Can you draw every flag in Powerpoint?

Essay by Taewon **"Zye"** Ham

Introduction.

197 flags.

We draw all of them in Powerpoint.

Or die trying.

1. Problem Specification

■ Flags

How many nations are there in the world? What are their official flags?

These questions are oftentimes central to serious cultural and political dispute. Unfortunately I cannot resolve all conflicts around the globe within the span of this essay, so I took a layperson's approach: The nations and flags present in **Sporcle's "Flags of the World" quiz**[1] are considered to be canon.

■ Tools

Although the first page says Powerpoint for brevity, our actual drawing tool is **Google Slides.** Google Slides has the advantage of being free and on the web[2], so it's easier to try the instructions yourself. Most instructions in this essay are performable in both programs.

This essay is **interactive**, meaning it contains some exercises for you. I recommend you open a blank presentation slide now and play with it as you read. If you get stuck, you can find the answers to exercises in the link below.

Answers to exercises (https://bit.ly/flag_in_ppt)

■ Rules

How do we know if we've drawn a flag correctly? Who will judge us?

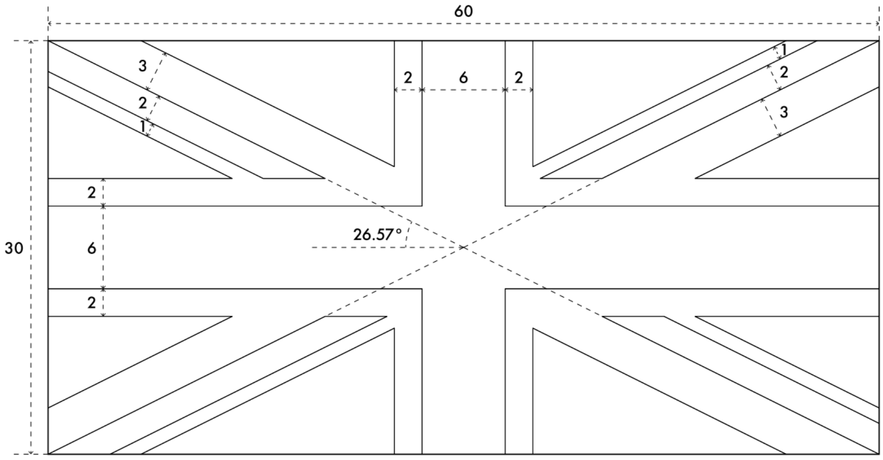
There can be multiple ways to define how a flag should look. For example, the design of the flag of the United Kingdom was described in 1801 as follows:[3]

Azure, the Crosses Saltires of St. Andrew and St. Patrick quarterly per saltire, counterchanged argent and gules; the latter fimbriated of the second, surmounted by the Cross of St. George of the third, fimbriated as the Saltire.	*saltire*: diagonal cross *argent*: white *gules*: red *fimbriated*: outlined *of the n-th*: the *n*-th color that appeared in the description

Though this classy display of words gives me — and probably you — an urge to learn heraldry, it does not contain the specific dimensions of the flag.

What we need is an exact **construction sheet**. For instance, the following design is an accurate representation of the Union Flag, approved by the Garter King of Arms:[4]

[1] "Flags of the World," https://www.sporcle.com/games/g/worldflags
[2] There's also a version of Powerpoint for the web, but with crucial functions (regarding flag drawing) missing.
[3] "Union Flag: approved designs," https://www.college-of-arms.gov.uk/resources/union-flag-approved-designs
[4] same source as above.



We will use official specifications like these to draw our flags, if they are provided.[5]

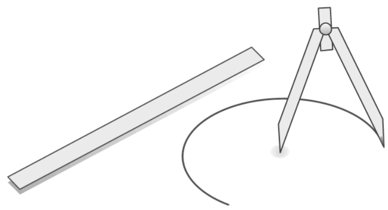
Fortunately, the hardworking people at Wikipedia have gathered a lot of these diagrams. If you go to any **'Flag of [*country*]' article on Wikipedia**, there's usually an image attached showing the construction sheet, or at least the best proportions deduced from official images and laws.

Exercise 0.

Search the flag of your country (or any country of your interest) in Wikipedia. Is there a construction sheet? Does it teach you something new about the flag?

Now for the most important part. **What features** in Google Slides may we use to draw?

To answer this question, we refer to a time-honored mathematical amusement called **straightedge-and-compass construction**. This field of geometry basically asks what shapes you can draw by using a straight stick, a compass, and nothing else.



Here are some important philosophies behind straightedge-and-compass construction.

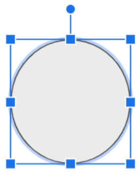
- **(a) Perfect tools**: Assume that the lines drawn with the stick are perfectly straight, and the circles drawn with the compass are perfectly circular.
- **(b) No eyeballing**: If you're meant to draw a square, you need to draw a mathematically perfect square. You can't just draw a shape that *looks* like a square.
- **(c) No measuring**: The stick does not have any length markings. You cannot use it like a ruler.

We'll use these as references.

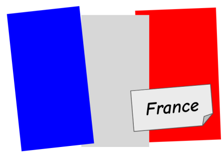
[5] There are 2 possible proportions for the Union flag, namely 2:1 and 5:3. I've shown the 2:1 flag, since that's the one displayed on Sporcle's quiz.

For our challenge, the **basic philosophies** are:

(a) Perfect tools: Assume that the shapes in Google Slides are perfect. For instance, select the `Oval` shape. Hold `shift` and drag on the slide. We assume the resulting shape to be a perfect **circle**. We ignore the fact that your monitor has square pixels, or that your computer can't perform infinitely precise calculations.



(b) No eyeballing: We can't just draw a blue, white, and red rectangle and call it **France**. The proportions of the flag have to be mathematically exact.



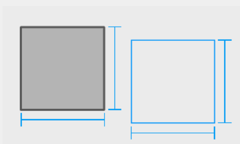
(c) No measuring: We can't input numbers while drawing. For instance, we're not allowed to right-click a shape, go to `Format options`, and then set the `Width` and `Height` of the shape to some exact centimeter values.

These guidelines are a bit abstract, but you'll probably get the gist of it as you get used to Google Slides. The following exercise deals with some key features of the program.

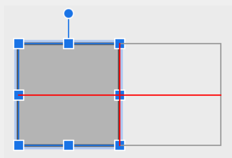
Exercise 1.

1.1. Try creating other shapes while holding `shift`. What mathematical shapes can we assume them to be? For example, creating the `Rectangle` shape this way results in a **square**.

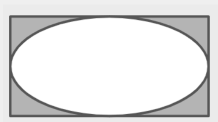
1.2. Draw a square as shown in exercise 1.1.
Then click anywhere else. This **de-selects** the square.
Next, create a `Rectangle` again, without holding `shift`.
Try to make its size identical to the first square.
Notice how the width and height of the second shape '**snaps**' to those of the first.
This way, we can make the width or height of any shape identical to some other shape we've already created.



1.3. Try to move a square while holding `shift`. What happens?
Next, move it while holding `ctrl`. What happens?
What if you hold **both** `ctrl` and `shift`?



<u>1.4.</u> Can you use the features from exercises 1.2 and 1.3 to draw **an** `Oval` **whose width is twice its height**?



2. The Warp and Weft

In exercise 1.4, you needed to draw an oval with an **aspect ratio of 2:1** — that's the ratio of its width to its height.[6] One way to do this is the following.

(a) Draw a square.

(b) Copy it by holding `ctrl` and dragging it.

(c) Drag the copied square to the right of the original square.
If you bring it close enough, it '**snaps**' to the original square.
The [two squares combined] is a shape with an aspect ratio of 2:1.

(d) Draw an `Oval`.

(e) Drag the corners of the oval to the corners of the [two squares combined].
This '**snaps**' the corners of the shapes.

Take note of the steps (c) and (e). In both steps, you '**snap**' a shape to something else. According to our philosophy, when Google Slides — our **perfect tool** — does these snaps, the result is mathematically exact.

So given a square, we can create a shape with double the length. But is that all we can make?

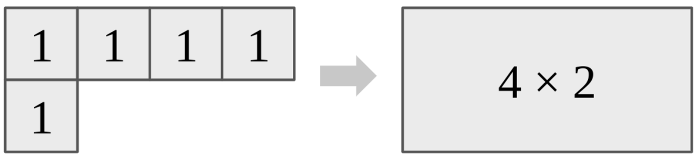
Theorem 1. Given a square with length 1, we can create [a rectangle with width n and height 1] for all positive integers n.

Proof. Just do the same process as above but snap n squares together. □

One may laugh at using such bold words as **theorem**s in an essay about Powerpoint vexillology, but this is serious business. Since [Theorem 1] can be applied for heights as well, this means that:

Theorem 2. Given a square with length 1, we can create [a rectangle with width w and height h] for **any two positive integers** w and h.

Proof. Snap w squares together horizontally, and h squares together vertically. Draw a new rectangle and set the width and height to equal the w squares and h squares, respectfully. □



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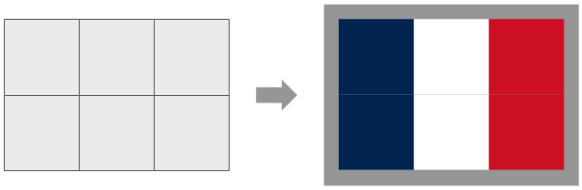
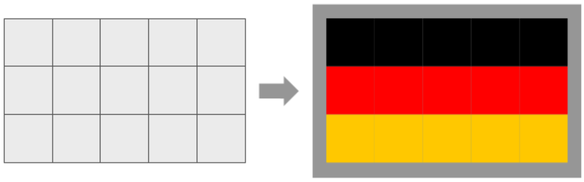
[6] Whether to write the aspect ratio as 'width : height' or 'height : width' is not unified between sources. I've chosen 'width : height' since its more familiar for math students if the x value comes first.

With this method, we can create any flag which solely consists of rectangles (if all the proportions are integers)! Let's try drawing a few easy flags.

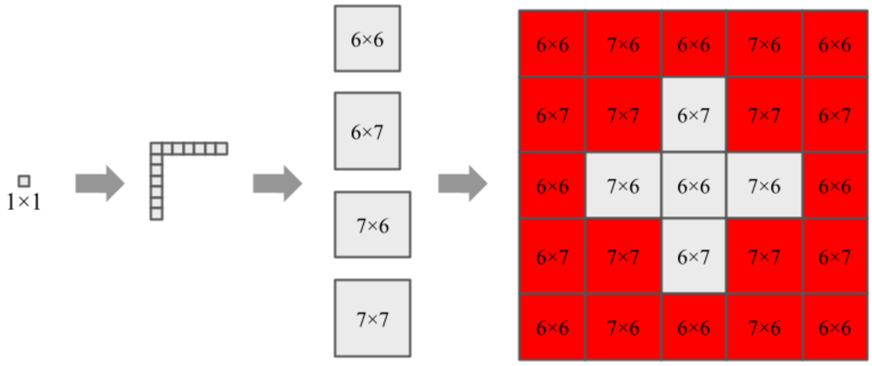
France. The aspect ratio is 3:2. There are three vertical stripes, so we just draw 6 identical squares(3×2) and color them.



Germany. The aspect ratio is 5:3. There are three horizontal stripes, so we just draw 15 identical squares(5×3) and color them.



Switzerland. The aspect ratio is 1:1. The construction sheet says that we need to divide the width and height of the flag in ratios of **6:7:6:7:6** to draw the Swiss cross. So we first create rectangles with dimensions 6×6, 6×7, 7×6 and 7×7. Then we arrange them as below.



It's easy to see which flags are constructible this way. All of them are listed below.

We cleared **45** out of the 197 flags rather quickly, which is about **22%** of the job done! Some flags like **Greece** or **Tonga** are trickier than others, but not by a huge margin. While we celebrate reaching this first milestone, let me set something up real quick.

Definition. A square with length 1 is called a **unit square**.

Definition. Suppose we have a unit square. A number x is ***constructible*** if, and only if, we can create a square with length x.

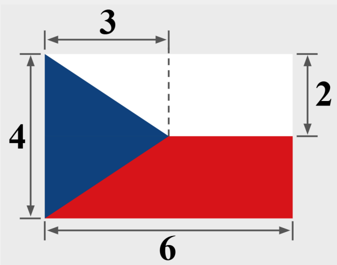
So for example, 10 is a ***constructible*** number.[7] From our theorems we can also say,

"All numbers in $\mathbb{N}$ are ***constructible***."

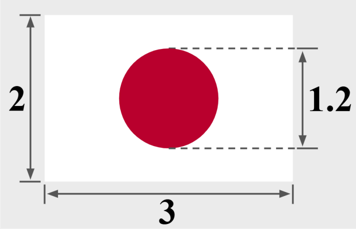
where $\mathbb{N}$ is the set of all natural numbers(non-negative integers).

Exercise 2.

2.1. Show that if we have a unit square and a square with length x ($x > 1$), then we can create a square with length $x - 1$. We will shorten this to the phrase "-1 is a ***constructible*** number." All negative integers are ***constructible*** by this definition.



2.2. Let's draw a flag with something other than squares! Draw the flag of **Czechia**. Which shapes did you use? Did you use features like `Rotate` or `Flip`?



2.3. Draw the flag of **Japan**. …*Uh oh!* For some reason, the construction sheet I've got contains decimal numbers. Can you still draw the flag?
(※ The circle is centered on the flag.)

<u>2.4.</u> That gives me an idea…
Given a square with length 1, can you draw a square that is **one fifth** as long as that square?

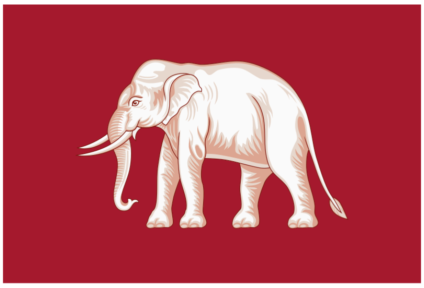
Note that exercise 2.4 is not trivial. You can draw a square **5 times as long** as a certain square with our previous method, but **not the other way around**. For exercise 2.4, you need to utilize other features in Google Slides, such as multi-select(`Group`) or `Distribute`.

While you try that, I need to talk about the elephant in the room.

[7] ***constructible*** is written in bold and italicized to distinguish it from the term 'constructible number' in algebra.

3. The Elephant in the Flag

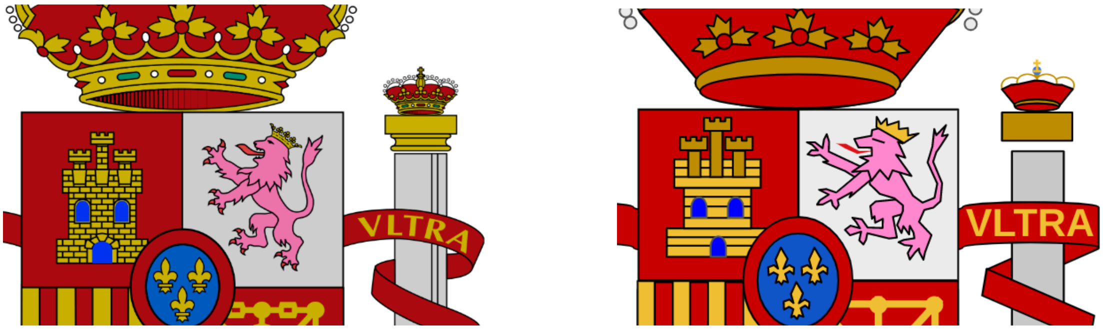
There is an obvious problem with our goal: **Emblems**.



We may be able to draw some simple shapes, but how would we ever draw the coat of arms in the flag of **Spain**, the dragon in the flag of **Bhutan**, or an actual elephant such as the one to the right?

That's the flag of 19th century Siam(now **Thailand**) by the way, unfortunately not in our flag list. Anyway, how would we ever draw *an actual drawing* in Google Slides?

Well, I have an idea. An emblem simply has to "resemble" the official one to count as correct, if the official specification **never mentions its exact dimensions**. For example, the Spanish coat of arms is not defined by math, but by descriptions like "a standing purple lion in the top-right quadrant," which leads to many valid interpretations.



The official Spanish coat of arms(left) vs. my rendition in Google Slides(right)

I don't know which international committee on Google Slide flag designs will be judging us, but what's important is that whoever is judging, there will be **some margin of error allowed** — since there is no exact answer to begin with. Then, drawing such a flag becomes a trivial problem for a mathematician: We simply hire an artist good enough to satisfy that judge. No math required!

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*No matter how small the **allowed margin of error** (ε) is,*
*there exists a **level of artistry** (X) such that*
*all **artists** (x) on that level or higher can draw a **flag** ($f(x)$ — short for 'flag of x')*
*whose difference with the **original flag** (α) is below the margin of error.*

No matter what ε is($\varepsilon > 0$), there exists an $X > 0$ such that whenever $x \geq X$, we have $\lvert f(x) - \alpha \rvert < \varepsilon$.	$\lim_{x \to \infty} f(x) = \alpha$

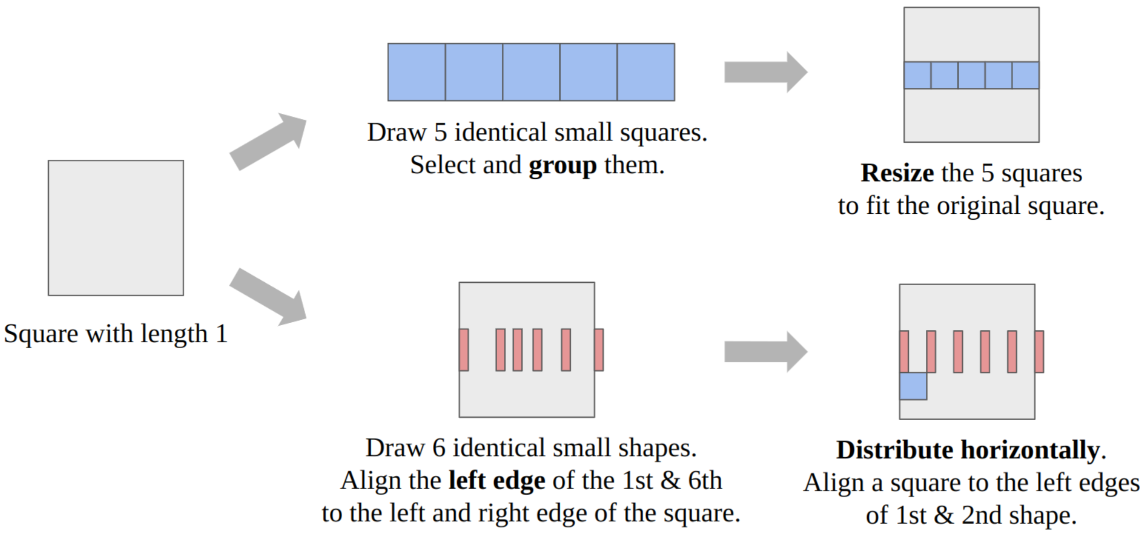
I call this 'the definition of an artist's limit.'

Anyways, by taking advantage of massive loopholes in heraldry(not that I feel good about it), we've cleared another big chunk of flags. Now, we can draw any flag consisting of rectangles **and** any mathematically ambiguous drawings!

That's **46** out of the remaining 152 flags, which is about **46%** of the job done! Some flags — ones with asterisks — *seem* to have mathematically describable elements but don't have enough specifications. Other than that, constructing the rectangular parts of these flags is pretty standard.

4. Breakthroughs

Here are two possible answers to exercise 2.4 — How to draw a one fifth-long square.



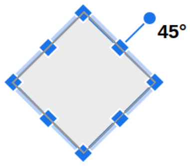
It should be easy to draw any square with length $\frac{1}{n}$ using this method, for all integers n.

Combined with the theorems from chapter 2, we can draw any square with length $\frac{m}{n}$ where m and n are integers. In other words, we can say "all numbers in $\mathbb{Q}$ are ***constructible***," where $\mathbb{Q}$ is the set of all rational numbers.[8]

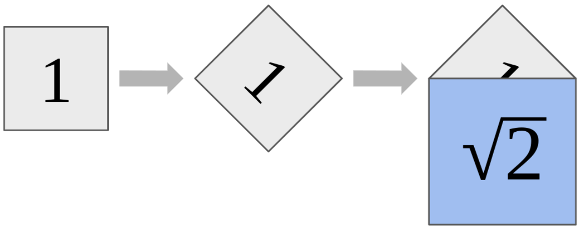
[8] 'Rational numbers' also include negative numbers like -0.5, but that's okay. Refer to exercise 2.1.

Now let's find out what other numbers are ***constructible***. For instance, what about $\sqrt{2}$?

The diagonal of a 1-by-1 square is $\sqrt{2}$. So let's try rotating the unit square. If you hold `shift` while rotating, the angle of the shape 'snaps' to multiples of **15 degrees**($\frac{\pi}{12}$ radians). Since we never input any number for this, our philosophy considers this a valid move.



After the 45° rotation, we can draw a new square and 'snap' its sides to the two corners.

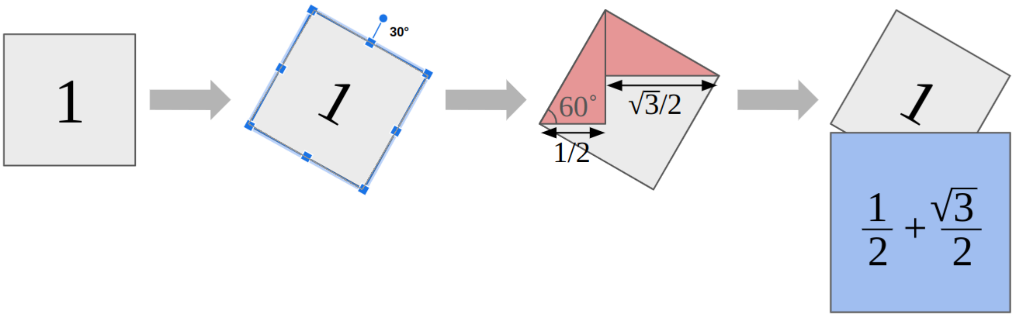


So, $\sqrt{2}$ joins the party! Now we can make a square with length $3\sqrt{2}$, or $\frac{\sqrt{2}}{5}$, or $100\sqrt{2} - 70$. We can even make $\frac{1}{\sqrt{2}}$ since that is just equal to $\frac{\sqrt{2}}{2}$. So, we can ***construct*** any number made by using addition, subtraction, multiplication and division on [rational numbers and $\sqrt{2}$].[9] In algebra, the set of all such numbers is usually denoted as $\mathbb{Q}(\sqrt{2})$. Thus:

"All numbers in $\mathbb{Q}(\sqrt{2})$ are ***constructible***."

We have **adjoined** $\sqrt{2}$ to $\mathbb{Q}$, in mathematic terms.

Next up is $\sqrt{3}$. This time we rotate the unit square by 30 degrees and use some trigonometry. (The red triangles are unnecessary in the actual construction.)



After obtaining $\frac{1}{2} + \frac{\sqrt{3}}{2}$, you can easily double the length and subtract 1 to obtain $\sqrt{3}$.

"The set of numbers made by adding, subtracting, multiplying and dividing [rational numbers, $\sqrt{2}$ and $\sqrt{3}$]" is denoted as $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ in algebra.

But hold your horses before saying "all numbers in $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ are ***constructible***," because there's a small problem. By its definition, $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ includes $\sqrt{2} \times \sqrt{3}$ too, which is $\sqrt{6}$.

[9] Except dividing by zero of course, since that's not possible.

So we first have to ask: can we make $\sqrt{6}$ in Google Slides?

There are a few ways to handle that question. The first one requires more trigonometry:

Exercise 4.1. Calculate $\sin 15^\circ$ and $\cos 15^\circ$ to prove that $\sqrt{6}$ is ***constructible***.

Somehow, turning the unit square by 15 degrees is the solution. But that feels a bit like a hack, doesn't it? Can't we just make $\sqrt{6}$ using the two squares with length $\sqrt{2}$ and $\sqrt{3}$?

That is indeed possible, using a method that generalizes to any two numbers.

Theorem 3. If a and b are ***constructible***, then ab is also ***constructible***.

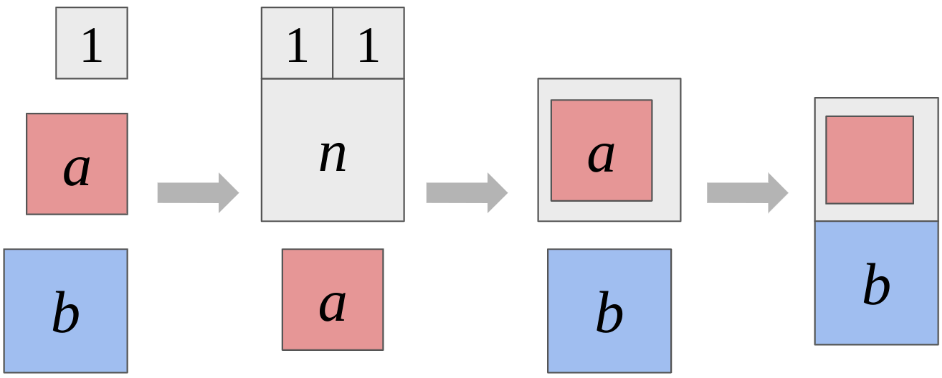
Proof. Suppose we had a unit square, and we created two squares with lengths a and b. Then we can do the following.

(a) Pick an integer larger than a. Let's call it n.

(b) Draw an n-by-n square and place the a-by-a square inside it.

(c) Select the n-by-n square and a-by-a square together and `Group` them.

(d) Resize them so that the total size becomes identical to the b-by-b square.



After (d), the white square(length n) becomes length b. So what does the red square become?

$$n : b = a : x$$

Solving this gives $x = \frac{ab}{n}$. So, we can copy this red square n times to get length ab. □

Exercise 4.2. Prove that if a is ***constructible***, then $\frac{1}{a}$ is also ***constructible***. ($a \neq 0$)

[*Tip*: First, think of the case when $a > 1$.]

This is a major breakthrough. We found a property that applies to everything: If a and b are ***constructible***, then $a + b$, $a - b$, ab and $\frac{a}{b}$ are all ***constructible***. The last — if $b \neq 0$ — can be done by making $\frac{1}{b}$ first, and then making $a \times \frac{1}{b}$.

In algebra, there's a fancy word for this:

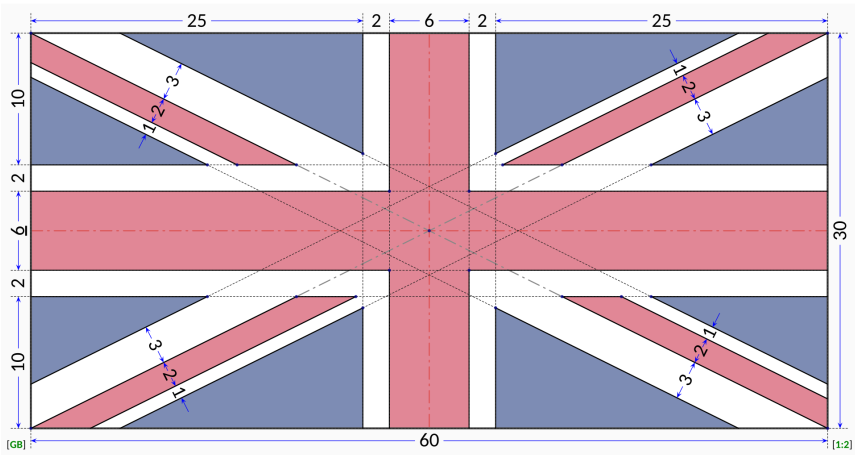
"The set of ***constructible*** numbers is a **field**."

We've attained this without even knowing what other irrational numbers are ***constructible***!

That's nice, but it's been too long since we drew a flag. Let's return to that.

5. Roadblocks

I'm beginning to realize I'm well above the word count that I'd originally envisioned, but it's too late to stop now. You can't stop either, because how could you stop without even drawing the flag of the **United Kingdom**?[10]

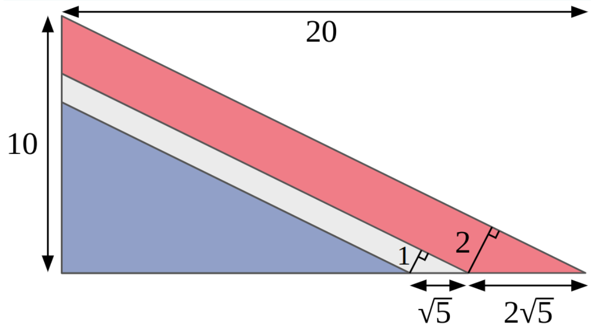


The only non-trivial part of this design is the diagonal stripes. The widths of the stripes are integers, so can't we just draw some rectangles and rotate them?

No, because the angle of rotation is not a multiple of 15 degrees. Instead, we'll need to use `Right Triangle`s and overlap them to make it look like stripes.

For example, the top left quadrant would be drawn like the figure to the right. Each triangle has a width-to-height ratio of 2:1, and the difference between the width of trangles are multiples of $\sqrt{5}$ by the Pythagorean theorem.

Conversely, if we manage to construct $\sqrt{5}$, we would be able to draw the entire flag.



[10] Construction sheet by user 'MapGrid' on Wikipedia. https://en.wikipedia.org/wiki/Flag_of_the_United_Kingdom#/media/File:Flag_of_the_United_Kingdom_(1-2)_(construction_sheet).svg

So, can we construct $\sqrt{5}$?

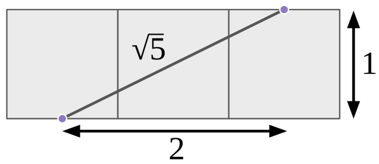
Exercise 5.

5.1. Euclid's first postulate states that we can always draw a line from any point to any point. Can Google Slides do the same? Draw some shapes like a `Rectangle`, an `Oval`, and a `5-Point Star`. Next, select the `Line` tool and hover over the shapes. Which points are **highlighted**? Do the endpoints of the `Line` 'snap'?

5.2. Draw a `Line` segment that is $\sqrt{5}$ units long, only using the **highlighted** points of some unit squares. Try the same thing with square roots of different numbers.

5.3. After exercise 5.2, is it possible to draw a `Rectangle` with a width of $\sqrt{5}$?

Exercise 5.3 is very important. If we have a `Line` of some length, can we draw a `Rectangle` whose width is equal to that? If that were possible, then we would be able to construct not just $\sqrt{5}$, but a square root of any integer.[11]



However, this is not trivial because once you complete exercise 5.2, you get a slanted line segment, and **there is no way to rotate the line** into a horizontal one.

I encourage you to try solving exercise 5.3 for at least 5 minutes. It seems like it **should** be doable, but it's not easy at all.
In fact, I couldn't find a way to construct $\sqrt{5}$ or **any** other square roots of [prime numbers greater than 5] with this method after **days** of trying! Here are some of the attempts that didn't work.

Exercise 5.4. Place a circle centered on one endpoint of the `Line` shown above. grab the corner of the circle, hold `ctrl` and `shift`, and drag. Does the arc of the circle 'snap' to the other end of the `Line`? If so, the circle's radius would be $\sqrt{5}$.

Exercise 5.5. Draw a `Line` as shown above. Rotate the line clockwise by 45 degrees. What is the difference between the x- or y-coordinates of the two ends?[12] If those numbers contain $\sqrt{5}$, we would've succeeded. What if we rotate by 30 degrees?

As far as my research goes, there is no foreseeable way to construct square roots of [prime numbers greater than 5] in google slides. Thus I leave this as an **open problem**:

Q. Which square roots of integers are ***constructible*** numbers?

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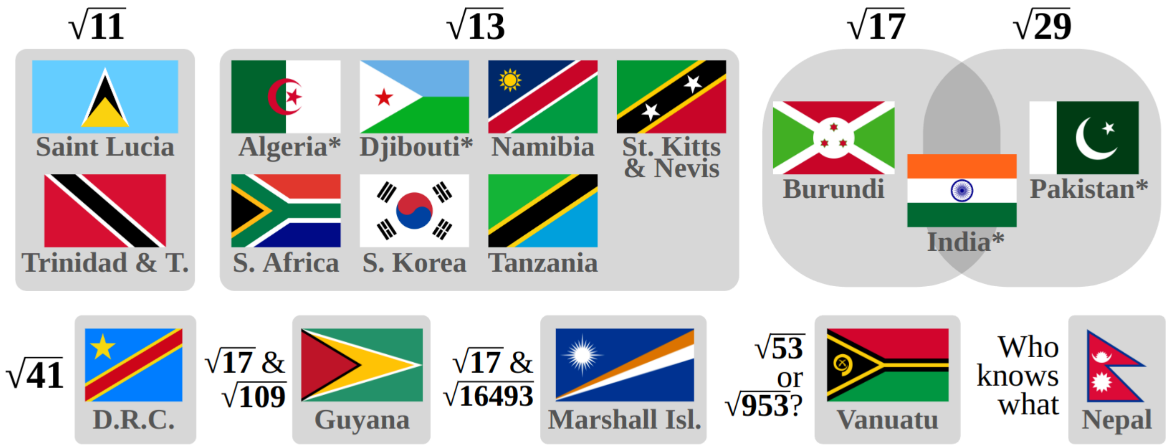
[11] Why? That is left as exercise.
[12] Some trigonometry will be helpful, such as the **sum and difference identities for sine and cosine**.

For now, I will assume — with great regret — that all square roots of primes greater than 5 are ***not constructible***. There are some flags that use such numbers in the construction, so the answer to the title of this essay is a **No** for now. At least, we can show which flags we cannot draw, and what square roots we need in order to draw them:



That's **17** flags we give up on drawing, so our goal of 197 total flags now becomes **180**. Some flags with asterisks have no visible slanted stripes, but there are square roots hidden in their construction — believe me, I tried hard to draw each of them.

On a brighter note, have you noticed that we never marked **Czechia** as a 'drawable' flag even though we drew it in exercise 2.2? That's because I first had to explain why some flags with triangles are doable and some aren't. But now that's done!

Here are all the flags with squares, triangles, and circles, with **no square roots** in the construction. (Notably, the crescent in the flag of **Maldives** is just a green circle partially covering a white circle.)

That's **20** flags, so the amount of flags constructed is 111 out of 180. We are **62%** done.

6. Reach for the Stars

I'm getting close to the 3,500-word mark and I should definitely stop. Even the premise of this essay is meaningless now because I had to give up on all those square roots!

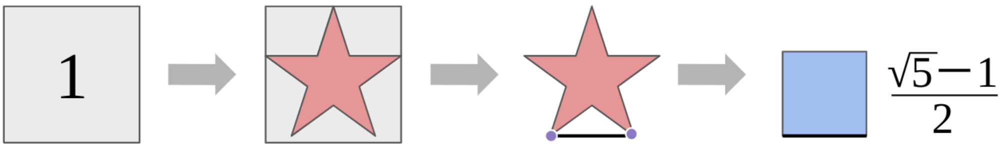
But you know what I haven't given up on? The glorious flag of the **United Kingdom**.[13]

[13] I'd like to inform that I'm not a citizen of the UK. In fact, I'm a citizen of South Korea, the flag of which I already had to give up on.

Theorem 4. $\sqrt{5}$ is a ***constructible*** number.

Proof. The ratio between a regular pentagon's side and its diagonal is 1 to the **golden ratio**, which is $\phi = \frac{\sqrt{5}+1}{2}$.

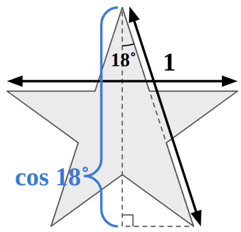
Suppose we have a unit square. Draw a `Regular Pentagon` or a `5-Point Star` with width 1. Then, the distance between the two bottom points is $\frac{1}{\phi} = \frac{\sqrt{5}-1}{2}$. Connect the two points with a `Line`, and draw a square with that length. Then, constructing $\sqrt{5}$ is trivial. □



Again, we have extended our field of view to new shapes and numbers. Now we say:

"All numbers in $\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$ are ***constructible***."[14]

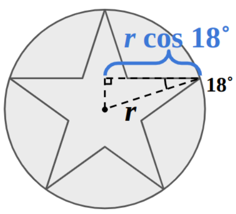
And thus, the **Union Jack** can be drawn. The actual construction is just a repetition of right triangles and rectangles, so I will not go through the whole process here. Not only that, we can even add more numbers like $\cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$ to our repertoire, which is the ratio between the width and height of a regular 5-point star.



But can we go further? Can we now draw every flag containing a 5-pointed star such as that of the **USA**? Indeed, and we just have to overcome one more step: **Inscribed stars**. In most construction sheets, we are instructed to draw [a star inscribed in a circle with a certain diameter], which is different from just specifying the width of the star.

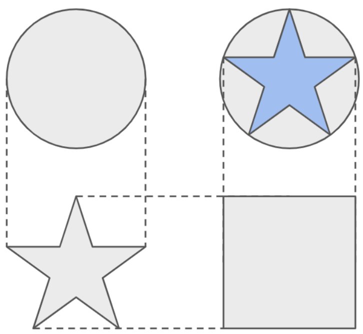
Thankfully, we can easily do this with the following theorem, which has a *very neat* proof.

Theorem 5. Given a circle, it is possible to draw an inscribed 5-point star.



Proof. Let r be the radius of the circle. Then the inscribed 5-point star must have a width of $2r\cos 18^\circ$, as shown to the left.

Recall that the ratio between the width and height of a 5-point star is also $1 : \cos 18^\circ$.



Thus we can draw an inscribed 5-point star with the (*very neat*) process shown to the right. The square in the bottom right has a length of $2r\cos 18^\circ$. □

[14] We have **extended** the **field** $\mathbb{Q}$ by **adjoining** the square roots of 2, 3 and 5.

So, along with flags containing $\sqrt{5}$, we can now draw **any flag with 5-point stars**, if the position and size of the star are ***constructible*** numbers!

That's **37** more flags, so we're at 148 out of 180 — **82%** done.

Exercise 6.

6.1. Perhaps the flag of your country can be drawn by now. If so, search up the construction sheet and draw it! (If you're from the USA, good luck.)

6.2. It is possible to construct the square root of any given number using a straightedge and a compass. That may mean Google Slides is an inferior tool in some ways, but Google Slides is also superior for one thing: **Heptagons**.

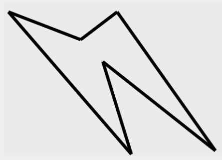
Try replicating the process of theorems 4 and 5 with a `7-Point Star`. You will find that $\cos\frac{90^\circ}{7}$ and $\sin\frac{90^\circ}{7}$ are ***constructible***. These are not constructible with a straightedge and a compass!

7. Endgame

The 32 remaining flags are quite special, in that drawing each of them is a head-scratching problem on their own. They were the most interesting ones to think about while writing this essay, and I encourage you to try tackling them yourself. Thus, this last chapter will not consist of specific constructions, but of challenging exercises regarding some ideas in those constructions.

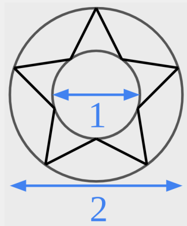
Exercise 7.

7.1. Suppose you are drawing a flag containing the polygon shown to the right, and you've succeeded in drawing that polygon with `Line`s. To complete the flag, the interior of this polygon should be colored blue, and the exterior white. Can you do this only using `Right Triangle`s? Note that the corners of `Right Triangle`s will snap to the endpoints of `Line`s.



7.2. A **simple polygon** is a polygon which does not intersect itself. Do you think you can apply exercise 7.1 to all simple polygons?

7.3. All stars in the flags of chapter 6 are **regular**, meaning that all 5 points have angles of 36°. However, some flags have stars defined with **two diameters** instead — those of its 'circumcircle' and 'incircle.' For example, can you draw the star to the right?



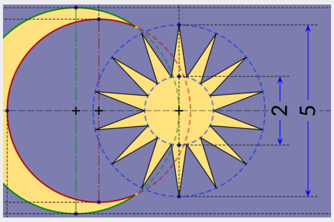
The intended solution for exercise 7.3 requires two applications of theorem 5, a rotation, ten `Line`s, and perhaps an application of exercise 7.1 — not trivial at all! Imagine showing that shape to someone who has only just read chapter 2. They'd think it's downright impossible.

However, now we have the power of **generalization**: Any shape can be drawn if all the points have ***constructible*** coordinates. In exercise 7.3, all 10 points of the star are in some way 'pentagonally related' with the two circles, so all coordinates of the points in this star are [some combinations of cos 18°, sin 18°, and other ***constructible*** numbers]. So we can firmly declare its possibility even before we find any actual way to do it! This power of our mathematical tool is probably the most important takeaway of this essay.

Below are the rest of the flags that I found to be constructible in Google Slides. The soyombo symbol in the flag of **Mongolia**, the 14-point star in the flag of **Malaysia**, the complex 8-ray sun in the flag of **Philippines**… I could talk about each of them for 30 pages, but alas, as the tradition goes in math, the most interesting parts are left as exercise for the reader.

That's **24** flags, bringing us to 172 out of 180, a whopping **96%**!

Exercise 7.4. (**Malaysia**) Prove that a 14-point star can be drawn if its two diameters (of its ‘circumcircle’ and ‘incircle’) are ***constructible***.



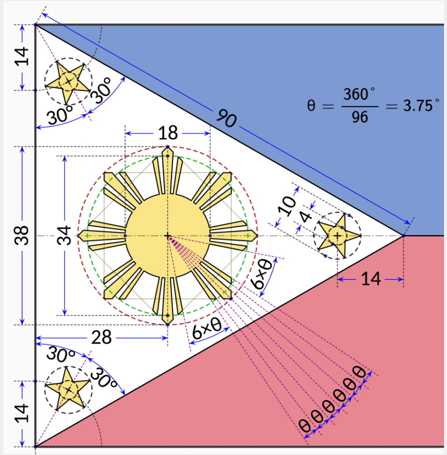
Exercise 7.5. (**Philippines**) Prove that:

(a) cos 22. 5° and sin 22. 5° are ***constructible***.
[*Hint*: Use some kind of star.]

(b) A 24-point star can be drawn if its two diameters are ***constructible***.
[*Hint*: You need cos & sin of (360°/48).]

(c) Given two `line`s that form an angle, you can draw a `line` **bisecting** that angle.

(d) If two ***constructible*** `line`s meet at one point, then the coordinates of that point are also ***constructible***. (A ***constructible*** `line` is a `line` whose endpoints have ***constructible*** coordinates.)[15] [16]



The 8 remaining flags are interesting too, each with its own reason. I’m not sure if some of them can be drawn or not, and there’s a chance you may spot a breakthrough that I couldn’t find. So take the following list with a grain of *saltire*.[17]


Angola
Needs cos 360°/32 & sin 360°/32


Kazakhstan
Possible if the drawings allow some margin of error


Antigua & B.
Needs cos 360°/32 & sin 360°/32


Kiribati
Needs 17-point star (possible in *s. & c.*)
Shape of waves is ambiguous


Georgia
Possible with `Merge Shapes` in Powerpoint


Kyrgyzstan
Possible with `Merge Shapes` in Powerpoint, and if the drawings allow some margin of error


Iran
Possible in *s. & c.* construction


New Zealand
Needs cos 8° & sin 8°

[15] Both construction sheets by user ‘MapGrid’ on Wikipedia.
https://en.wikipedia.org/wiki/Flag_of_Malaysia#/media/File:Flag_of_Malaysia_(construction_sheet).svg ,
https://en.wikipedia.org/wiki/Flag_of_the_Philippines#/media/File:Flag_of_the_Philippines_(construction_sheet).svg
[16] Some numbers in Philippines’ flag are approximations. I chose to think everything other than ‘34’ is exact.
[17] ‘*s. & c.*’ stands for straightedge-and-compass construction. For Angola and [Antigua and Barbuda], note that the `32-Point Star` does not have 32 ‘snappable’ points. For the construction of the national emblem of Iran, check https://ajammc.com/2019/02/11/iran-flag-unique-symbol-revolution/ .

Conclusion.

No, you cannot draw every flag in Powerpoint / Google Slides.

Out of 197 flags in a Sporcle list, 172 are definitely drawable, and a few are ambiguous. I would probably put **Georgia**, **Kazakhstan** and **Kyrgyzstan** in the 'drawable' category, making our total coverage $\frac{175}{197} = \mathbf{89\%}$. Also, we can conclude that the set of ***constructible*** numbers includes the following field:

$$\mathbb{Q}\left(\cos\frac{\pi}{840},\ \sin\frac{\pi}{840}\right)$$

This is because $\frac{\pi}{840} = \frac{3^\circ}{14} \approx 0.21^\circ$ is the smallest angle we can get by linearly combining 18°, $\frac{90^\circ}{7}$ and $\frac{360^\circ}{48}$, whose cosines and sines were proven to be ***constructible*** in exercises. Conversely, all numbers we have constructed until now are combinations of rational numbers, $\cos\frac{\pi}{840}$ and $\sin\frac{\pi}{840}$. However, these answers are susceptible of improvement, as some questions in the *prestigious* field of Google Slide vexillology are left unanswered.

What started out as a funny random question has become quite a journey through arithmetic and geometry. Rather than just trying out each flag one by one, we asked a general question of 'which numbers are ***constructible***?' and began thinking of the **properties** that the set of ***constructible*** numbers maintains. This let us more easily answer whether certain flags can be drawn or not.

This overall structure is quite similar to how a first chapter of an algebra textbook would go. A set of numbers, fields, groups and such are at first defined by how they behave — **what property applies to the operations on them**. This groundwork is what gives way to all kinds of proofs and theorems.

Some exercises in this essay were much harder than others. I think the most challenging and interesting problem for a math enthusiast would be the one hinted in exercise 7.2, for it is independent from the specificity of Google Slides. Of course, I also hope someone with a deeper understanding of Google Slides can answer our glaring open question of which square roots are constructible.

Further References

"Flags of the World," https://www.crwflags.com/fotw/flags/

— a website devoted to vexillology. Contains lots of valuable information along with decades-old quips of many enthusiasts.

Austria-Forum, "Konstruktionsmuster internationaler Flaggen," https://austria-forum.org/attach/AEIOU/Vexillologie/construction%20sheets%20A-Z.pdf

— construction sheet of 229 flags provided by Austria-Forum. I unfortunately don't know who compiled this database. Has a few inaccuracies but mostly very reliable.