an engine generates thrust.

So what is needed to calculate the specific impulse?

Luckily for my limited Aerospace mechanical knowledge, NASA has made a comprehensive website page on how to calculate this, which we can use as a process guide.

Rocket Thrust Equation $F = \dot{m} V_e + (p_e - p_0) A_e$

where p = pressure, V = velocity, A = area, $\dot{m}$ = mass flow rate, F = thrust

Define: Equivalent Velocity: $V_{eq} = V_e + \frac{(p_e - p_0) A_e}{\dot{m}}$ $\quad F = \dot{m} V_{eq}$

Define: Total Impulse: $I = F \Delta t = \int F\,dt = \int \dot{m} V_{eq}\,dt = m V_{eq}$

Define: Specific Impulse: $\frac{\text{Total Impulse}}{\text{Weight}}$ $\quad Isp = \frac{I}{m g_0} = \frac{V_{eq}}{g_0}$ $\quad$ **units = sec**

$$\boxed{Isp = \frac{F}{\dot{m} g_0}}$$

[1]

So the variables we need to determine are:
- Exhaust Velocity
- Area
- Mass Flow Rate
- Exhaust Pressure
- Ambient Pressure

Exhaust Velocity
In the Star-Trek enterprise, the impulse engine is powered by a Deuterium-Deuterium fusion reactor that produces electro-plasma[2], a highly energetic form of plasma.

Deuterium-Deuterium fusion reactors are currently only theoretical. However, if one did exist we can estimate that it would produce a neutron with the kinetic energy of approximately 3.2MeV[3].

1 MeV = 1.6×10^{-13}J

So $3.2 \times 1.6 \times 10^{-13}$ = 5.12×10^{-13}J

We can now use this equation to calculate the velocity of the neutron as:

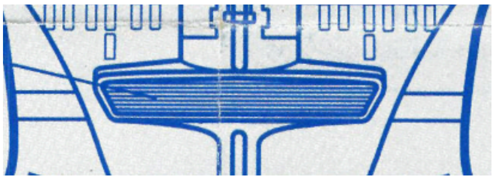
$Kinetic\ Energy = \frac{1}{2} \times mass \times velocity^2$

The mass of a neutron is approximately $1.675 \times 10^{-27} kg$[4] then

$v^2 = (5.12 \times 10^{-13}) \div (0.5 \times 1.675 \times 10^{-27}) = 6.113432836 \times 10^{14} \Rightarrow v = 2.4725357 \times 10^7$
$\approx 2.47 \times 10^7 ms^{-1}$

Area

The area needed for the equation is the exit area of the rocket nozzle. Usually, this looks similar to a child's drawing with a simple circle with flames coming out. The enterprises exit nozzle however looks a little different.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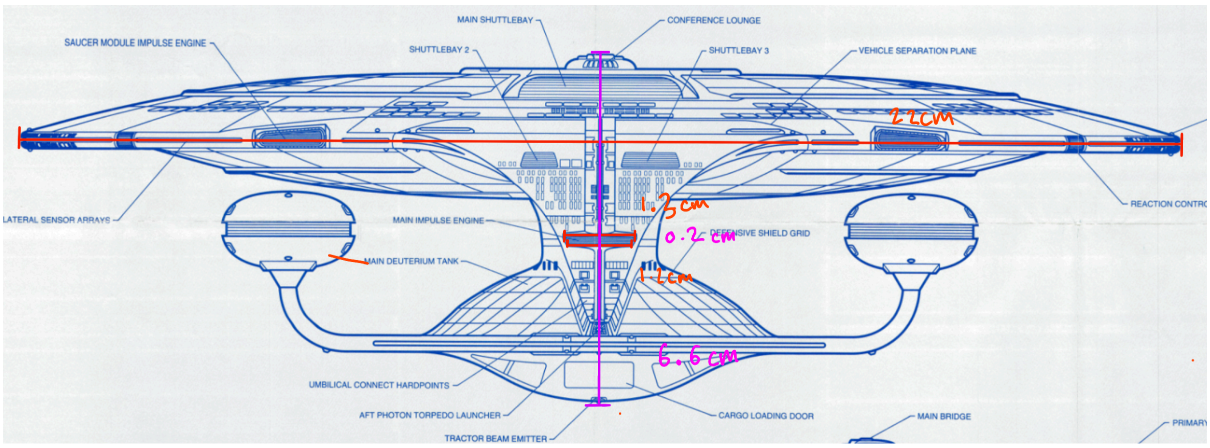
We can model it as an upside down trapezium, and as these are schematic diagrams they should be proportionate, so we can use ratios to determine its dimensions.



In my image the width is 22cm, and the edges of the impulse engine 1.2 and 1.3 cm. The full height is 6.6cm and the height is 0.2cm.

In reality the height of the starship is 137.5m, and the width is 467.0m[6].

Hence to find the "real" dimensions of the engine:

$\frac{1.2}{22} \times 467 \approx 25.47$m

$\frac{1.3}{22} \times 467 \approx 27.60$m

$\frac{0.2}{6.6} \times 137.5 \approx 4.167$m

As the area of a trapezium is 0.5(a+b)h, the exit area of the impulse engine is:

$\frac{1}{2} \times (25.47 + 27.60) \times (4.167) \approx 110.6m^2$

Mass Flow Rate

Mass flow rate is the mass of fluid that passes per unit time through a vessel, and $= density \times cross\ sectional\ area \times velocity$

The cross sectional area and velocity are the same as the ones we calculated previously. The density of the plasma was mentioned in the TNG episode, “Masterpiece society,” where its density is said to be "one hundred billion kilograms per cubic centimeter," (which is approximately $1 \times 10^{17} kgm^{-3}$)[7].

So mass flow rate = $10^{17} \times 110.6 \times 2.47 \times 10^{7} \approx 2.73 \times 10^{26}\ kgs^{-1}$

Pressure

Ambient:

As space is a vacuum, the ambient pressure will be negligible so we can just use 0 for this value.

Exhaust Pressure:

We can calculate the Exhaust Pressure using the two equations $Power = Force \times Velocity$ to find the force and $Pressure = Force \div Area$.

$Power = 8 \times 10^{24} W$[8]

$Velocity = 2.47 \times 10^{7} ms^{-1}$

$Power \div Velocity = (8 \times 10^{24}) \div (2.47 \times 10^{7}) = Force = 3.24 \times 10^{17} N$

Pressure = Force/Area = $(3.24 \times 10^{17}) \div 110.6 = 2.93 \times 10^{15} Pa$

Thrust

Now that we have all our variables we can calculate the thrust.

$$Force = mass\ flow\ rate \times exhaust\ velocity + (exhaust\ pressure - ambient\ pressure) \times area$$

$$F = 2.73 \times 10^{26} \times 2.47 \times 10^{7} + (2.93 \times 10^{15} - 0) \times 110.6 = 6.74 \times 10^{33} N$$

Finally: Specific impulse

We have now done the hard part of collating the variables, and can now calculate the specific impulse, by following the instructions outlined by the NASA image above.

$Force = mass\ flow\ rate \times equivalent\ velocity \Rightarrow equivalent\ velocity = Force \div mass\ flow\ rate$

$V_{eq} = (6.74 \times 10^{33}) \div (2.73 \times 10^{26}) = 2.47 \times 10^{7} ms^{-1}$

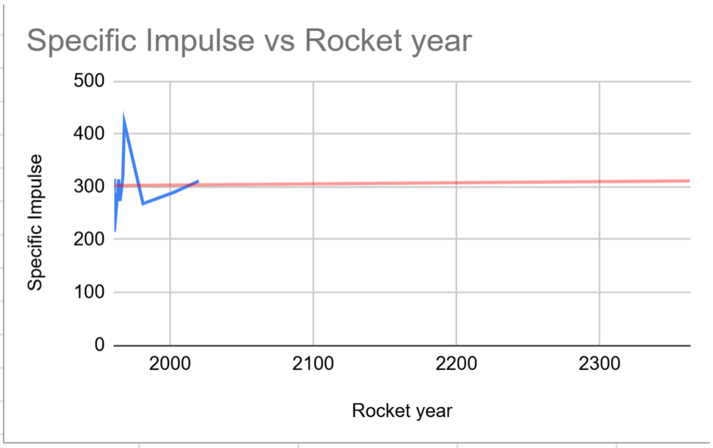
$Total\ impulse = V_{eq} \times mass = 2.47 \times 10^{7} \times 3.98 \times 10^{8} = 9.83 \times 10^{15} Ns$ where mass is the ships total mass which is $3.98 \times 10^{8} kg$[6].

$Specific\ Impulse = Total\ Impulse \div Weight\ (using\ g_0 = 9.81)$

$= (9.83 \times 10^{15}) \div 9.81 = 1.00 \times 10^{15}$

Which is 2.38×10^{12} greater than the Apollo mission Saturn V rocket's specific impulse in a vacuum of 421s[9]. And if we plot every rocket that's taken a human into space against their specific impulse, we can see that if rockets continue the current trend, specific impulse will not have increased to this by 2363[10] when the Enterprise D is supposed to be built.

Every rocket that's carried a human into space		
Rocket	Rocket year	Specific Impulse
Vostok	1961	315
Mercury	1961	215
Voskhod	1964	314
Gemini	1965	273
Soyuz	1967	319
Apollo	1968	421
Space shuttle	1981	268
Shenzhou	2003	290
Crew Dragon	2020	311



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This doesn't look good for my ship becoming a reality in the same way it did in Star Trek. However, humans are still very new to the space world, and despite the nice, neat line of best fit it is clear the graph is not linear, so there could be some exponential growth with the development of methods of harvesting energy like deuterium-tritium nuclear fusion which researchers have already carried out[12], rather than the deuterium-deuterium nuclear reactor in the Enterprise's impulse engine. Furthermore, developments in Artificial Intelligence and Virtual Reality are making Star Trek's "Holodeck" look more and more plausible. So, while I may not get to *really* ride a spaceship at ¼ of the speed of light in my lifetime, I may be able to *feel* like I may *boldly go where no one has gone before…*

References

[1] https://www.grc.nasa.gov/www/k-12/airplane/specimp.html
[2] https://memory-alpha.fandom.com/wiki/Electro-plasma
[3] https://www.ditl.org/scitech-page.php?ScitechID=7&ListID=Sci-tech
[4] https://pmt.physicsandmathstutor.com/download/Physics/A-level/Past-Papers/AQA/Data%20Sheet.pdf
[5] https://www.cygnus-x1.net/links/lcars/star-trek-the-next-generation-enterprise.php
[6] https://www.kasper-online.de/en/docs/startrek/ncc1701d.htm
[7] http://www.starfleetjedi.net/forum/viewtopic.php?t=6651
[8] https://scifi.stackexchange.com/questions/99179/how-much-energy-could-the-enterprise-d-produce
[9]https://en.wikipedia.org/wiki/Saturn_V#:~:text=November%209%2C%201967%20
[10] https://en.wikipedia.org/wiki/USS_Enterprise_(NCC-1701-D)
[11]https://www.space.com/every-crewed-spacecraft-human-spaceflight-history.html
, wikipedia

[12]
https://ccfe.ukaea.uk/fusion-energy/fusion-in-brief/#:~:text=Researchers%20have%20overcome%20many%20of,scale%20of%20a%20power%20plant.