# The Painter's Palette

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*A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas.*

— G. H. Hardy

Picture a painter standing before a vast canvas, not with traditional pigments, but wielding an *infinite* palette of hues. In the 24-bit RGB colour space, every imaginable hue is meticulously determined by three fundamental components which are red, green, and blue. With 256 possible intensity levels for each, the painter has exactly $256 \times 256 \times 256 = 16,777,216$ distinct colours at his fingertips.

In the RGB model, every colour is represented by a triplet $(R,G,B)$, where each component ranges from 0 to 255. We can think of each colour as a vector in $\mathbb{R}^3$, with the coordinates corresponding to the intensities of the red, green, and blue components. For instance, we can represent pure red, pure green, and pure blue by the following vectors respectively:

$$\begin{bmatrix} 255 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 \\ 255 \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 \\ 0 \\ 255 \end{bmatrix}$$

We can divide each component by 255 (this process is known as normalisation). In doing so, we bring our coordinates into the interval $[0,1]$. This means that every colour becomes a point inside a cube in $\mathbb{R}^3$ — often referred to as the RGB colour cube. The vertex $(0,0,0)$ represents black (signifying the the absence of light) while the opposite vertex $(1,1,1)$ corresponds to white (where all colours are present in full intensity). Although the cube can be regarded as a geometrical object, one should also interpret it as a powerful illustration of some fundamental Linear Algebra properties.

The painter's primary colours — pure red, pure green, and pure blue, act as the standard basis vectors, i.e.

$$\mathbf{e}_1 = (1,0,0), \quad \mathbf{e}_2 = (0,1,0), \quad \mathbf{e}_3 = (0,0,1).$$

Each brushstroke on the canvas is a unique linear combination of these basis vectors. Just as the painter blends pigments on a palette, every digital colour is formed by mixing these three fundamental hues. For instance, by combining red $(1,0,0)$ and green $(0,1,0)$, the painter creates yellow $(1,1,0)$! Scaling these vectors changes the intensity, allowing the artist to craft subtle variations — from the deepest crimson to the faintest blush. In particular, multiplying a colour vector by a scalar $k$ such that $0 \leq k \leq 1$ dims its intensity — a vivid red $(1,0,0)$ can be transformed into a more subdued due $(k,0,0)$.

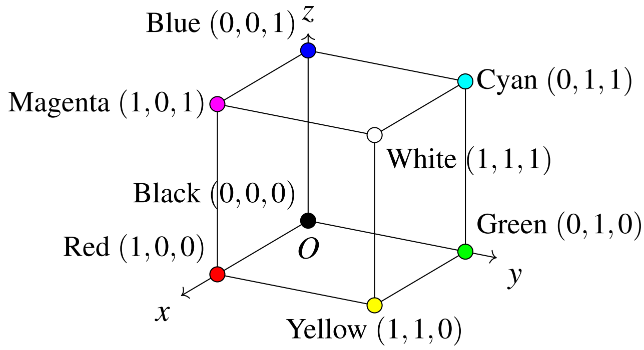



Figure 1: The RGB colour cube

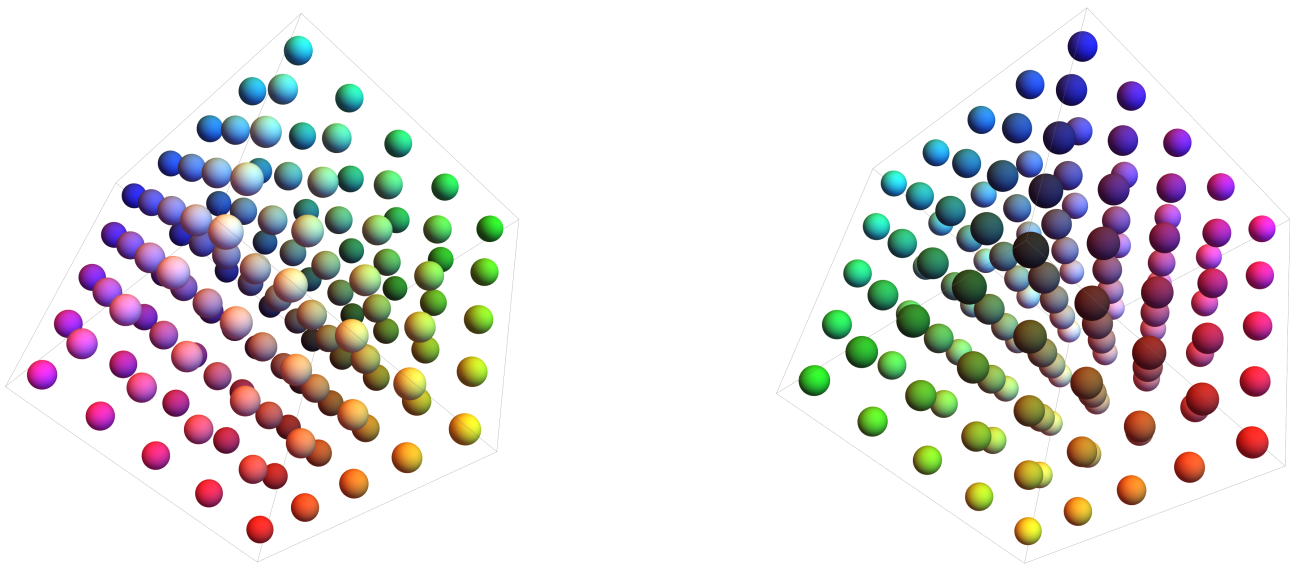


Figure 2: The RGB colour cube from two different perspectives

Each section of the painter's artwork is encoded in matrices representing the intensities of red, green, and blue across the canvas

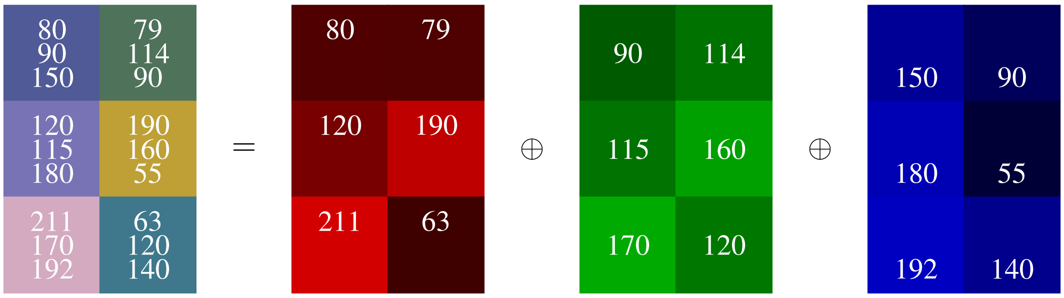
$$\mathbf{R} = \begin{bmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} g_{11} & \cdots & g_{1n} \\ \vdots & \ddots & \vdots \\ g_{m1} & \cdots & g_{mn} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{m1} & \cdots & b_{mn} \end{bmatrix}.$$

By combining these matrices element-wise via an operation denoted by $\oplus$, the painter reconstructs images where every pixel is a blend of colours

$$\mathbf{R} \oplus \mathbf{G} \oplus \mathbf{B} = \begin{bmatrix} (r_{11}, g_{11}, b_{11}) & \cdots & (r_{1n}, g_{1n}, b_{1n}) \\ \vdots & \ddots & \vdots \\ (r_{m1}, g_{m1}, b_{m1}) & \cdots & (r_{mn}, g_{mn}, b_{mn}) \end{bmatrix}.$$

This process mirrors the natural way our eyes perceive a seamless scene from millions of individual points of light.

You might have come across what is known as a HEX code in digital designing software such as Adobe Photoshop. It turns out, each RGB component ranging from 0 to 255 can be converted to a two-digit hexadecimal number. For example, the colour $(80, 90, 150)$ corresponds to the HEX value #505$A$96 after converting 80, 90, and 150 into hexadecimal (50, 5A, and 96 respectively). This conversion emphasises on the unique and reversible nature of colour representation — every HEX code corresponds to one and only one colour within the vast RGB spectrum.




$$\mathbf{R} = \begin{bmatrix} 80 & 90 \\ 120 & 190 \\ 211 & 63 \end{bmatrix} \qquad \mathbf{G} = \begin{bmatrix} 90 & 114 \\ 115 & 160 \\ 170 & 120 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 150 & 90 \\ 180 & 55 \\ 192 & 140 \end{bmatrix}$$

Figure 3: Unique decomposition into RGB components

In our world, the number of possible colours is determined by the intensities of red, green, and blue. Each of these colours can be represented by 256 different intensity levels. To find the total number of possible colours, we multiply the number of choices for each colour. That is,

$$256 \text{ choices for red} \times 256 \text{ choices for green} \times 256 \text{ choices for blue},$$

which yields $256^3 = 16,777,216$. So, there are approximately 16.7 million possible colours!

As the painter steps back, he marvels at the fusion of art and Mathematics. Just as he mixes colours to evoke emotion and meaning, Linear Algebra provides a precise and elegant framework to describe every hue in our digital world. The ability to represent colours as vectors, to add and scale these vectors, and to decompose images into matrices of RGB values is a testament to the power of mathematical thinking. Indeed, we are reminded that beauty lies not only in the colours we see but also in the structure that underpins them.

But for the grand finale, who is this painter? The truth is, we are all painters. Everyday, you wield your smartphone like a brush, capturing moments that range from stunning landscapes to intimate close-ups, from radiant sunrises to serene sunsets. Each image transforms into a canvas, as you artfully paint your screen with the colors and emotions of the world around you.