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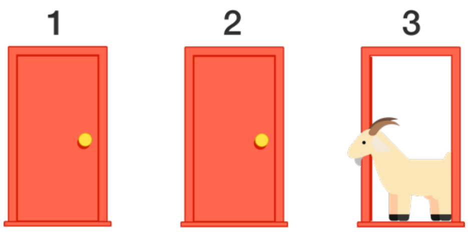
The Monty Hall Problem: A Guide to Besting the Show: “Let’s Make a Deal”

Introduction

The Monty Hall Problem is a seemingly paradoxical puzzle that has stumped both mathematicians and game show enthusiasts alike. The Monty Hall Problem originates from the game show ‘Let’s Make a Deal’ and was named after the gameshow’s host, Monty Hall. The actual statistical problem originates from 1975 where Steve Selvin sent in a letter providing the idea and solution of his statistical brain teaser to the ‘American Economist’, which is a scientific journal covering statistics and statistical problems.

The Game Rules



The game involves a contestant standing before three doors, two of which hide goats, while the third hides a car. The contestant must then choose one of the three doors in hopes of winning the car. The host, Monty Hall, will then always open one of the two remaining doors, revealing a goat. The contestant is then offered the choice to swap or stay.

Statistics behind the Original Monty Hall Problem

A common misconception is that changing would have no impact on the likelihood of successfully choosing the car as it would be a **50-50** chance as there would be one goat and one car left behind the doors after one of the goats are revealed. However, after taking a look at the probability in the form of a table it is clear to see that the probability is indeed swayed towards success if the contestant chooses to swap doors.

Contestant Picks	Prize is behind	Host Opens	Result if Stay	Result if Swap
1	1	2 or 3	WIN	LOSE
1	2	3	LOSE	WIN
1	3	2	LOSE	WIN
2	1	3	LOSE	WIN
2	2	1 or 3	WIN	LOSE
2	3	1	LOSE	WIN
3	1	2	LOSE	WIN
3	2	1	LOSE	WIN
3	3	1 or 2	WIN	LOSE
		Win probability	1/3	2/3

Thus, even with more doors, swapping to one of the two remaining doors still increases the odds of winning. While this does not guarantee a successful choice it does double the likelihood.

This can be explained as follows: when first choosing a door, the contestant has no prior knowledge of what is behind each door. Therefore, the chance of picking a goat is $^2/_3$, while the chance of picking the correct door is only $^1/_3$. Once Monty opens a door revealing a goat, it might seem like the odds become **50-50**, but this is not actually the case. There are two doors that remain and only two other choices it being a **50%** chance should be the reasonable answer. This is not the case, however, as once the contestant knows where one of the goats certainly is (behind the opened door) the chance of having picked a door with a goat will remain $^2/_3$ while swapping to the only other door would halve it.

Variations of the Monty Hall Problem

Monty Hall also appears in other forms in which the number of doors varies and can be discussed and looked at a number of different ways.

Variations of the problem involving more doors generally follow the same key principle: swapping doors after receiving more information increases the player's odds of success. For example, if there were suddenly five doors and after choosing one the host opens two others revealing the goats. The statistical explanation behind this logic is that, with your initial choice, you have a $^1/_n$ chance of choosing the correct door, where **n** represents the number of doors. After two of the doors are excluded from possible options (after the host reveals two goats) the odds become concentrated as n decreases while the initial choice holds the same statistical probability. Thus, even with more doors, swapping to one of the two remaining doors still increases the odds of winning.

Bayes' Theorem and Conditional Probability

Bayes' Theorem is a key concept in the Monty Hall Problem. It describes how to update the probabilities of hypotheses based on new evidence. This follows the laws of conditional probability, which describe the likelihood of one event occurring given that another event has already occurred. Conditional probability differs from joint probability, which measures the likelihood of two events occurring together without knowing which one is true. The Bayes Theorem has such a formula:
[1]

$$P(H \mid E) = \frac{P(E \mid H)}{P(E)} P(H) = \frac{P(E \mid H) \times P(H)}{P(E \mid H) \times P(H) + P(E \mid \text{not}H) \times P(\text{not}H)}.$$

Application to the Monty Hall Problem

[1] https://brilliant.org/wiki/monty-hall-problem/#using-bayes-theorem

In the Monty Hall Problem, Bayes' Theorem can be used to calculate how the probability of the original door being correct changes after Monty reveals a goat behind one of the other doors.
In the equation shown above the probabilities can be inputted where H is the hypothesis that "The car is behind door 1" while E is the evidence that Monty reveals where one of the goats was behind.
The probabilities will have such values:

P (H) = $\frac{1}{3}$ → the initial probability of correctly choosing the correct door (door no. 1).

P (H') = $\frac{2}{3}$ → the probability of not having chosen the right door which can also be shown through the equation P (H') = 1 – P (H).

P (H|E) = 1 → the probability that Monty will show a goat when opening one of the doors if the hypothesis is correct (due to the game rules, this will always be 1).

P (H'|E) = 1 → the probability of Monty showing a goat if the hypothesis is false (the car isn't behind door 1) – this will also always be 1 due to the game rules.

When inputting these values in the previously shown equation:
[2]

$$P(H \mid E) = \frac{1 \times \frac{1}{3}}{1 \times \frac{1}{3} + 1 \times \frac{2}{3}} = \frac{\frac{1}{3}}{1} = \frac{1}{3}$$

This is the equation showing the probability of correct door being chosen the first time, which remains unchanged. However, since one door is no longer a choice after Monty revealed the goat behind it the remaining door has a probability of $^2/_3$.

Application in variation

The Bayes theorem can also be used to calculate the probability in any variation of the Monty Hall Problem.

1. What if Monty Hall didn't know where the goat was?

If Monty didn't know where the goat was there would be no guarantee that Monty would open a door with a goat behind it when revealing one of the unchosen doors contents. This would affect the value in one key detail: the probability of Monty choosing a goat door if the initial hypothesis (chosen door) is incorrect.

P (H) = $\frac{1}{3}$ → This would remain the same as there is still the same number of doors.

P (H') = $\frac{2}{3}$ → This would also remain the same due to P (H') = 1 – P(H).

[2] https://brilliant.org/wiki/monty-hall-problem/#using-bayes-theorem

P (H|E) = 1 → If the hypothesis is correct, the other two doors will both have a goat behind it regardless of which Monty chooses.

P (H'|E) = $\frac{1}{2}$ → This value would change as if the hypothesis were wrong, either doors 2 or 3 could have a car behind them making the probability of Monty showing a goat **50-50.**

When these values are inputted into the Bayes equation:[3]

$$P(H \mid E) = \frac{1 \times \frac{1}{3}}{1 \times \frac{1}{3} + \frac{1}{2} \times \frac{2}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

This shows that when the Monty Hall Problem is altered in this way, the assumed likelihood of success is that it is neither swayed here nor there. However, one must remember that in half the cases of Monty Hall revealing the contents of one of the remaining doors, the location of the prize is shown, guaranteeing a win for the contestant. Therefore, the equation does show the likelihood of successfully having chosen the right door, the first time if a goat is revealed by Monty.

2. What if there were 100 doors to choose from?

In a popular variation of the game is one in which the number of doors is increased, for example 100 doors. In this variation, following the contestant choosing a door initially from the 100 choices, Monty opens 98 of the remaining doors all showing goats. The contestant is then asked to swap or stay from their initial choice and one other. In this variation the statistical probability of swapping increases exponentially compared to the doubling of the chance of success in the original. This can also be proven through Bayes' theorem, in which the hypothesis is "The prize is behind door 1". In this case the probability values will once again be different.

P (H) = $\frac{1}{100}$ → the initial probability of correctly choosing the correct door (door no. 1)

P (H') = $\frac{99}{100}$ → the probability of not having chosen the right door which can also be shown through the equation P (H') = 1 – P (H)

P (H|E) = 1 → the probability that Monty will show a goat when opening 98 of the doors if the hypothesis is correct (due to the game rules, this will always be 1)

P (H'|E) = 1 → the probability of Monty showing a goat if the hypothesis is false (the car isn't behind door 1) – this will also always be 1 due to the game rules

[3] [3] https://brilliant.org/wiki/monty-hall-problem/#using-bayes-theorem

When applied to the theorem:

$$P(H \mid E) = \frac{1 \times \frac{1}{100}}{1 \times \frac{1}{100} + 1 \times \frac{99}{100}} = \frac{\frac{1}{100}}{1} = \frac{1}{100}$$

The Bayes Theorem shows that if the contestant remains with their original choice of door 1, the probability of having initially chosen the right door is **1%** whilst choosing the only other available door would lead to a **99%** chance of success. Whilst no version of this type of variant will guarantee a win, it can be seen that is this format is kept the same increasing the number of doors would only continuously increase the probability of success after swapping doors.

The Psychology of the Monty Hall Problem

A major reason the Monty Hall Problem confuses people is due to cognitive biases. People tend to assume that when presented with two remaining options, the odds should be **50-50**. This is an example of the equiprobability bias, where we mistakenly assume all remaining options have equal chances of success. Additionally, the endowment effect makes people reluctant to switch choices because they feel a sense of ownership over their original decision. This psychological resistance is one reason why many contestants could incorrectly choose to stay rather than switch, even when the math behind the problem suggests otherwise.

How to use this in day-to-day life

While the chances of appearing on a Monty Hall-style game show are slim, the lessons from this statistical problem remain valuable. This problem teaches an important lesson about probability: new information can dramatically shift the odds in decision-making. Whether in business, medicine, or everyday choices, understanding conditional probability helps us make better, data-driven decisions. It teaches us that probability is not always **50-50**; additional information can significantly impact decision-making.