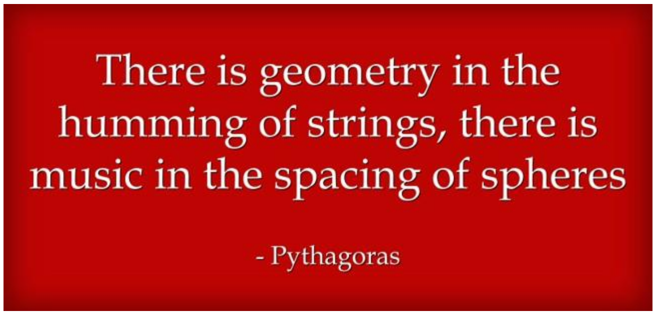
The Nerds Behind the Notes

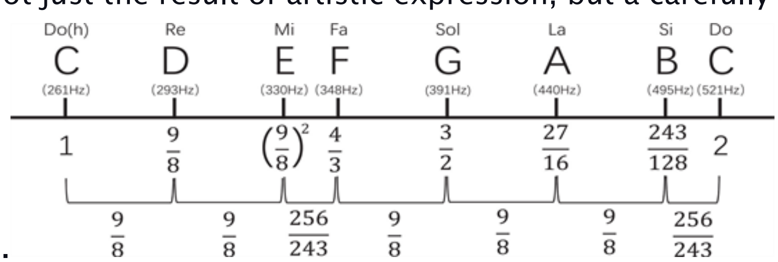
Music and mathematics may seem like two opposite worlds—one driven by emotion, the other by logic. Yet, beneath every melody and rhythm lies a foundation of numbers, patterns, and symmetry.

Do Re Mi Fa So… Even the most amateur musician has pondered: why is there a whole-tone difference between most notes, except E to F and B to C? How do we measure a tone? How did we arrive at semitones? Why does it all work so seamlessly? Who established these rules? Mathematicians, believe it or not.

In the modern world, it is widely accepted that mathematics contains an element of art. What remains difficult to grasp is how such seemingly distinct fields—music and mathematics—are fundamentally intertwined. This article will not delve into any complicated mathematical or musical concepts but rather show you a more musical side to history's biggest nerds. So, hum with me, as we explore the contributions of history's greatest mathematicians in shaping how we understand and create music.



Considering he was regarded as both the “Father of Mathematics” and the “Father of Music”, it is no surprise that ancient Greek mathematician and philosopher, Pythagoras of Samos, makes it to the top of our list. Pythagoras is often credited as the first person to strike a chord between music and mathematics, unravelling the mathematical structure of sound. Over 2,500 years ago, he made a discovery that would change the way we understand sound. The story goes that while walking past a blacksmith’s forge, Pythagoras heard the rhythmic hammering of anvils, each producing a different pitch. Intrigued, he decided to string together some of his own experiments. By varying the lengths of strings on a lyre, he found that the pitch of a string depended directly on its length. When a string is halved, the pitch jumps by one octave—a sound we still hear today. But it didn’t stop there. As Pythagoras tinkered with different string lengths, he discovered that certain numerical ratios—like 2:1 for an octave, 3:2 for a perfect fifth, and 4:3 for a perfect fourth—produced harmonies that the human ear finds pleasing. These ratios, the building blocks of our musical scales, were music to his ears, revealing that the harmony of sound is not just the result of artistic expression, but a carefully calculated symphony of numbers.



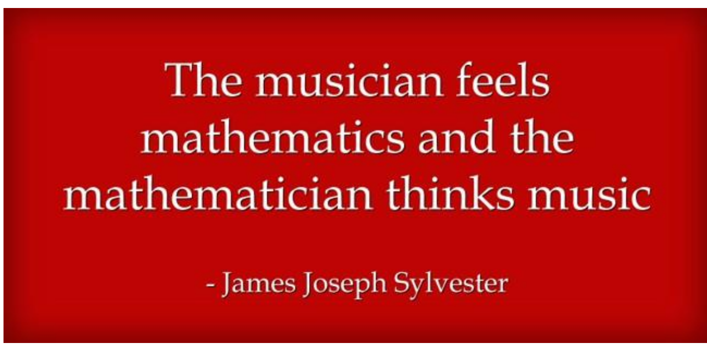
So, whenever you play a chord on the piano, remember, it all started with a blacksmith’s hammer, some curious string lengths, and a radical discovery that made both mathematical and musical history.

Marin Mersenne, as it turns out, happens to be the inspiration behind this topic. I unironically came across him for the first time in a book called “The Music of Primes”, where his love for music also, his love for music was revealed to me. Known as the “Father of Acoustics”, Mersenne improved on Pythagoras’ ratios and transformed music theory into a science. While Pythagoras unwrapped the fascinatingly simple ratios behind harmony, Mersenne went even further and discovered precise mathematical rules that govern how strings vibrate to produce sound. He revealed that the pitch of a string depends on its length, tension, and mass per unit length. So, when you take a string and you shorten its length or tighten it, or make it lighter, you are increasing the pitch of the sound it creates. These are known as “Mersenne’s Laws” and. Mersenne even boiled it down into a formula that is still being used today as the blueprint for tuning instruments:

$$f = \frac{1}{2L}\sqrt{\frac{F}{W}}$$

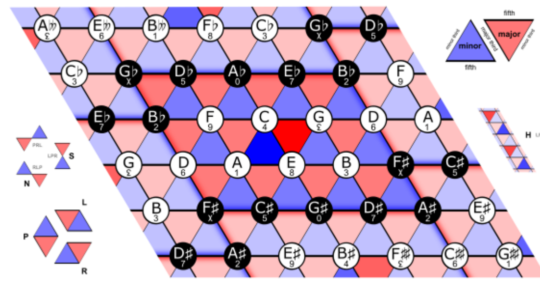
f = fundamental frequency
L = length of string
F = tension in string
W = mass per unit length of string

Mersenne’s work really strikes a chord with instrument makers today. Thanks to him, tuning guitars, violins, and pianos isn’t just guesswork, but a precise science. So, the next time you pick up your guitar and tune each string to pitch-perfect harmony, spare a thought for Mersenne—he’s the reason your notes hit just right.



The man I like to call “the half-blind genius”, Leonhard Euler, is hands down my favourite mathematician ever; so, you can imagine my excitement when I discovered how alike we are in terms of our passion for both mathematics and music. I’m sure you know most of his lore—how he published close to a thousand papers and books, covering almost every branch of mathematics—well, a good chunk of his work was in music. His mathematical prowess coupled well with his musical talent so that as he was playing several musical instruments, he was also redefining music theory, infusing mathematical logic into music, and exploring the influence of musical beauty in mathematics.

One of his most fascinating inventions was the Tonnetz—tone network—which mapped out harmonically related notes in a kind of musical grid. It is created such that notes that are pleasing together are next to each other. It is not only used in



transformational music analysis but also in helping to visualise how chords can flow or modulate mathematically.

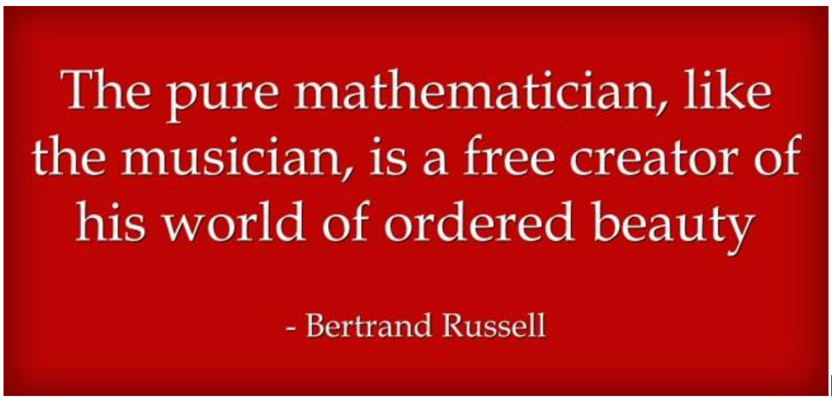
This dude didn't stop there. Euler also developed a formula to measure how pleasant an interval sounds. He called it the gradus suavitatis or "degree of sweetness", using factorisation and logarithms (because, of course, he did), he derived the formula:

$$n = p_1^{a_1} p_2^{a_2} \ldots p_r^{a_r} \quad (p_i \text{ distinct primes, } a_i \geq 1)$$

$$E(n) = 1 + \sum_{k=1}^{r} a_k (p_k - 1)$$

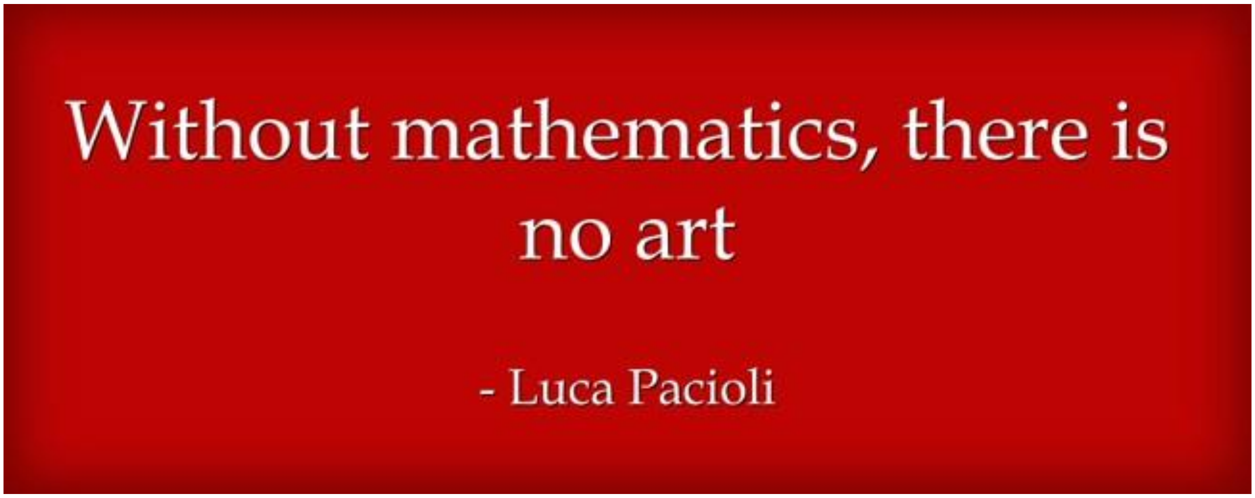
Using this formula, Euler mathematically confirmed that intervals made from simpler ratios like 2:1, or 3:2 are more consonant—and thus more satisfying musically—than the ones with more complex ratios. So, he basically just proved why some chords just sound right.

Euler's work might not be as well-known to musicians today as Mersenne's, but it still quietly underpins parts of modern music theory—and even digital sound analysis. So, whether you're crafting a melody, switching keys, or building a chord progression, chances are, the half-blind genius already saw it coming.



Last but most definitely not least, is the man who taught us that every sound is secretly a sum of sine waves. Jean-Baptiste Joseph Fourier is a more recent of the bunch and his work is very likely the most applicable in modern sound production. If Pythagoras was the tourist, Mersenne was the tuner, and Euler was the performer, then my good man Joseph was the sound engineer—just 2 centuries early. Fourier didn't just listen to music, he decoded it. He had a theory: any periodic sound—be it a piano note, a human voice, or a full-blown symphony orchestra—can be expressed as the sum of pure tones with different frequencies, amplitudes, and phase relationships. This "Fourier Series" is now the blueprint of sound. When we hear a note, what we are really hearing is a stack of different frequencies playing together—like a musical smoothie of sine waves. This is what gives instruments their timbre, which is why a flute and a trumpet sound different even when playing the exact same pitch. Fourier made it so that we could mathematically see sound.

Fourier's influence today is OFF THE CHARTS!! Fourier analysis powers anything from MP3 file compression to voice recognition, to pitch correction, and real-time audio effects. Any time you use Auto-Tune, see a spectrogram or even tweak your EQ in a music app, you're standing on the shoulders of Fourier. So, while he might not have composed a single symphony, his math is behind almost every track you've ever heard. Fourier didn't just change music theory—he rewrote the science of sound itself.



If you asked me a few weeks ago if maths and music were connected at all, I would give a half-hearted answer: "Everything is connected at some level" I wouldn't be able to fathom that these two fields would be so intertwined at the fundamental level.
The mathematical minds we've just discussed didn't just dabble in music, they orchestrated the very principle that allows us to tune, compose, and enjoy the sounds we love today.
I hope reading this as had as much an effect on you as writing it has on me. As someone who finds joy in both numbers and notes, it's been inspiring to discover that the same minds who cracked equations and rewrote mathematical theory were also compositing the foundations of music. I hope, as I have been able to that you can start to see hidden rhythms in formulas and the logic behind the beauty of a chord.
Next time you're lost in your favourite song, remember—behind every note, there's a number. Behind every chord, a calculation. And behind every melody, a mathematician

humming along, proving that sometimes, the most beautiful music begins with a formula.

References

- https://www.youtube.com/watch?v=EdYzqLgMmgk&list=WL&index=2
- https://www.youtube.com/watch?v=1DUZsQ2by2s&list=WL&index=3
- https://www.cam.ac.uk/research/news/pythagoras-was-wrong-there-are-no-universal-musical-harmonies-study-finds
- https://www.nature.com/articles/s41467-024-45812-z
- https://www.smackmypitchup.com/smpu/?sec=content&id=78&topid=123#:~:text=In%20other%20word%2C%20everything%20we,physicist%20and%20a%20great%20mathematician.
- https://www.youtube.com/watch?v=B6Dvfv_ASVg
- http://www.science4all.org/article/math-music/
- https://www.youtube.com/watch?v=nK2jYk37Rlg&list=WL&index=1
- https://plato.stanford.edu/entries/mersenne/
- https://thonyc.wordpress.com/2021/09/29/musical-mathematical-minim-marin-mersenne/
- file:///C:/Users/20juw/Downloads/E315en.pdf